MATHEMATICS

Paper 9709/11
Pure Mathematics 1

Key messages

The question paper contains a statement in the rubric on the front cover that ‘no marks will be given for unsupported answers from a calculator.’ This means that clear working must be shown to justify solutions, particularly in syllabus items such as quadratic equations and trigonometric equations. In the case of quadratic equations, for example, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for full marks to be awarded. It is also insufficient to quote only the formula: candidates need to show values substituted into it. When factorising, candidates should ensure that the factors always expand to give the coefficients of the quadratic equation.

Using a column vector to describe translations is the most effective approach, as it allows candidates to be clear and concise.

General comments

Some very good responses were seen but the paper proved very challenging for many candidates. In AS and A Level Mathematics papers the knowledge and use of basic algebraic and trigonometric methods is expected, as stated in the syllabus.

Comments on specific questions

Question 1

Successful candidates cleared the fraction to obtain a 3-term quadratic in sin θ which they solved in a variety of ways, factorising being the most common. Candidates who obtained a correct 3-term quadratic in sin θ were usually able to solve the equation to find $\sin\theta = -\frac{1}{2}$ or $\sin\theta = \frac{2}{3}$ and hence find a correct angle. Many candidates did not obtain the full set of solutions, –150°; –30°; 41.8°; 138.2°, often because they failed to recognise that the domain of the question was from –180° to 180°.

Question 2

(a) Successful candidates substituted $x = 4$ into $\frac{dy}{dx}$ to give $4(2 \times 4 - 5)^3 - 9 \times 4^{\frac{1}{2}}$, obtained $\frac{dy}{dx} = 90$ and hence found that the gradient of the normal = $-\frac{1}{90}$. Some candidates stated that $\frac{dy}{dx} = -\frac{11}{2}$ or integrated $\frac{dy}{dx}$ and were unable to find the correct answer.

(b) Most candidates recognised the need to integrate, with occasional algebraic slips at times. Candidates who expanded the brackets before integrating were generally unsuccessful in obtaining the correct equation. Candidates who integrated the expression usually equated their result to $-\frac{11}{2}$, substituted $x = 4$ and found the constant of integration. A few candidates found that the constant of integration = 2 but did not give the full equation as their final answer.

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Question 3

(a) This proved to be a difficult question for most candidates. Successful candidates stated that $ar^2 = 18$ and also $a + ar + ar^2 = 26$ or $a + ar + 18 = 26$ or $26 = \frac{a(1-r^3)}{1-r}$, obtained a 3-term quadratic or cubic, solved that equation to find $r = -\frac{3}{4}$ and hence $a = 32$, tenth term = −2.40 .

Most candidates stated $ar^2 = 18$, $26 = \frac{a(1-r^3)}{1-r}$ and many then obtained $8r^3 - 26r^2 + 18 = 0$. Successful candidates solved this cubic equation using their calculator. Candidates who attempted to solve the cubic equation by factorisation were generally unsuccessful.

(b) Candidates who found values for a and r in **3(a)** were generally able to substitute into $S_\infty = \frac{a}{1-r}$.

Question 4

Candidates subtracted either before or after integration, both methods being equally successful. Stronger candidates integrated and subtracted in either order then substituted the correct limits to obtain the correct answer. Other candidates made algebraic or sign errors in the integration and substitution. A small number of candidates found the area of the triangle using area of triangle = $\frac{1}{2} \times 15 \times 15 = 112.5$. Some candidates gave the final answer as a negative area which did not score the final mark and a few candidates used the wrong limits for the integration.

Question 5

(a) (i) Most candidates understood the process required and many candidates scored full marks. There were some sign errors and some candidates expanded $80(px)^2$ to $80px^2$.

(ii) As with **5(a)(i)**, most candidates understood the process required and many candidates scored full marks. There were some sign errors.

(b) Successful candidates used their answers to **5(a)** to multiply the relevant three pairs to form a 3-term quadratic in p and equated this to 93 to obtain $48 + 160p + 80p^2 = 93 \Rightarrow 5(4p + 9)(4p - 1) = 0 \Rightarrow p = -\frac{9}{4}$ and $p = \frac{1}{4}$.

Some candidates made sign errors and some candidates did not show clearly how they solved the quadratic.

Question 6

(a) This question was accessible to most candidates and many scored full marks. Successful candidates substituted $k = 2$, $p = 11$ to eliminate y and obtain $2x^2 - 2x(11x + 3) + 2 = 0$ or $\frac{2x^2+2}{2x} = 11x + 3$ and then simplified to a 3-term quadratic, which they then solved. Some candidates made sign errors or algebraic slips.

(b) Most candidates substituted $y = 4x + 3$ and reduced to $(2 - 4k)x^2 - 3kx + 2 = 0$ or equivalent, with occasional sign errors. Most candidates then used $b^2 - 4ac = 9k^2 - 4(2 - 4k) \times 2$ and simplified to obtain a 3-term quadratic which they solved. Some candidates solved their quadratic equation but did not express the answer as an inequality whilst other candidates made sign errors.

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Question 7

(a) Many candidates stated $\frac{dy}{dx} = 8x - \frac{18}{x^3}$ and some then substituted $x = 2$, to find $\frac{dy}{dx} = \frac{55}{4}$. Many candidates were unable to give a correct chain rule equation. Some candidates found that $\frac{dx}{dt} = \frac{4}{11}$ but did not then give the final answer as $\frac{-4}{11}$.

(b) Successful candidates stated $8x - \frac{18}{x^3} = 0$, solved this equation to obtain $x = \pm\sqrt{\frac{3}{2}}$ or $\pm\frac{\sqrt{6}}{2}$ and hence $y = 4$ at both points. These candidates then found $\frac{d^2y}{dx^2} = 8 + \frac{54}{x^4}$ and concluded that both points were a minimum. Some candidates did not simplify their answer to x from $\pm\sqrt[4]{\frac{9}{4}}$ to $\pm\sqrt{\frac{3}{2}}$ and some gave only one value of x.

Question 8

Most candidates found the centre of the circle and the coordinates of the point of intersection of the circle and the line x = –2 and it was common for candidates to score the first three marks. Strong candidates, who understood the need to find the point of intersection of the two tangents, often went on to provide near perfect solutions. Some used implicit differentiation and others simultaneous equations as alternative methods. A number of candidates would have benefitted from a sketch diagram to understand more clearly from where the ‘base’ and ‘height’ of the required triangle could be obtained.

Question 9

(a) This question proved to be challenging for most candidates. Successful candidates used $\frac{1}{2}r^2a = 8a \Rightarrow r = 4$ and then $\frac{1}{2}r^2 \sin\alpha = 4 \Rightarrow a = \frac{\pi}{6}$. Candidates who found the correct values of r and α invariably went on to find the correct exact area. Some candidates found the correct values of r only.

(b) This question proved to be challenging for almost all candidates. Successful candidates used the cosine rule on triangle ABC or split ABC into two right-angled triangle to find α. Candidates who found the correct value of α usually went on to find the correct area of the segment. Candidates who rounded early in the question could not obtain the answer to the accuracy required.

Question 10

(a) Most candidates scored some marks on this question with only the strongest candidates scoring full marks. The stretch was described correctly by many candidates. In general, candidates who used vector notation to describe the translation scored higher marks than those who used words to describe the translation. A common mistake was to state e.g. translation left 2 units, down 5 units rather than translation left 2 units horizontally, down 5 units vertically.

(b) Many candidates scored full marks on this question. Some candidates did not show the line $y = x$ some candidate’s $g^{-1}(x)$ was not a reflection of $g(x)$ in the line $y = x$.

(c) There were many fully answers correct to this part. Where mistakes occurred, they were most commonly due to incorrect handling of the square root, such as forgetting to square the left-hand side or forgetting to square the denominator.

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(d) Successful candidates stated that the range of g^{-1} is $g^{-1}(x) \geq -2$. Other candidates did not use the correct inequality or stated $x \geq -2$.

(e) Successful candidates obtained $g^{-1}h(4) = g^{-1}(2) = \frac{31}{9}$.

(f) This question proved to be challenging for most candidates and there were few correct answers. Successful candidates stated that hg^{-1} is impossible since the range of g^{-1} is $x \geq -2$ and is not within the domain of h which is $x \geq 0$, or an equivalent statement.

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MATHEMATICS

Paper 9709/12
Pure Mathematics 1

Key message

Candidates would benefit from more practice on basic algebraic techniques. 4 mistakes which were commonly seen were:

1. the use of the quadratic formula to solve quadratic equations with only two terms and then making a mistake with it
2. the error of thinking $(–a – b)^2 = – (a + b)^2$
3. the error of thinking $(–a – b)^2 = a^2 –2ab + b^2$
4. the error of thinking that $4.5a – 12 = –7.5a$.

General comments

The paper was generally found to be very accessible for most candidates. Many very good scripts were seen, and candidates generally seemed to have sufficient time to finish the paper. Presentation of work was mostly good, although some answers still seem to be written in pencil and then overwritten with ink. This practice produces a very unclear image when the script is scanned and makes it difficult to mark. Consequently, appropriate marks may not be awarded. Centres should strongly advise candidates not to do this.

Comments on specific questions

Question 1

Nearly all candidates scored some marks on this question but not many scored full marks. Candidates generally knew that a stretch of factor 2 parallel to the *y*-axis and a vertical translation were required but the size of the translation was the main difficulty. Most candidates performed the stretch first but then a translation of $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ -11 \end{pmatrix}$ were very common rather than the required $\begin{pmatrix} 0 \\ -14 \end{pmatrix}$. The correct terminology was generally used but the word down (or up, left or right) is not considered to be appropriately mathematical language and should not be used.

Question 2

This question proved to be generally straightforward with a good number of candidates able to score full marks. Weaker candidates sometimes put both expressions equal to 0 and then equated them, rather than finding an expression for one variable and substituting it into the other equation. Algebraic misunderstandings were common when attempting to simplify the resulting equation (see key message).

Question 3

Similarly, this question proved to be generally straightforward and again a good number of candidates were able to score full marks. Some candidates listed all of the terms until they obtained one with a power of *x* equal to 7 whereas others used an algebraic method to obtain the required value of *r* in $\binom{5}{r}$. Those candidates using the algebraic approach should be encouraged to check that the answer which they obtain gives the required power of x. A common mistake was finding *r* = 1 rather than 3 and then deciding that $(x^2)^4 = x^6$ in order to obtain the correct power. Other common errors were failing to square the *p* in $(px^2)^2$ or

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failing to cube the 4 or the *p* or both in $(\frac{4}{p}x)^3$. Some candidates misread $\frac{4}{p}x$ as $\frac{4}{px}$ and would perhaps benefit from more practice on expressions denoted in this way.

Question 4

Most candidates realised that **4(a)** was a rates of change question and were able to differentiate correctly. Some then simplified incorrectly, see key message, but the vast majority were able to substitute for *x* and apply the chain rule correctly. Weaker candidates sometimes simply substituted into the original equation. In **4(b)** the majority of candidates realised that $\frac{dy}{dx}$ was needed but this was sometimes set > 0 rather than = 0, possibly because the question referred to the curve having a minimum point rather than a stationary point. (This may also be the reason why some weaker candidates worked with the second differential rather than the first.) The substitution and required solution were usually done correctly.

Question 5

This question produced a mixed response with many candidates scoring full or nearly full marks but others missing it out completely or making a very poor attempt at it. In **5(a)** some candidates substituted 0 and 2π into the equation and obtained 7 for both answers. Some of these then stated that the greatest and least possible values were both 7. Candidates should always be encouraged to think about whether or not their answers are sensible. Others obtained decimal answers presumably because they were in the wrong mode on their calculator. One approach which seemed to work well for candidates was to consider the greatest and least values of cos2*x*, then 4 cos2*x* and finally 4cos2*x* + 3 in a step-by-step approach. Many fully correct graphs were seen but some were spoiled by them not levelling off at 0 and 2π. **5(c)** was found to be very challenging for many candidates with some attempting to solve the equation rather than considering the number of solutions and others just omitting it. The expected approach was to draw the line $y = 2x - 1$ and then consider the number of points of intersection. Those who did this were usually successful although some lines were inaccurately drawn and so an incorrect answer was obtained. Candidates would benefit from more practice on this type of question.

Question 6

Please note that due to a series-specific issue with **Question 6**, full marks have been awarded to all candidates for this question.

Question 7

7(a) was generally well done with most candidates working from the right-hand side of the identity. Replacing $\tan\theta$ with $\frac{\sin\theta}{\cos\theta}$ was generally the correct first step followed either by using a common denominator or multiplying the numerator and the denominator by $\cos^2\theta$, and finally replacing $\sin^2 J$ with $1-\cos^2\theta$. It is important with proof questions that all of the required steps are clearly seen. Candidates should note that treating the identity as an equation and cross-multiplying was not accepted as an appropriate method of proof. In **7(b)** the question states ‘Hence solve the equation’. Many candidates ignored this instruction and spent a good amount of time trying to solve the equation as it was given in the question. Candidates should be encouraged to read the questions carefully before they start, perhaps twice, to ensure that time is not wasted unnecessarily. Those candidates who used **7(a)** were usually able to simplify the resulting equation to a quadratic equation, although a mark was often lost for using the calculator function rather than factorisation or other accepted method of solution. Candidates were then usually able to find one correct answer from $\tan^{-1}3$ on their calculator, but when $\tan^{-1}-1.25$ gave an answer outside of the required range they often incorrectly thought that that meant that there were no solutions.

Question 8

In **8(a)** successful candidates usually substituted $y = -2$ into the circle equation and solved the resulting quadratic. The question asked for the co-ordinates of the points so it was expected that the final answers were given as $(2,-2)$, $(12,-2)$, rather than simply stating $x = 2,12$. The centre was found by completing the square of the original equation. In **8(b)** candidates would have benefited from drawing a diagram using the 3 points found in **8(a)** or at least using the given diagram. Successful candidates usually split the triangle in

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half to form a right-angled triangle or used the cosine rule. Common errors included finding the wrong angle or misquoting the cosine rule or using it with the incorrect lengths. Many candidates unnecessarily used the formula for the distance between the points A and B, and made a mistake with it, rather than considering that they were on the same horizontal line. Again, in **8(c)** candidates would have benefitted from using a diagram. Most candidates who were successful found the area of the large sector and then added on the area of the triangle. Common errors included forgetting to add on the triangle or using $\pi -$ rather than $2\pi -$ for the angle in the sector.

Question 9

9(a) was generally well done by candidates but as they worked through the question an increasing number missed out the subsequent parts, especially **9(d)**. A significant number of candidates ignored or mis-read the minus sign in the question. Candidates would benefit from working on old exam question papers in order to become more familiar with how functions are presented. Some weaker candidates differentiated in **9(a)** but the vast majority knew that integration was required. A significant number either did not consider the constant of integration or used 19 rather than 0 in order to find it. In **9(b)** candidates generally knew that $\frac{dy}{dx}$ needed to be set $= 0$ and those who had found the constant of integration correctly in **9(a)** were then able to solve it and determine its nature. Those who had forgotten about the constant of integration or had used 19 in order to find it, were left with an unsolvable equation as x^2 would be negative. Some realised that this would be impossible, but did not seem to then consider if this was due to **9(a)** being incorrect. Candidates should be encouraged to check their earlier work if this type of situation occurs. Because they could not solve **9(b)** some candidates unnecessarily missed out the rest of the question. In **9(c)** many candidates were able to integrate correctly and use the given point to find the equation of the curve, although weaker candidates sometimes attempted to use the equation of a straight line. Again, those candidates who had made a mistake in **9(a)**, were then left with an unsolvable equation when they equated $\frac{dy}{dx}$ to $-\frac{9}{4}$ in **9(d)**. Other common errors were using the given point $(2,19)$ in the equation of the normal or using the given value of $\frac{dy}{dx}$ as the gradient.

Question 10

This question was generally very well done, especially **10(a)**. In **10(a)(i)** candidates usually successfully formed an equation by equating the difference between the second and first term with the difference between the third and the second. This equation was usually simplified to $2k^2 - 12k = 0$ but some candidates then unnecessarily used the quadratic formula to attempt to solve it and made an error. In **10(a)(ii)** candidates were usually able to use the formula for the sum of an arithmetic progression in order to find the required answer. In **10(b)** most candidates were able to write down the two required equations: $ar^3 = 36$ and $ar^5 = 6$ and attempt to solve them, although some wrote $\frac{x}{36} = \frac{6}{x}$ but were then confused about what the *x* was that they had found. Candidates were familiar with the sum to infinity formula but were very often unable to write the answer exactly, or in the required form.

Question 11

Candidates were very familiar with the required technique in **11(a)**. Some though, did not think about the potential link between the different parts of the question, and did not use the completed square form in **11(b)**. Candidates should be encouraged to think about potential links between different parts of a question and to ask themselves, why was I asked to express **11(a)** in this form? Those who did use the completed square form in **11(b)(i)** were usually able to obtain $y = \pm\sqrt{x-2} - 2$, but many either chose the positive value rather than the negative or left the $\pm$ in their answer. **11(b)(ii)** proved to be one of the most challenging parts of the paper with few fully correct answers seen. Most candidates were able to find $gf(x)$ although some incorrectly found $fg(x)$ or $g^{-1}f^{-1}(x)$ instead. Those who used the completed square form were able to make some progress towards the inverse function and obtain $y + 2 = -(x+2)^2$, but many then incorrectly stated that $-\sqrt{y+2} = x + 2$. Those who carried out the correct steps obtained $y = \pm\sqrt{-x-2} - 2$, but again many chose the positive rather than the negative value.

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MATHEMATICS

Paper 9709/13
Pure Mathematics 1

Key messages

Candidates' inability to perform basic algebraic manipulation was responsible for them losing a considerable number of marks. Centres should provide more practice in dealing with algebraic solutions rather than numerical ones that can be evaluated on calculators. Many candidates did not understand the need to preserve extra places of decimal in their working in order to obtain answers of the required accuracy.

General comments

Many very good scripts were seen, and candidates had sufficient time to finish the paper. Presentation of work was mostly good, although some answers still seem to be written in pencil and then overwritten with ink. This practice produces a very unclear image which is difficult to mark on screen. Centres should strongly advise candidates not to do this.

Comments on specific questions

Question 1

The majority of candidates found this question straightforward. Common errors included $(-2)^3 = 8$ when substituting into their expression for $\frac{dy}{dx}$. A significant number of candidates found the negative reciprocal of their gradient value and used that to find the equation of a straight line.

Question 2

This question was answered correctly by most candidates. Finding the common ratio and using it in the formula for the sum to infinity was generally completed successfully. Some candidates made algebraic errors in simplification but most arrived at a correct quadratic equation. The most common error was failing to correctly identify the one solution that satisfied the requirements of the question.

Question 3

Candidates generally completed the process of integration correctly, failing to divide by 4 being the most common error. After having correctly substituted limits, a significant minority of candidates were unable to progress accurately.

Question 4

(a) All but the very weakest candidates were awarded full marks for this part of the question.

(b) It appeared that many of the candidates had no experience of tackling this type of question and consequently made no attempt at it. Equating 1.985^5 to their response to **4(a)** was a common wrong approach.

Question 5

Most candidates were able to obtain the correct quadratic equation by using the appropriate trigonometric identities and this got them the first three marks. A significant number of candidates then failed to show a correct method of solving their equation to produce decimal answers to a sufficient level of accuracy.

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Candidates who arrived at the two correct answers between 0 and 180 degrees often failed to find the other two answers correctly.

Question 6

Candidates found an equation in a and N by using the expression for the N^{th} term successfully. However, when using the S_n formula many candidates failed to replace n with 3N and consequently were not able to obtain the three accuracy marks that were available. Many otherwise successful responses failed to disregard the negative solution for N.

Question 7

(a) It was not apparent that all candidates realised that here $\frac{dy}{dx}$ needed to be negative.

(b) Most candidates were able to integrate the expression for $\frac{dy}{dx}$ successfully but, having arrived at a cubic equation, were not able to identify the lower of their values from **7(a)** as the *x* co-ordinate of the maximum point from their knowledge of the shape of the curve.

Question 8

(a) Failure to use decimal values to 5 places in order to arrive at an answer correct to 4 places often resulted in candidates getting few marks.

(b) This was generally well answered. A correct method was used for finding the required arc length by the majority of candidates although some used an incorrect value for *r*.

(c) This was generally well answered. Candidates either subtracted the sector and two triangles from the square or they worked out the area of the segment and subtracted it from the isosceles right-angled triangle. The most common error was using an incorrect value for *r*.

Question 9

(a) Candidates who used the product of the gradients of *PR* and *RQ* being -1 were generally successful. Candidates who chose the equally valid method of using Pythagoras were less successful because of algebraic errors, statements such as $(k - 5)^2 = k^2 + 25$ were common.

(b) Recognition that, because of the right angle, PQ is a diameter of the circle and, therefore, its midpoint is the centre of the circle made this part of the question straightforward. Candidates who did not realise this either made little progress or tried to find the centre by another method – unfortunately the extra algebra involved often led to errors. A significant minority of candidates made no attempt at this part of the question.

Question 10

(a) All but the very weakest candidates recognised the need for differentiation, omission of the ×2 was the most common error. Although most candidates dealt successfully with the negative power to arrive at a quadratic, it was disappointing to see so many who arrived at $(2x-5)^2 = 9$ then go on to expand the brackets rather than take the square root of both sides.

(b) Once again, the omission of ×2 was the main reason for incorrect differentiation. Candidates generally knew to use the *x* coordinates from **10(a)** in order to evaluate $\frac{d^2y}{dx^2}$ and hence establish the nature of the turning points.

(c) (i) This proved to be the least well answered question on the paper. Most candidates found the new coordinates of the original maximum point.

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(ii) Full marks for this part were most often awarded to candidates who clearly showed their working for the translation before then completing the reflection. Unfortunately, a significant minority of candidates carried out the translation and then forgot the reflection. Attempts to carry out both transformations at the same time were often unsuccessful. This part of this question was the most frequently omitted on the paper.

Question 11

(a) At least three different approaches were successfully used by candidates. Candidates, having found the correct values of *a*, lost credit for giving their final answer as a range of values.

(b) Most candidates started correctly, generally by finding $g(x)$ from its inverse. The more successful candidates were the ones that continued by evaluating $g(0)$ and then $gg(0)$ and then equating $fgg(0)$ to 96. Other more algebraic approaches were, once more, hampered by inaccurate application of algebraic processes.

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MATHEMATICS

Paper 9709/15
Pure Mathematics 1

Key messages

Centres are reminded that candidates need to show full working to justify their answers. This is essential in questions such as **10(b)** where the final result is given. Often, in these cases, working leading to the final answer is omitted and method marks lost. A key strength in high scoring scripts is thorough working, substitutions and formulae shown clearly and the solutions to equations shown fully. Where there is evidence a calculator equation solver or calculator definite integration function has been used full marks will not be awarded.

Candidates should be encouraged to sketch functions, graphs and coordinate grids where these will be helpful in problem solving

Comments on specific questions

Question 1

Many candidates approached this question with confidence, demonstrating a secure understanding of fundamental integration techniques. Successful responses were well-structured with most candidates integrating term by term rather than expanding the first term and combining it with the second term prior to integration. A few answers showed uncertainty in dealing with the constants when integrating the first term but they were in a minority. The use of the given point to find the constant of integration was rarely omitted or incorrect and most final answers were correctly presented as an equation.

Question 2

This question was generally well-attempted, with many candidates demonstrating a sound understanding of the binomial expansion. Strong responses systematically identified and correctly evaluated only relevant terms from each expansion. Very few errors were seen in the evaluation of combinations. The most successful candidates clearly matched coefficients and presented their work in a logical and structured manner to obtain a quadratic equation in a. As ever it was expected that the solution of this equation would be presented in the candidates' answers. Those who failed to show a correct method of solution could not gain full credit for their answers.

Question 3

3(a) The technique of completing the square was well understood and applied correctly in many answers. Various correct and acceptable forms were seen. When marks were lost this was often due to sign errors when calculating the term outside the square. Those candidates who were unable to complete the square often used the quadratic formula correctly to find both solutions and gained one mark only. Those who simply stated the solutions gained no credit. Most answers reflected a good understanding of the requirement of the question to provide answers in exact form.

3(b) The link between the quadratic equation in **3(a)** and the successfully rearranged equation in **3(b)** was usually seen although a few candidates ignored their **3(a)** answers and restarted by solving the trigonometric equation. The two solutions were used effectively to find the acute angle but the obtuse angle was sometimes ignored as candidates either did not appreciate the given range or that the tangent of obtuse angles are negative. Full marks could not be awarded when and extra obtuse angle was found from the use of an incorrect relationship between $\tan\theta$ and $\tan(180-\theta)$.

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Question 4

Finding the volume of rotation of an area under a curve was well understood by most candidates and the calculation of the lower limit was nearly always successful. The squaring process for the curve equation caused some problems but the resulting integrals were usually found correctly. The best answers showed each step in the process clearly and showed the values reached after substitution of the limits. There was evidence that calculators had been used to complete the definite integration. Without algebraic solutions these gained no credit.

Question 5

5(a) Very many completely correct answers were seen with working mainly in radians but some very good answers also used degrees. The appropriate arc length formula, Pythagoras' theorem and simple trigonometry were all used effectively to find the required lengths. The use of appropriate accuracy within the working to obtain three significant figure accuracy in the final answer was appreciated by most of the cohort.

5(b) Again, very many completely correct answers were seen to this part. The formulae for the area of a sector and the area of a triangle together with the required subtraction being effectively used. Some chose the extended route to the solution by dividing the required area into a segment and triangle. This produced more opportunities for errors but was usually successful.

Question 6

6(a) A straightforward question requiring only the calculation of the eleventh terms of an arithmetic and a geometric progression. The use of the required formula was nearly always seen although some chose to find all the terms up to the eleventh in both series. Use of formulae allowed quick calculations and resulted in fewer errors. Some, however, lost these marks because they mistakenly calculated the sum of the first eleven terms for each series.

6(b) This question was answered correctly by those who interpreted it correctly. Finding the sum of the first eighteen terms for two series whose parameters had been found in **6(a)** caused few problems. The use of the correct formula with $n = 18$ was seen in most answers. When a question is as straightforward as this, candidates should ensure they are using the correct value of *n* and not assume it is unchanged from **6(a)**

Question 7

7(a) This part was completed correctly more often than any other part. The most popular approach was to use the coordinates of A and B to form an equation with the given gradient. Some candidates chose to form two straight line equations and solve these simultaneously. This was equally effective but did provide more opportunities for sign errors.

7(b) Only half the candidates who completed **7(a)** correctly were able to complete this part. The simplest route to the solution using vector $\overrightarrow{AD} = \overrightarrow{BC}$ with the coordinates of D found from the value of p from **7(a)** was only seen occasionally. The use of a simple diagram featured in the better answers and enabled the correct orientation of the points to be seen. Some correct answers were seen from the solution of the equations of lines BC and DC but errors were often seen in the formulation of the equations. Without a diagram the incorrect assumption $\overrightarrow{AD} = \overrightarrow{CB}$ was often used.

7(c) Finding the perpendicular bisector of AB caused few problems and many correct answers were seen where this was used correctly to find the sides of the triangle. Although calculation gave a negative y coordinate this was usually interpreted as a positive length and very few negative areas were seen.

Question 8

8(a) The requirement to find the gradient of the curve was appreciated by nearly all candidates as was the need to form a second equation in a and b using the curve equation and the given point. Setting the gradient to zero, substituting the given point and solving the simultaneous equations was routine for most candidates and many correct answers were seen.

8(b) This part was equally well answered with nearly all candidates realising their gradient equation should be set to zero with their values of a and b and solved. As previously stated, method marks can only be

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awarded when a full solution is seen and those candidates who went from the quadratic equation to it's roots without showing a clear method could not gain the method mark.

8(c) The use of the chain rule to link the rates of change of x and y and the gradient was well understood by most of the cohort. The gradient from **8(a)** was mostly used correctly with the given x value, and the given rate of change of the y coordinate was interpreted as $\frac{dy}{dt}$. A few answers found $\frac{dt}{dx}$ and quoted this as the required rate of change.

Question 9

9(a) Very well answered with a range of correct descriptions showing that the understanding of the link between transformations and the equations of curves was a strength. Many candidates gained at least three of the four available marks. The most common reason for losing a mark was showing the translation in the y direction before the stretch. The descriptions of the transformations were generally clear. Examiners were able to award marks for phonetically correct spelling but descriptions such as translocation, transmute, stench and stich for translation and stretch could not be accepted.

9(b) Examiners were looking for sketches with a reasonably accurate scale on the y-axis, graphs with a clear tendency towards a gradient of zero at maximum and minimum points and the correct domains and ranges for the given functions. The best sketches showed these features but many used zero to 2 pi as the domain of both functions and many did not show gradients of zero at turning points. The use of the range of g(x) as -1 to 4 was nearly always seen and ensured most candidates gained some credit in this part.

9(c) Most answers showed attempts to change the subject of the formula for g(x) to find its inverse and then went on to substitute $f\left(\frac{\pi}{3}\right)$ to reach a final answer. Some answers reflected a deeper understanding of functions and used the solution of $g(x) = f\left(\frac{\pi}{3}\right)$ to find the required answer. In the first case it was not always clear that the candidate had used $f\left(\frac{\pi}{3}\right) = \frac{1}{2}$ and marks were unnecessarily lost. Another example of the need to show all steps in the working clearly.

9(d) The depth of knowledge required to answer this part was seldom evident in candidate responses. The best answers stated that the range of g exceeded the domain of f or showed this by means of an example. Some answers confused range and domain. The incorrect answer 'fg is not a one-to-one function' was often seen.

Question 10

10(a) Most candidates successfully identified the centre of the circle and demonstrated good understanding in finding the gradient of the tangent using the centre and the given point.. High-scoring responses showed fluent use of the point and gradient form of a straight line, derived either through geometric reasoning or differentiation. A few candidates knew the quick method of substituting the given point into the completed square form of the circle equation to obtain $(x+2)(2+2)+(y-4)(8-4)=32$ which simplifies to the required tangent equation. While the majority completed this part accurately, a small number of candidates omitted the final answer in the required format. It is advisable that candidates read each question carefully and highlight or underline specific instructions regarding the form of the answer.

10(b) Many candidates began well by correctly substituting the equation of the line into the circle's equation. Those who maintained a clear algebraic structure and avoided sign errors were able to manipulate the resulting expression to derive and simplify the required inequality. Candidates who used the circle equation given in **10(a)**, rather than the completed square form, often found the algebra more manageable, as it led to a more streamlined and effective solution process. The requirement for a line to be outside the circle was well-understood. The most straightforward method of setting the distance to the line from the centre greater than the radius was rarely seen. A clear logical flow in working and thoughtful organisation of steps were common features of full-mark responses This type of question encourages candidates to maintain accuracy in expansion and to check for algebraic consistency.

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MATHEMATICS

Paper 9709/21
Pure Mathematics 2

Key messages

Candidates should ensure that they read each question carefully and give the answer in the required form or show sufficient steps in the solution of a problem that requires a candidate to show a given result. Candidates should also be aware of the importance of the word 'hence' and also consider that previous parts in some questions may still be used in subsequent parts without the use of the word 'hence' for example in **Questions 7 (b)** and **7(c)**.

General comments

Many candidates had prepared well and were able to show a good understanding of the syllabus requirements. Some candidates needed further study of the Numerical Methods part of the syllabus, since they did not attempt **Question 3(c)**. Most candidates appeared to have sufficient space for their solutions and there did not appear to be any timing issues.

Comments on specific questions

Question 1

Most candidates attempted to differentiate using the product rule and were given credit for this. However, most candidates did not attempt to use the chain rule when differentiating $\cos\left(x^2+1\right)$.

Question 2

(a) Many candidates were able to obtain the value -2.16 by correct application of logarithms to both sides of the given inequality and subsequent calculator use, or by using their calculator to evaluate $\log_4 0.05$. Most were able then to give the correct result of $x < -2.16$. Most errors resulted from taking the logarithm of 4^x only, or by writing 0.05 subsequently as 0.5.

(b) Many correct solutions were seen with candidates using either two linear equations or a squaring method to obtain the critical values. There was no overall preference for a particular method.

(c) Very few correct solutions were seen with many candidates just quoting their result or part of their result for **2(b)**. Candidates should have taken note of the use of the word 'integers' and considered its definition.

Question 3

(a) Few completely correct sketches were seen. Some candidates were able to sketch the graph of $y = 3e^{-2x}$ correctly but very few were aware of the shape of the graph of $y = \sec x$ and the fact that the curve tended towards an asymptote at $x = \frac{\pi}{2}$.

(b) Many candidates equated $3e^{-2x}$ and $\sec x$ but were unable to manipulate the terms correctly to obtain the given result. Some candidates were unable to deal correctly with the term of 3 arriving at the given result incorrectly.

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(c) It was essential that candidates have their calculator in the correct mode, the range of x being given in radians at the beginning of the question. Of the candidates that produced correct iterations, some stopped an iteration too soon. It appeared that having seen that the value on their calculator rounded to 0.487 as the previous value did, they omitted to write it down.

Question 4

(a) Most candidates were able to differentiate correctly and equate the result to zero. However many candidates were then unable to simplify the exponential terms correctly to obtain $e^x = 4$ and subsequently $x = \ln 4$.

(b) It was essential that candidates find the upper limit of the integral needed to find the area, but many did not do this and often used the answer to **4(a)** instead. Many candidates made a reasonable attempt at integration, but few correct answers were seen due to an incorrect limit being used, or incorrect simplification.

Question 5

(a) Many candidates were able to produce a correct solution using both the remainder theorem and the factor theorem correctly. Errors usually came from arithmetic slips in simplification or the solution of the resulting simultaneous equations or from not making the correct use of the remainder of -15.

(b) Provided a correct **5(a)** had been achieved, most candidates were able to obtain a correct quadratic factor and hence the correct linear factors for $p(x)$. Candidates using incorrect values from **5(a)** were able to gain a method mark for using an appropriate method to find a quadratic factor provided the method went as far as obtaining the term in *x*.

(c) In spite of the use of the word ‘Hence’ at the start of the question, some candidates treated the question as a new question and did not attempt to make use of the work done in the previous parts. Candidates gained a method mark if they were able to obtain a valid equation in $\sin\theta$ using linear factors from **5(b)**. Of the candidates who obtained a correct equation of $\sin\theta = -\frac{3}{4}$, many obtained $-48.6°$ but fewer obtained the correct solution in the given range.

Question 6

(a) Many candidates were careless with the brackets in the numerator when differentiating *x* with respect to *t*, consequently obtaining an incorrect $\frac{dx}{dt}$. Most made an attempt at $\frac{dy}{dt}$, but many did not obtain the correct numerical numerator of 6, often obtaining 2 instead. Most candidates were able to show how $\frac{dy}{dx}$ was obtained but there were fewer fully correct solutions.

(b) Candidates first needed to use the given y coordinate to find the value of the parameter at the given point. Few candidates obtained a correct value of 2 for the parameter t and thus few correct solutions were seen.

(c) Many candidates stated that the curve represented an increasing function, giving a correct reason.

Question 7

(a) There were many successful proofs by a variety of methods. The method shown on the mark scheme was the most compact but writing the left-hand side of the expression fully in terms of $\cos x$ was the most common method. It was very often successful, but the increased number of steps made errors more likely.

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(b) Some candidates managed to produce one correct inequality usually from considering $4\cos^4\theta \geqslant 0$, and so there are no real solutions if $k < 5$. Few candidates considered $4\cos^4\theta \leqslant 1$, and by applying similar logic, obtained $k > 9$.

(c) Too many candidates did not relate the integrand to **part (a)** and write it as $\sqrt{4\cos^4\left(\frac{t}{2}\right)}$ or $2\cos^2\left(\frac{t}{2}\right)$. As a result, very few correct exact solutions were seen.

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MATHEMATICS

Paper 9709/22
Pure Mathematics 2

Key messages

Candidates should ensure that they read each question carefully and give the answer in the required form or show sufficient steps in the solution of a problem that requires a candidate to show a given result. Candidates should also be aware of the importance of the word 'Hence' and also consider that previous parts in some questions may still be used in subsequent parts without the use of the word 'Hence' for example in **Questions 5(c)**.

General comments

Many candidates had clearly prepared well for the examination and were able to attempt most questions showing a good understanding of the syllabus requirements. Most candidates appeared to have sufficient space for their solutions and there did not appear to be any timing issues.

Comments on specific questions

Question 1

Many candidates were able to obtain an integral in the form $k\ln(4x+1)$, where k was an integer. The fact that candidates were given the form of the answer required may have helped. Most were able to apply the limits correctly and then make use of the laws of logarithms.

Question 2

(a) Most candidates attempted a correctly positioned V shaped graph for the graph of $y = |2x-9|$. When sketching the straight line graph, it was essential to consider the comparative gradients of the modulus graph and the straight line graph. The straight line needed to be positioned correctly so that there was only one point of intersection with the modulus graph.

(b) Candidates made use of two linear inequalities or a squaring method to obtain 2 critical values. Many correct solutions obtaining the critical values were seen. The sketch in **2(a)** was meant to highlight the fact that there was only one point of intersection. Too many candidates included an incorrect solution of $x < -2$.

Question 3

In general, the quotient rule was made use of correctly and the resulting quadratic equation from equating the first derivative to zero was usually obtained. Success at obtaining both stationary points was mixed due to arithmetic errors. Candidates who started off by using the product rule did so correctly, but the additional algebraic steps often led to errors.

Question 4

(a) Most candidates correctly equated $4e^{-2x}$ and $1+0.5\sin 3x$. Dividing by 4 as the initial step usually led to a complete proof. Candidates who took the natural logarithm of both sides of the equation $4e^{-2x} = 1+0.5\sin 3x$ first, were usually unable to use the rules of logarithms successfully to complete the proof correctly.

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(b) Most candidates were able to use an iterative process. However, in this question, it was essential that candidates had their calculator in radian mode to complete the process successfully.

(c) It was pleasing to see that candidates were aware of the integration necessary to find the given shaded area and apply the process successfully. The most common errors were with the coefficient of $\cos 3x$, or its sign. An incorrect **4(b)** meant that a maximum of 3 marks was available. Candidates should be aware that when performing calculus, angles are in radians.

Question 5

(a) A seemingly popular question with candidates resulting in many fully correct solutions. Arithmetic errors were few. The most prevalent aspect was a failure to show $f\left(\frac{1}{2}\right)=0$ and $f(3)=0$ at the start of the question, although this was usually implied at some later stage.

(b) Most candidates appreciated the use of the word 'Hence' and made use of their values obtained in **5(a)**. Many candidates attempted to divide successively by the two given factors in the stem of the question and obtain the correct quadratic quotient of (x^2-5). Those candidates who formed a quadratic expression using the two given factors and then used this to find the quadratic quotient were equally successful.

(c) Many candidates recognised that there were two possibilities for $\text{p}(\cot 2\theta)=0$; $\cot 2\theta=\frac{1}{2}$ and $\cot 2\theta=3$. Successful solution of the equation $\tan 2\theta=\frac{1}{3}$ usually followed.

Question 6

(a) A substitution of $y=\text{e}^{-1}$ into the given equation was needed and most candidates did this and obtained a correct quadratic equation as a result. Showing that the discriminant was negative usually followed as a correct conclusion.

(b) It was essential that candidates recognise that implicit differentiation was needed. There were two possible methods of finding the gradient function. The first involved differentiation of the given equation taking into account a product or differentiation of $\ln y=\frac{14-6x}{x^2-3}$, taking into account a quotient. Many candidates were able to obtain the first 3 marks available but often errors in simplification and substitution meant that the equation of the tangent was incorrect or in the incorrect form.

Question 7

(a) Most of the candidates who attempted this question were able to use the identity for $\sin(\theta+30°)$ and subsequently obtain the expression $2\sqrt{3}\sin\theta\cos\theta+2\cos^2\theta$. Few candidates then used the appropriate double angle formulae to obtain $\sqrt{3}\sin 2\theta+\cos 2\theta+1$. The first two terms of this expression could then be used to write in the form $R\cos(2\theta-\alpha)$.

(b) Few candidates recognised the need to use $\cos(4\theta-\alpha)=k$. Those candidates who did, usually applied a correct method to find one angle but often failed to find or ignored a second angle. There were very few fully correct answers.

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Paper 9709/23
Pure Mathematics 2

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(b) Few candidates recognised the need to use $\cos(4\theta-\alpha)=k$. Those candidates who did, usually applied a correct method to find one angle but often failed to find or ignored a second angle. There were very few fully correct answers.

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MATHEMATICS

Paper 9709/25
Practical Test

There were too few candidates for a meaningful report to be produced.

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MATHEMATICS

Paper 9709/31
Pure Mathematics 3

Key messages

- In the exam, give your full attention to the questions asked.
- Practice the basic methods so that you recognise the standard patterns.
- Develop problem solving skills so that you are aware of the methods relevant to a given situation.
- Set your work out clearly so that the Examiner can follow what you are trying to do.

General comments

The mark profile for the candidates for this paper was very different from that for candidates in other time zones. There were some candidates who had clearly studied the syllabus and had prepared thoroughly for the examination. However, the median score of 10 marks reflects the large number of scripts where candidates demonstrated little understanding of the topics covered. The questions that commonly gained no response were **Question 6** – complex numbers in polar form, **Question 8** – vectors, **Question 9** – integration by parts and numerical methods and **Question 11b** – integration to obtain a volume.

In contrast to the large number of candidates who struggled to answer the questions, there were a minority who demonstrated understanding at a sophisticated level and demonstrated a high level of insight in their responses.

Comments on specific questions

Question 1

(a) The good candidates understood that the sketch should be a symmetrical V shape. Some drew this with no indication of the coordinates of the points of intersection on the axes. Some sketches were of the correct shape but with the vertex in the wrong place (and not always on the *x*-axis). Several sketches did not consist of a pair of straight line segments.

(b) The simplest way to do this was to consider where the straight line $y = 3x - 1$ intersects the straight line $y = 3 - 2x$. The majority of candidates who attempted a solution preferred to consider where $(3x-1)^2 = (2x-3)^2$. Having squared these terms, the final answer frequently included the false critical value $x = -2$.

Question 2

The responses were either concise and fully correct, or they involved false methods from the outset. The most common errors were to use incorrect statements such as $\ln(p-1) = \ln p - \ln 1$ and/or $\ln(q+1) = \ln q + \ln 1$. Many solutions missed the crucial step $3 = \ln(e^3)$.

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Question 3

Some candidates had a correct method for starting on this question, but the most common start was to consider $\frac{z+5i}{z-5}\times\frac{z+5}{z+5}$, which led nowhere. The candidates who attempted $\frac{x+\mathrm{i}y+5\mathrm{i}}{x+\mathrm{i}y-5}\times\frac{x-5-\mathrm{i}y}{x-5-\mathrm{i}y}$ made good progress with the multiplication but often got no further due to the complicated fraction and slips in the working. A much simpler approach was to consider $\frac{x+\mathrm{i}y+5\mathrm{i}}{x+\mathrm{i}y-5}=k,$ where k is real, multiply both sides by $x+\mathrm{i}y-5$ and compare the real and imaginary parts.

Question 4

Several candidates started with attempts to differentiate x and y with respect to t, and to use the results to obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Those who used the correct method to differentiate a function of a function often obtained a correct answer. If a candidate was able to deduce that $\tan t=1$ at the given point then they usually obtained a correct equation for the tangent.

Some of the stronger candidates chose to convert the parametric equations of the curve to a cartesian equation. If they obtained the form $y=3(\ln x)^2$ they usually completed the question correctly.

Question 5

Many candidates made hard work of this question. The simplest approach is to use the remainder theorem, equating $\mathrm{f}(-2a)=-22a^3$ and $\mathrm{f}\left(\frac{a}{3}\right)=-a^3.$ Although care needs to be taken with the algebra, the correct equations are simple to solve. The more common approach was to attempt to use algebraic division. Many candidates did not get as far as a constant remainder before attempting to use the given remainders. There were also many errors in applying the division process.

Question 6

This question produced a high proportion of blank responses, with a third of candidates not attempting any part of it.

(a) Candidates need to know the basic results for the modulus and argument of the product of two complex numbers. Several candidates were able to find the product of the moduli. Adding the arguments proved to be more difficult – some candidates were trying to add, but made errors.

(b) All candidates should have been able to score the first mark, which was for showing z_1 and z_2 on an Argand diagram. There were a few clear answers, but the points plotted were often not in correct positions relative to one another, for example, the argument of z_1 was often smaller than the argument of z_2.

(c) Answers were expected to include the correct terms, enlargement and rotation. The responses commonly lacked any correct terminology for the transformations.

Question 7

(a) The candidates who started with a correct expansion of $\sin\left(x+\frac{1}{6}\pi\right)$ often made good progress with this part of the question. The majority of candidates did not attempt the expansion and scored no marks.

(b) Candidates with an answer to **7(a)** were able to score two marks for the correct use of their answer to that part. Several candidates did demonstrate a correct understanding of how to solve this equation, but many started by stating $\sin(2\theta-\alpha)=\sin 2\theta-\sin\alpha$ and scored no marks.

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Question 8

(a) There were some correct answers to this part. Many candidates scored no marks because they did not understand the difference between the equation of the line passing through A and B and the vector $\overrightarrow{AB}$. Some candidates who understood the method did not score the accuracy mark because they did not use the correct format for the vector equation of a line: it is expected to start **r** = ….

(b) The way to show that two lines do not intersect is to look for the point of intersection and show that there are no values for the parameters that satisfy all three equations. Several candidates did attempt to do this. Many solutions failed due to slips in the basic arithmetic when adding and subtracting terms.

(c) The most obvious method to use here is a scalar product because the direction vector for l_2 is given and the direction vector of l_1 was found in **8(a)**. Some candidates did this very proficiently. Some candidates were confused between the directions of the lines and the position vectors of points on the lines.

Question 9

(a) Several candidates started correctly by attempting to use integration by parts to evaluate the integral. This was often completed correctly, but some candidates then stopped and did not attempt to substitute limits and equate their answer to 4. The rearrangement of the equation was sometimes muddled and often contained algebraic errors.

(b) This part of the question was independent of the first part; it could be done by using the given result from **9(a)**, although several candidates preferred to go back to using the result of their integration. The basic method of looking for a sign change when using $\mathrm{f}(x) = 0$ was widely understood.

(c) Most successful candidates started with an initial value between 2 and 2.1 and used the iterative formula correctly. It is a simple calculator task, but almost half of the candidates offered no response.

Question 10

(a) There were several correct answers to this part of the question. Some candidates used algebraic division, and some preferred the method of comparing coefficients.

(b) Only a minority of candidates made any progress with this part of the question. Many did not appear to recognise that they needed to start by separating the variables. If the variables were separated correctly then the integration, using the result of **10(a)**, was very straightforward. Without separation of the variables no progress was possible.

Question 11

(a) Several candidates recognised the function as a product of two terms. The challenge was to differentiate $\sqrt{\sin 2x}$. The common incorrect answers were $\sqrt{2\cos 2x}$ and $\frac{1}{2\sqrt{\sin 2x}}$. Without a derivative of the correct form, further marks were not available because the work was not equivalent to that expected.

(b) The majority of candidates offered no response to this question. Some candidates did not have a correct method for integrating to obtain a volume of revolution. Of those who attempted the correct method, only a minority had a correct method for finding $\int \cos^2 x \sin 2x \, dx$. The most common successful approaches were to convert the integral to $\int 2\cos^3 x \sin x \, dx$ or to $\int \frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x \, dx$.

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MATHEMATICS

Paper 9709/32
Pure Mathematics 3

Key messages

- Read the question carefully and make sure that you have found what you were asked to find. In this paper many candidates lost two marks in **Question 2(a)** because they found the term in x^2, but not the expansion 'up to and including the term in x^2'.
- When a question requests a particular method then marks are not available for an alternative method. This was particularly relevant in **Question 5**, which requested the formation of a quartic equation, and **Question 9(b)** which asked for the use of a scalar product.
- If a question starts 'Hence …' then this question requires you to use the outcome of a previous part. In **Question 7(b)** marks were available for the correct use of an incorrect answer from **7(a)**, but not for a fresh start using a completely different method. In **Question 9(c)** marks were available for the correct use of an incorrect answer from **9(b)** but only if this was based on using the angle *BAC*.
- Write clearly and ensure that numerals are clearly distinguishable from one another – in some instances candidates misread their own writing.
- When a formula is given on the formula booklet, no credit will be earned if the formula is misquoted. This applies particularly to using correct trigonometric formulae, and correct methods for differentiation and integration.

General comments

Many candidates made a strong start to this paper, finding the early questions familiar and accessible. Towards the end of the paper candidates found the tasks more challenging because they did not recognise basic forms; this then caused significant loss of marks because what they did was not equivalent to what was required. For example, in **Question 8**, many candidates did not recognise the form $\int \frac{\cos 2\theta}{\sin 2\theta} \mathrm{d}\theta$, and in **Question 11(a)** candidates did not have a correct method for finding $\frac{\mathrm{d}}{\mathrm{d}x}\left(\cos^2 x\right)$.

The stronger candidates used mathematical notation correctly and set their work out clearly. Poor notation often resulted in the loss of marks – candidates who omitted brackets or did not process them correctly could not earn subsequent accuracy marks. In **Question 4**, it was common for $-2\left(1-\tan^2 x\right)$ to become $-2-2\tan^2 x$. In **Question 11(a)**, the incorrect use of trigonometric terms created difficulties; what should have been $10\cos 2x \times \cos^2 x$ was written as $10\cos \times \cos^2$ and became $10\cos^3$ which is meaningless.

Comments on specific questions

Question 1

Those candidates who realised that this equation could be rewritten as a quadratic in e^x often obtained full marks. The most common errors were to claim that $4\left(e^x - 3\right) = 4e^x - 3$ and to give the final answer correct to 3 significant figures rather than correct to 3 decimal places.

Candidates who did not attempt to form a quadratic equation usually made inappropriate and incorrect attempts to use logarithms.

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Question 2

(a) The majority of candidates gained the first 2 marks for the correct unsimplified terms, although this was often followed by careless errors in the arithmetic. The most common error was to expand using powers of *x*, rather than powers of (–2*x*). A small number of candidates demonstrated a lack of understanding by trying to use the binomial theorem to expand $(6-x)^1$.

A significant minority of candidates, then went on to give just the term in x^2, rather than the full expansion up to and including the term in x^2.

(b) Although this is quite a common task, many candidates seemed to be completely unaware that there was a restricted set of values for which the expansion was valid.

Question 3

This proved to be an accessible question with very few candidates scoring no marks or giving no response. There were some excellent diagrams that were both neat and accurate. Some candidates had clearly used a ruler and pair of compasses. In contrast, many candidates did not take sufficient care to ensure that the key features on the diagram tied in with their scales on the axes. Indeed, some candidates did not give an indication of scale at all. The radius of 2 for the circle was often problematic. Whilst labelling of 1 and 5 on the imaginary axes helped, the horizontal direction was not always considered. Candidates who used different scales on the axes or indeed non-linear scales found it difficult to produce a correct diagram.

The majority or diagrams involved a circle of the correct size and with an attempt at the correct radius. It was common for it to be recognised that the point (1, 2) was involved, but some diagrams did not have the two required half-lines. Where the half-lines were drawn, candidates often did not consider key points that they should pass through, such as (0, 3) and (2, 3).

Sometimes the candidates went to the effort of making the circle and/or half-lines dotted rather than solid. Strict inequalities are represented by dotted lines; if 'equal to' is needed then solid lines should be used.

Question 4

There were two main approaches to this question. Those who used a formula for tan 2*x* to form an equation in tan *x* were generally the more successful. The most common error in this method involved an incorrect sign when multiplying out $-2\left(1-\tan^2 x\right)$. When combining two fractions into one, some candidates used a more complicated denominator than necessary, resulting in a cubic equation. Both of these groups of candidates were able to gain the second method mark for solving a quadratic equation, but an extra solution from the cubic equation resulted in the loss of the final mark.

A small minority of candidates were successful in forming and solving a quadratic equation in $\cot x$.

Some candidates chose to write the original equation in terms of sin x and cos x. They rarely progressed beyond obtaining a correct horizontal equation such as $2\sin^2 x-3\sin x\cos x+\cos^2 x=0$. Although this can be factorised and solved quite simply, the candidates did not recognise this.

The most common false start was to rewrite the equation as $\frac{1}{3}\tan x-\frac{1}{4}\tan 2x=\frac{1}{3}$.

Question 5

Many candidates knew what was required here, possibly because they had seen similar questions on past papers. There were some sign errors in the course of forming equations comparing the real and imaginary parts of the equation $(x+\mathrm{i}y)^2=-1-4\sqrt{5}\mathrm{i}$, including several arising from the error $\mathrm{i}^2=1$. Having obtained a correct quartic equation, there was widespread confusion about how to use the solutions, often because solutions to the quartic were treated as values of *x* or *y* rather than values of x^2 or y^2. This was not helped by unwise substitutions such as '$x=x^2$' when rewriting the quartic as a quadratic. The final answers were expected to be in the form *x* + i*y*.

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A small number of candidates did not follow the wording of the question and attempted to use a modulus and argument approach.

A significant minority of candidates got no further than copying the question or attempting to use $\left(-1-4\sqrt{5}\mathrm{i}\right)^2 = x + \mathrm{i}y$.

Question 6

(a) Many candidates did produce an acceptable pair of graphs, but then failed to gain the second mark by omitting to mention or even indicate where the root was to be found. In general, the quality of the sketches was not good. The modulus graph was usually the more successful of the two, but there was a lot of carelessness regarding the symmetry, and the points of intersection with the axes were often not marked or incorrect. Some candidates produced a sketch with a gentle rounding at, or even just above, the point (2,0). On the *x*-axis the positions of 2 and π were often inconsistent. For the trigonometric graph the maximum was often in the wrong position.

(b) There were many fully correct solutions. Some candidates tried to use $x-2 = 2\sin\frac{1}{2}x$ in place of $\left|x-2\right| = 2\sin\frac{1}{2}x$, and others avoided the problem by using $\left(x-2\right)^2 = 2\sin^2\frac{1}{2}x$. A small number of candidates with correct working did not go on to say how this demonstrated the presence of a root. There are still some candidates not working in radians.

(c) The majority of candidates scored full marks for this question. The most common errors involved working in degrees, or not giving values to the required degree of accuracy. A small number of candidates started at a value other than 1.03. Some candidates stated all the iterates required but never stated a conclusion about the value of the root. While it is a good idea to continue for one or two iterates after the point where the root seems to have been confirmed, some candidates continued well beyond what was expected.

Question 7

(a) The majority of candidates obtained the correct value for *R*. A few obtained a decimal approximation to *R* because they found *α* first and then used inexact working.

Many candidates stated $\tan\alpha = \dfrac{24}{7}$, which effectively meant that they had used the wrong identity. This usually arose from simply memorising a method rather than actually using a trigonometric identity.

More candidates than usual started with the incorrect statements $\cos\alpha = 24$ and $\sin\alpha = 7$, resulting in the loss of two marks. Although the range of values for *α* is stated in radians, several candidates gave an answer in degrees.

(b) Many candidates understood what was required here, and the majority scored at least two marks for the correct use of their answers to **7(a)**. The most common reason for not gaining the method mark was not dealing correctly with the $\dfrac{1}{3}x$. Most candidates did consider the possibility of a second solution, but relatively few considered the possibility of $\dfrac{1}{3}x - 0.2838 = -0.2003$.

Question 8

The majority of candidates recognised the need to separate the variables, and most completed this step correctly. When attempting to integrate, $p\ln\left(4x+3\right)$ was obtained more reliably than $q\ln\sin 2\theta$. Errors in the coefficients were common, particularly in the trigonometric term.

Some candidates were not sure how to proceed with the theta integral, not recognising it as a form they could integrate directly, and tried to use identities to convert it into a different form. This was sometimes

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successful, for example using double angle formulae to separate the integrand into $\frac{1}{2\tan\theta} - \frac{\tan\theta}{2}$, leading to $\frac{\cos\theta}{2\sin\theta} - \frac{\sin\theta}{2\cos\theta}$ (or obtaining the latter directly using double angle formulae for sine and cosine). However, there were frequent errors with this approach, often meaning that a solution of the required form was not obtained.

The method of finding the constant of integration was well known to candidates. This was usually done correctly, with just the occasional case of incorrect manipulation before the constant was found (for example, trying to remove the logarithms but not maintaining a correct form for the constant).

Those with correct integration and a correct solution then had the challenge of finding x in terms of θ. This was by no means straightforward for many candidates who struggled with the manipulation of the logarithms. Those candidates who had retained an exact value for the constant of integration were more likely to achieve a tidy simplified answer.

Question 9

(a) Many candidates knew what was required and obtained a correct solution. The main problem was with the ' **r** =', which was missing in a large number of solutions. This is a key part of the structure for the vector equation of a line. For several candidates there was confusion between the vector $\overrightarrow{AB}$ and the equation of the line passing through *A* and *B*; it was common for candidates to find the former, but not the latter. There were several arithmetic slips when subtracting the two position vectors.

(b) Most candidates were familiar with the process of using the scalar product of two vectors to find the angle between them. This question required the cosine of the angle, and several candidates did not score the final mark because they gave the size of the angle without ever stating a value for the cosine of the angle. In order to find the angle *BAC*, it is necessary to work with $\overrightarrow{BA}$ and $\overrightarrow{CA}$ or with $\overrightarrow{AB}$ and $\overrightarrow{AC}$, not with a mixture of the two. Some candidates found an angle, but not the required angle.

The question asks for the use of a scalar product, so use of the cosine rule was not acceptable.

(c) There were few fully correct solutions. There are still candidates who interpret the word 'exact' as meaning to give as many decimal places as possible. The use of $39.6°$ was a common error, limiting the score to a maximum of 1 mark.

The better candidates did attempt to find the exact value of sine of the angle, but there was a lot of confusion over the expression $\sin\left(\cos^{-1}\left(\sqrt{\tfrac{35}{59}}\right)\right)$.

Question 10

(a) Many candidates found the quotient and the remainder with no difficulty, most by using algebraic division. Some candidates were confused because they were not used to dividing to obtain a fraction as a quotient, and some divided the wrong way round. There were some sign errors in completing the division, resulting in a sign error for the remainder.

(b) Those candidates who recognised the need to use integration by parts usually obtained at least the first method mark. Many candidates obtained the correct form $\int x^2 \frac{B}{C + Dx^2}\,dx$ in the first stage of the integration, but only a few obtained the correct term $\int \frac{x^2}{1+4x^2}\,dx$. Depending on the nature of the error, candidates still had the opportunity to see the connection with **10(a)** and use their answer to go on to complete the integration. Even those candidates with the correct form usually did not make the connection. For those who got that far, the second stage in the integration often produced a term involving a logarithm. Those candidates who completed the integration with terms of the correct form often had an error in the coefficient of the second trigonometric term.

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In order to score the final M1, candidates needed to be working with terms of the correct form, and to give a clear indication that they had considered the lower limit.

Question 11

(a) Most candidates recognised this function as a product of two terms and attempted to differentiate it using the product rule. There were a few sign errors in applying the rule, but by far the greatest problem was in differentiating $\cos^2 x$. The common incorrect answers were $2\cos x$ and $\sin^2 x$. Some candidates expressed $\cos^2 x$ in terms of $\cos 2x$ before attempting to differentiate. Another alternative was to write the function as $10\sin x\cos^3 x$ but this too created problems with differentiating the power.

There was a lot of incorrect notation in use, with, for example, candidates expressing the derivative as '$-10\sin\sin\cos + 10\cos\cos^2$' which is not only incorrect but it also loses the mixture of single and double angles and makes the remaining marks unavailable.

For candidates who obtained the correct derivative, there were many possible routes to obtaining the *x*-coordinate of *M*, one of the simplest of which was to recognise $10\cos 2x\cos x - 10\sin 2x\sin x$ as $10\cos 3x$.

A small minority of candidates thought that the task required integration.

(b) There were several fully correct solutions. Some candidates obtained a 'correct' answer, but they did not use the required method. Some also used an alternative substitution to the one specified. The common errors included $\frac{du}{dx} = \sin x$, losing the 5, an error in the double angle formula and arithmetic/algebraic slips in the substitution. Many candidates then made errors in changing and substituting the limits. Some candidates were clearly aware that if the order of the limits was changed then the sign of the integral should also change. Some candidates lost marks through trying to 'fudge' the working to obtain a positive answer. Some candidates used the limits for *x* as limits for *u*. Some candidates changed the variable correctly but never actually integrated.

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MATHEMATICS

Paper 9709/33
Pure Mathematics 3

Key messages

Candidates should always check that their final answers are valid for the question they are solving as in **Question 2**. In questions that require a given result to be shown, it is essential that each step in the process be shown as in **Question 5(a)**, **Question 8(a)**, **Question 11(a)** and **Question 11(c)**. Candidates should also be guided by the wording used in a question which may lead to an appropriate approach to take as in **Question 6**, which states in the stem of the question that the answer should be given in a specific form. As always, candidates should read the question carefully and check that they have fully met the demands of the question.

General comments

Many candidates were able to show that they understood and were able to apply appropriate methods to many of the learning objectives outlined in the syllabus. It was evident that the topic of complex numbers was problematic for many candidates as shown by the solutions for **Questions 4** and **6**. In **Questions 4(b)** and **Question 6**, writing a complex number in the form $x + iy$ would have made the solutions more accessible.

Most candidates had sufficient room for their solutions and there did not appear to be any timing issues with most candidates having attempted all the questions.

Comments on specific questions

Question 1

(a) Many candidates were able to sketch a correct graph. It was essential that the graph extended into the second quadrant and did not finish at the *y*-axis. The intercepts with the coordinate axes also needed to be marked in order for a meaningful sketch to be obtained.

(b) Candidates chose either to consider 2 linear inequalities or equations or use a squaring method to obtain a quadratic inequality or equation in order to obtain the critical values. Most candidates were able to obtain both values and hence the required solution. Some candidates rejected the negative root in error. It was intended that the graph in **1(a)** could guide candidates in their final solution, but there was little evidence of the graph being used.

Question 2

Many correct solutions were seen, with candidates manipulating logarithms correctly to obtain a quadratic equation in *x*. Whilst many candidates obtained the solutions $\frac{3}{2}$ and -3, many did not reject the negative solution. Candidates should check that their answers are true solutions to the original question.

Question 3

Many correct solutions were seen with most candidates using the correct double angle formula to obtain the integrand $\frac{3}{2}(1+\cos 10x)$. Limits were usually applied correctly although there were sometimes arithmetic slips. It was pleasing to see that candidates gave their answer in the required exact form. Candidates who did not attempt to use the double angle formula were unable to make any progress.

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Question 4

(a) The most successful method involved using the given exponential form of the complex numbers. Other equivalent methods were acceptable. It was essential that terms involving indices be simplified, for example $z_1z_2 = r_1r_2e^{(\theta_1+\theta_2)}$ or equivalent if using other forms. Most candidates were able to gain marks by recognising the correct conjugates, but it was essential that a conclusion of some form be given as candidates were required to show the given result. Common errors included missing subscripts for r and θ, not simplifying indices correctly and not giving an adequate conclusion.

(b) It is essential that candidates read the question carefully. Too many did not state the other root of the given equation, with it sometimes appearing as an afterthought. Few completely correct solutions were seen as many were unsure of the correct approach to take. The most successful method was to re-write the roots of the equation in the form $3\left(\cos\frac{\pi}{4} \pm i\sin\frac{\pi}{4}\right)$ at some stage in the solution either using the sum and product of the roots or by expanding out and simplifying $\left(z-3e^{-\frac{\pi}{4}i}\right)\left(z-3e^{\frac{\pi}{4}i}\right)$.

Question 5

(a) Many completely correct solutions were seen, with candidates applying implicit differentiation to a product and manipulating terms appropriately. It is essential that sufficient steps be shown in a question that requires a result to be shown. Many candidates obtained the result $\frac{dy}{dx} = \frac{y^2e^{-x} - y}{x + 2ye^{-x}}$ correctly but then wrote down the given answer with no indication of how this was obtained. Some indication that both the numerator and the denominator of the fraction be multiplied by e^x or an equivalent valid method needed to be shown.

(b) Most candidates realised that they needed to find the values of y when $x = 0$ and used these values in the result given in **5(a)**. A surprising number of candidates made arithmetic slips in their calculations or only used the positive value of y.

Question 6

Very few completely correct solutions were seen. Too many candidates were unable to gain any marks in this question because they multiplied the denominator and numerator of the given expression by $z-4i$. This meant that there were still imaginary terms in the numerator and little progress could then be made. It was essential that the denominator and numerator of the given expression be multiplied by $x-i(y+4)$. Of the candidates that used this correct approach, few then made use of the fact that the imaginary part of the expression was zero and then go on to make use of the given modulus. Candidates should have been guided by the form in which the answer was required. Different initial approaches were acceptable but few candidates attempted them. Each of these methods still required candidates to use $z = x + iy$.

Question 7

(a) Most candidates were able to apply a correct method and obtain the correct partial fractions. There were occasional minor arithmetic or algebraic errors.

(b) It was evident that candidates appreciated the use of the word 'Hence' at the start of the question and made use of their answer to **7(a)**. Some errors occurred when candidates attempted to write $(3a+2x)^{-1}$ as $(3a)^{-1}\left(1+\frac{2x}{3a}\right)^{-1}$ and $(2a-x)^{-1}$ as $(2a)^{-1}\left(1-\frac{x}{2a}\right)^{-1}$. Most candidates were able to obtain the first two terms in the expansion of at least one the relevant expressions and often obtained a correct expansion to 3 terms of at least one of the relevant expressions, making use of their result from **7(a)**. Errors usually occurred during simplification and were usually sign errors or

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errors with the indices of the constant a. Some candidates realised they needed to subtract the two expansions but did not actually simplify the subtractions. It was pleasing to see many correct solutions.

(c) Few correct responses were seen. It was evident that candidates were aware that the expansion of $(3a+2x)^{-1}$ is valid for $|x| < \frac{3}{2}a$ and the expansion of $(2a-x)^{-1}$ is valid for $|x| < 2a$. However most candidates did not recognise that, for both expansions to be valid in an expression involving both, $|x| < \frac{3}{2}a$. Use of a simple number line using inequalities would have made this clear.

Question 8

(a) Many different approaches were acceptable, and it was pleasing to see many correct solutions. Candidates were able to start by using either the left-hand side or the right-hand side of the given expression. It was essential that each step of the process was shown as candidates were asked to 'Prove' the given identity, for example writing $\frac{4\cos 2\theta}{\sin^2 2\theta}$ as $\frac{4\cos 2\theta}{\sin 2\theta} \times \frac{1}{\sin 2\theta}$ or $\operatorname{cosec}^2\theta - \sec^2\theta$ as $(1+\cot^2\theta)-(1+\tan^2\theta)$.

(b) Many completely correct solutions were seen with most candidates using the result from **8(a)** to obtain the equation $\tan^2 2x = \frac{4}{5}$. Some candidates chose to simplify and obtain equivalent results, but these methods were usually lengthier and more prone to errors. Some candidates obtained one solution only, but most dealt with the double angle correctly.

Question 9

(a) The demand of the question was to find a vector equation, so the answer needed to be in the form of a vector equation which included the use of $r =$. Most candidates made correct use of a position vector and found a direction vector but did not give their final answer as a vector equation.

(b) Many candidates found the vector $\overrightarrow{AP}$ and made correct use of the scalar product to find the value of the scalar quantity involved. This often resulted in a correct position vector for P. Errors occurred when the vector $\overrightarrow{OP}$ instead of $\overrightarrow{AP}$ was used, meaning no further progress could be made.

(c) A diagram of the situation may have helped some candidates visualise the problem and then make use of simple displacement vectors to calculate the position vector of D. Many candidates did use a correct method and were able to gain at least one method mark for applying this method.

Question 10

(a) Most candidates separated the variables in the given differential equation correctly and then attempted to integrate each side of the resulting equation. It was pleasing to see that many candidates used integration by parts correctly and obtained $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$. There were some sign errors and some candidates, having obtained $-\frac{1}{3}x\cos 3x$, subsequently omitted the x term. Simplification of $\frac{\sin 4y}{\sin 2y}$ prior to integration proved to be more problematic for many candidates. The double angle formula for $\sin 4y$ was the only double angle formula that needed to be used with subsequent simplification to $2\cos 2y$ and then integration. Most candidates attempted to find the value of the arbitrary constant, but a completely correct method for both integrands and integrals leading to an expression in a correct form, was needed to gain a method mark and possibly the final accuracy mark. Many candidates were able to produce complete and correct solutions.

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(b) Marks for this question part could only be gained if the solution to **10(a)** was in the correct form. This usually resulted in candidates solving the equation $\sin 2y = \frac{11}{18}$. The method mark was available for those candidates whose solution was in the given form but with a sign error or algebraic error. Many candidates with correct solutions for **10(a)** were able to obtain at least one of the required solutions. Candidates should be aware of the wording in the question which implied there was more than one solution.

Question 11

(a) Most candidates made use of differentiation of a product and equated the result to zero. When simplifying such results, it is essential to show each step in the simplification process towards a given answer. Some candidates did omit essential working.

(b) Many correct solutions were seen with most candidates opting to make use of a change of sign method using $\tan 2\alpha + 4\alpha$. Other methods were acceptable provided sufficient detail and explanation was given.

(c) A variety of different responses was seen. It was acceptable to work using the given iterative formula as a starting point and simplify it showing sufficient detail to obtain the result $\tan 2\alpha = -4\alpha$. A reverse process was also acceptable provided the final result was an iterative formula.

(d) Most candidates were able to apply the given iterative formula correctly, giving the working and their final answer to the required level of accuracy.

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MATHEMATICS

Paper 9709/35
Pure Mathematics 3

Key messages

Candidates need to understand what the geometrical effects are of multiplying/dividing one complex number by another complex number. They also need to know the correct terminology to describe these operations, for example 'enlargement by a scale factor *k* about the origin' or 'rotation either clockwise or anticlockwise by α radians about the origin'.

Realise the importance of a sketch of the geometrical situation in any vector question. Be able to produce a sketch of the cyclic order *ABCD*. This avoids making the basic sign error that in a parallelogram vector ***AB*** = vector ***CD***.

General comments

When applying the scalar product formula, candidates are expected to show full and accurate working. This includes clearly stating the scalar (dot) product, explicitly substituting values when calculating the magnitudes of vectors, and demonstrating the steps taken to make $\cos\theta$ the subject of the formula.

When a question requires a sketch, key features must be included to gain full credit. These typically include the start and end points, turning points (maximum and minimum), and any asymptotes where relevant. Many candidates lost marks as they omitted any scale and almost all these essential features in their sketches.

The overall standard was good, but the inclusion of **Questions 3(b)** and **10(b)** caused difficulty for most. There were many errors in dealing accurately with **Questions 10(b)** and **(c)**, and subsequently very few managed to get the correct answer for **Question 10(c)**. However, **Question 11** was a very good source of marks for many.

Comments on specific questions

Question 1

Nearly all candidates converted immediately to lns and scored full marks. A few either failed to give their otherwise correct answer to 3 significant figures or made an arithmetical error in their working. Those candidates who decided to work with indices until they reached something of the form $a^{bx} = c$ were, however, not as successful since they usually incurred an error in their working before they reached this stage or failed to even reach this stage.

Question 2

Most candidates scored either 4 or 5 marks. Candidates usually formulated a quadratic equation in either $\cot\theta$ or $\tan\theta$, occasionally in $\cos 2\theta$. A few opted instead for a quartic in either $\sin\theta$ or $\cos\theta$, and the formulation $\sqrt{34}\sin\left(2\theta - \tan^{-1}\left(\frac{5}{3}\right)\right) = 3$ was also seen. However, a significant number of candidates missed the final A1 mark due to a range of issues. These consisted of rounding errors – for example, giving an answer of 1.81 instead of the required 1.82. Also, there were many incomplete solution sets – either by taking only one value for each trigonometric function, or by failing to consider the full domain, particularly the interval $(-\pi, 0)$, thereby missing valid solutions outside the principal range. Others used degrees instead of radians.

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Question 3

(a) Here, although the question asked for the answer in the form $re^{i\theta}$, marks were allowed when candidates stated r or modulus = $\frac{5}{6}$ and θ or argument = −2.75. Most candidates scored both marks, with any loss of mark usually occurring in the argument due to a sign or arithmetical error. However, some candidates struggled with converting from polar form to exponential form.

(b) This question was not well answered by most candidates. While many were able to describe the geometric effects in general terms, few used the correct mathematical terminology. Those who attempted the description often used informal words such as spin, turn, shrink, or compression instead of the appropriate terms such as rotation, enlargement or stretch. Some responses were overly descriptive, stating for example that 'the length is divided by 6' or 'the argument is reduced by 3,' without identifying the corresponding transformation.

Question 4

The correct use of the product rule was well known, and most candidates correctly dealt with it. Some candidates spotted that the ln term could be written as $4\ln x$ hence making the question much more straight forward. Most were able to eliminate the ln to arrive at an answer in terms of e but some were then unable to simplify the result to obtain the coordinates $(e^{-1/3}, -4e^{-1})$ and thus were unable to earn the last two marks. A significant number of candidates did not read the question carefully and were unaware that the y value was also needed. Those candidates who could not perform the differentiation correctly finished either with an equation where the ln terms could be cancelled, being present in both terms, or an equation that could not be solved exactly.

Question 5

(a) Most candidates calculated the distance from the origin to the centre of the circle and found the maximum and minimum distances by adding and subtracting 3 respectively. Others used a coordinate geometry approach to find the points of intersection between the circle with centre (–5, 4) and radius 3 and the line $y = -\frac{4x}{5}$, then calculated the distances from those points to the origin. A few found either just the maximum or the minimum. A small number of candidates did not know how to approach the question at all and were clearly confused about the representation of a complex number on a real diagram and ended up attempting to find lengths using Pythagoras' Theorem with complex numbers.

(b) Candidates found this part far harder than **5(a)** with many candidates clearly uncertain about exactly what is meant by argument and how to find its minimum. Several different methods were seen that could lead to a correct answer, but not all candidates drew the correct tangent and some did not annotate the diagram at all, which might have helped them. Many candidates identified one or more angles using trigonometry in the right-angled triangles formed between OC and the negative x-axis or between OC and the tangent, or for something similar. Fewer candidates were able to use a correct method to find the argument of the centre of the circle. Common errors were using a tangent in the wrong position, adding instead of subtracting, or vice versa. An alternative method used by a small number of candidates was to substitute $y = mx$ into the equation of the circle then find the discriminant of the resulting quadratic equation, solve for m and then calculate arg z. Candidates also often became confused about where the right angles were in their triangles and so used (usually) $\sin^{-1}$ instead of $\tan^{-1}$ or $\tan^{-1}$ instead of $\sin^{-1}$. A number of candidates used sin instead of $\sin^{-1}$.

Question 6

(a) The majority of candidates found and correctly used the given result but in some cases errors in manipulation led to an incorrect value for A. Others omitted to use the chain rule when differentiating either x or y (or both) or used the quotient rule incorrectly to differentiate, most often omitting 0 (the derivative of the numerator 2) which meant they could not reach a multiple of cosec $3t$. Some candidates omitted the 3 in '$3t$' or used x instead of t in places which meant they could not produce a complete argument. Arithmetical errors crept in at various points. Candidates would also be advised to be familiar with the standard derivatives given in the list of formulae.

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A surprisingly common error was to use the quotient rule to differentiate $\frac{2}{\cos 3t}$ (even though the chain rule would be better). Another error was not to show sufficient working when deriving $\frac{1}{2}$ cosec $3t$; since the trig function is given in the question. The onus is on candidates to show the derivation without jumps; the best strategy here is to reduce everything to $\sin 3t$ and $\cos 3t$ and exhibit cancellation explicitly. But simply to assert that, e.g., $\frac{3\sec^2 3t}{6\sec 3t\ \tan 3t} = \frac{1}{2}\operatorname{cosec} 3t$ without displaying any intermediate steps is insufficient to 'Show that $\frac{dy}{dx}$ can be written as $A\operatorname{cosec} 3t$'. A surprising number of candidates did not seem to know that differentiating $\sin 3t$ gives $3\cos 3t$ but instead simply gave $\cos 3t$. Some candidates thought that you could differentiate $\frac{2}{\cos 3t}$ just by differentiating the bottom and derived $\frac{2}{-\sin 3t}$. Several candidates did not handle the '2' on the bottom properly and so thought that $\frac{1}{2\sin 3t}$ equalled $2\operatorname{cosec} 3t$.

(b) Most candidates substituted into their gradient function then calculated the normal gradient, though some used the gradient of the tangent instead, or forgot to make the reciprocal negative. Many of the candidates found x and y correctly, although some introduced an error, or swapped x and y when finding the equation of the normal. Others muddled their substitution of $t = \frac{1}{12}\pi$, relating this value to x instead.

Question 7

(a) Some candidates did not appear to reaslise that the basic rule to allow them to differentiate $\tan^{-1} 4x$ is given in the list of formulae. While fully correct solutions were very common there were a number of candidates who omitted to use the chain rule or who having had the correct derivative gave only the positive value as the final answer.

(b) Several candidates knew that integration by parts was the appropriate approach but, as in **7(a)**, many of these could not differentiate $\tan^{-1}4x$ correctly and this limited them to a maximum mark of $\frac{3}{5}$. Some candidates used degrees instead of radians which cost them another mark. Other candidates carried out the integration in the y-space rather than the x-space but such candidates had two issues to contend with, as well as dealing with the change of variables in the integrand. Firstly they had to change the limits correctly and secondly, they had to know how to integrate $\sec^2 y$. Most candidates did not manage to deal with both correctly and hence fully correct solutions from this route were rare. Many candidates realised that they needed to integrate by parts but a significant number did not know the first step so could make no progress.

Question 8

(a) Many incorrect sec graphs were seen; some of them of roughly the correct shape but omitting any indication of scale horizontally or vertically. Others were of the incorrect shape or periodicity or were missing the asymptote. The graph $\sec x$ was often seen instead of the required graph of $\sec 2x$. Some candidates showed the linear graph with a positive gradient instead of a negative one. It was common for candidates to omit a conclusion to this by way of a dot, cross, statement, or an arrow at the point of intersection, any of which would have sufficed.

(b) Most candidates carried out correct calculations for a combined expression, showing that one value was positive and the other negative, or for two separate expressions and showed that the inequality reversed direction. Some candidates omitted to draw a conclusion from their values and others introduced arithmetical or rearrangement errors. Very few worked in degrees or obtained only 1 out of the 2 values correctly.

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(c) Some candidates started by substituting a consistent variable into the given formula and then rearranging to the equation in **8(a)**. These candidates tended to be more successful than those who started with the equation and rearranged to obtain the given formula – here a common mistake was to include suffixes in their final answer. Some arithmetical errors were seen, and others thought that $\cos^{-1}$ was equivalent to $\frac{1}{\cos}$ so could not produce a logical argument with the required intermediate step.

(d) Many candidates opted for $x = 1$ as their starting point. Only a few did not obtain the **A1** for the final answer as they still had a suffix in their $x = 0.992$ or did not show sufficient iterations. A very few candidates worked in degrees. Candidates should have realised from their sketch in **8(a)** that starting with a value of $x < \frac{1}{4}\pi$ was inappropriate since there was no guarantee that such a value would converge to the required root as the iteration process had to cross an asymptote.

Question 9

(a) Many candidates gained full marks on this question with division being a popular method for finding the constant A. They then used the fact that their remainder was of a simplified form to find the remaining constants. However, a significant number of candidates failed to recognise an improper fraction and so had an incorrect form, meaning the most they could score was 2 out of the 5 marks. Those who used this form could not then achieve full marks in **9(b)**.

(b) Most candidates attempted this part and many went on to achieve full marks. Common errors were not having the correct constant outside of the expansion or errors in manipulating their algebra. A few candidates thought that the question was simply asking for the coefficient of the term in x^2. Candidates' work was generally well presented and easy to follow.

Question 10

(a) This question was generally well attempted with most candidates gaining at least some marks. Many gained full marks and those only awarded 2 marks had generally made an error in their algebra. Most candidates knew how to find $|\mathbf{AB}|$ and $|\mathbf{BC}|$.

(b) Not many candidates gained full marks on this question and very often there was no attempt. One of the most common errors was to equate vector ***AB*** with vector ***CD***, rather than vector ***DC***. This meant that they gained no marks. A minority of candidates attempted to work with distances, or tried to find the centre of the rhombus and work with that, usually, not very successfully.

(c) Many candidates gained at least one mark on this part and a reasonable number gained full marks. Occasionally candidates used the wrong vectors, so did not gain the first M1. Although, when they used the vectors consistently, candidates achieved the second M1. Another common error was to use ***AB*** and ***BC***, rather than ***BA*** and ***BC***. These candidates usually obtained 2 out of the 3 marks. By now candidates should know the cyclic order ***ABCD*** and a quick diagram would have helped most candidates enormously.

Question 11

This proved to be quite an accessible differential equation question. Most candidates achieved some marks and many gained full marks. Generally, the candidates' presentation in this question was clear and their working was easy to follow. Some candidates lost a mark by not showing a relationship between x and y although they went on to find the correct value for $y(2)$. Some candidates were working in degrees and so even though they had integrated correctly, their constant term was incorrect. Other candidates struggled to integrate since they did not recognise that a $\tan^{-1}$ term was required and therefore lost the remainder of the marks.

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MATHEMATICS

Paper 9709/41
Mechanics

Key messages

- When answering questions involving any system of forces, a well-annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium situation or a Newton's law equation. Such a diagram would have been particularly useful in **Questions 1** and **4**.
- Non-exact numerical answers are required correct to 3 significant figures or angles correct to 1 decimal place as stated on the front of the question paper. Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- In questions such as **Question 7** on this paper, where displacement is given as a function of time, calculus must be used, and it is not possible to apply the equations of constant acceleration.

General comments

The questions were generally well answered by many candidates, although a wide range of marks was seen. The paper allowed candidates at all levels to show their knowledge of the subject, whilst still differentiating well between candidates of different abilities. **Questions 3(a)** and **5(a)** were found to be the most accessible questions, whilst **Questions 5(b)** and **6(b)** proved to be the most challenging.

In **Question 6**, the angle was given exactly as $\sin\theta = 0.4$. There is no need to evaluate the angle in situations such as this, and doing so can often lead to inexact answers; any approximation of the angle can lead to a loss of accuracy.

One of the rubric points on the front cover of the question paper was to take g = 10, and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

This question was answered well by most candidates. Successful candidates resolved forces parallel and perpendicular to the inclined plane and correctly applied Newton's second law with the given acceleration. The most common approach was to identify that friction acts down the plane (opposing the motion up the plane) and to use $\mu = 0.4$ correctly in calculating the tension in the rope. Common mistakes included mixing sine and cosine components, as well as errors with signs. Candidates who drew clear force diagrams were generally more successful and made fewer errors when resolving the forces.

Question 2

Most candidates successfully found the values of *P* and *Q*, showing a good understanding of the problem. Most correctly resolved the forces horizontally and vertically, while a few made sign errors when setting up their equations. Some candidates failed to obtain the equations by not setting their expressions equal to zero. Others incorrectly gave the magnitude of *Q* as a negative answer or simply wrote $\frac{5-10\sqrt{3}}{2}$, not realising it was negative.

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Question 3

This was found to be the most accessible question on the paper for most candidates.

(a) This part was answered particularly well by most candidates. The vast majority correctly used the information to find the velocity after the acceleration phase and then applied the appropriate constant acceleration equation to find the time for deceleration. Those candidates who used $\frac{1}{2}x8t = 80$ needed to also have $T = \textit{their}\ t + 40$ to earn the mark. Some candidates made arithmetic errors or incorrectly applied the kinematic equations.

(b) Fewer candidates attempted this part successfully. Many candidates understood that a displacement-time graph was required but struggled with drawing the correct shape, particularly during the acceleration and deceleration phases. Many candidates gained some credit for this question, and the most successful candidates calculated the distances travelled after 10 seconds and after 40 seconds. Some candidates, however, wrongly gave the distance travelled after 40 seconds as the distance travelled between 10 seconds and 40 seconds.

Question 4

(a) (i) This part was relatively straightforward and was well answered by most candidates.

(ii) This part proved more challenging. Many candidates correctly used $P = Fv$ to find the driving force at the given speed, then applied Newton's second law to find the acceleration. Common errors included omitting (or using an incorrect) constant resistance to motion force.

(b) This part was generally well answered by candidates who understood that at constant speed on an incline, the driving force must equal total resistance. This includes both the given resistance and the component of weight acting down the slope. Some candidates forgot to include the weight component or made errors in calculating $\sin\alpha$.

Question 5

(a) This part was answered well by many candidates. Most successfully set up simultaneous equations using momentum ($mu = 4$) and kinetic energy ($\frac{1}{2}mu^2 = 16$) and then solved these to find m and u. A few candidates made algebraic errors when manipulating the equations.

(b) This collision problem proved much more challenging, with fewer candidates achieving full marks. While many candidates correctly applied conservation of momentum to the collision, fewer were able to effectively use the loss of kinetic energy. Some candidates failed to recognise that both conservation of momentum and the given energy loss needed to be solved simultaneously. Common errors included sign mistakes and incorrect squaring of $2w$ in the kinetic energy equation.

Question 6

This was found to be the most challenging question on the paper for most candidates.

(a) Most candidates who attempted this part made good progress. The common approach seen was to attempt Newton's second law equations for particle P and for particle Q and solve simultaneously to find the required acceleration and tension. Some candidates made errors by incorrectly resolving forces on the inclined planes or by not properly accounting for the different masses.

(b) This energy method approach was found to be very challenging, with many candidates making little progress. Those who understood the work-energy principle generally performed better, correctly identifying the work done against friction, changes in kinetic energy, and changes in potential energy. However, a significant number of candidates either did not attempt this part or failed to account for the energy loss due to friction or made fundamental errors in applying energy conservation principles.

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Question 7

(a) Many candidates correctly found when particle *X* was resting using differentiation but struggled to correctly compute the total distance (as opposed to displacement), which required the candidates to consider the motion in separate intervals.

(b) This part required an analysis of two particles' motion, which was a struggle for most candidates. The question required finding when particle *Y* has a specific velocity and then demonstrating that both particles have equal displacements at that time. Many candidates were unable to integrate the acceleration function of particle *Y* correctly, and even fewer were able to complete the comparison of displacements. Those who recognised the questions intention to compare the positions of the two particles made the most progress, but poor handling of integration constants limited many responses.

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MATHEMATICS

Paper 9709/42
Mechanics

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. This was particularly noticeable in **Question 3**.
- In questions such as **Question 6** on this paper, where a velocity is given as a function of time, then calculus must be used, and it is not possible to apply the equations of constant acceleration.
- Non-exact numerical answers are required correct to 3 significant figures or angles correct to 1 decimal place as stated on the front of the question paper. Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1(a)** and **2** were found to be the easiest questions whilst **Question 7** proved to be the most challenging.

In **Question 3**, an angle was not given, but distances are, so that exact values of sine and cosine can be found. If the angles are found, then they should be used to at least 4 significant figures to maintain accuracy to 3 significant figures for a final answer.

Comments on specific questions

Question 1

(a) This was a well answered question by the majority of candidates. The most common approach being to use speed = $\frac{\text{distance}}{\text{time}}$. Another approach was to use constant acceleration formulae with an acceleration of 0 m s^{-2}. A common error seen was for the use of constant acceleration formulae with a non-zero acceleration, usually 10 m s^{-2}.

(b) Only a minority of candidates were successful in answering this question. The most common wrong answer seen was $25 \times 12 = 300$ J, with no account taken for the fact the 25 N force was not acting in the direction of motion and so a component of this force was required.

(c) There were two approaches seen in answering this question. Some candidates attempted power = force × velocity, but the majority used 25 N as the force rather than $25\cos(20)$. The other, more common method, seen was to use power = $\frac{\text{energy}}{\text{time}}$.

Question 2

Candidates are now quite adept at applying conservation of momentum to a given situation. However, a significant number thought that the particles were initially moving towards each other by including a negative sign with one of the initial velocities. This often resulted in the speed after collision as $\frac{8}{3}$ m s^{-1}. Having found a speed after collision, the majority of candidates were able to use a correct formula for kinetic energy to find

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the loss in kinetic energy during the collision. A minority of candidates did not realise that momentum was needed in this question and found the speed of the coalesced particle to be 2 + 5 = 7 m s^{-1}.

Question 3

This question proved a challenge for a significant number of candidates. Resolving in two directions was required with the majority of candidates attempting this horizontally and vertically and a minority attempting this parallel and perpendicular to the string. The main issue for those who did not have correct equations was that no angles are given in the question, so trigonometry is required to find at least the sine and cosine of an appropriate angle.

Question 4

Due to a series specific-issue with this question, full marks were awarded to all candidates for this question to make sure that no candidates were disadvantaged.

Question 5

Due to a series specific-issue with this question, full marks were awarded to all candidates for this question to make sure that no candidates were disadvantaged.

Question 6

The concept of variable acceleration being related to calculus rather than constant acceleration was well understood, so this question was a good source of marks for the most candidates.

(a) Many candidates appreciated the need to differentiate the given expression to get an acceleration function. then equate the acceleration to 0, solve for t and then use this value of t to find the maximum velocity of the particle. Errors seen by examiners were not using the chain rule when differentiating $(2t+1)^{\frac{3}{2}}$. Also, algebraic errors occurred in solving the acceleration function equal to 0. A minority of candidates attempted to use numerous values of t in the given function to find the maximum velocity, which may give a velocity of 3.5 m s^{-1}, but does not show that this value is a maximum.

(b) Candidates knew they had to integrate velocity to get displacement but found integrating $(2t+1)^{\frac{3}{2}}$ difficult and it was common to see $\frac{2}{5}(2t+1)^{\frac{5}{2}}$ rather than $\frac{1}{5}(2t+1)^{\frac{5}{2}}$. The resulting integration was often evaluated between $t = 0$ and $t = 3$ by a significant number of candidates. Those who did use limits of $t = 0$ and $t = 1.5$, often assumed that the value of the integrand was 0 when $t = 0$, so an incorrect answer of 4.15 m was seen often by examiners.

Question 7

This proved to be the most challenging question for the majority of candidates, with many ignoring the instruction to use an energy method throughout and using a constant acceleration method for part or all the question.

As the question did not give the distance AB, an energy method was required to find this distance. The frictional force needed to be calculated as $\frac{\sqrt{3}}{12} \times 3g\cos 30 = 3.75$, so that the resulting energy equation for the first part of the particles motion is $\frac{1}{2} \times 3 \times 8^2 - 3gd\sin 30 = \frac{\sqrt{3}}{12} \times 3g\cos 30 \times d$ (where d is the distance AB). This gives the distance AB to be 5.12 m. However, the distance was often seen to be omitted from the frictional force, resulting in a dimensionally incorrect work-energy equation.

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Energy had to be used for the second phase of the motion from *B* to *A*, using the distance found in the first phase. This gives the equation $\frac{1}{2} \times 3 \times v^2 + 5.12 \times \frac{\sqrt{3}}{12} \times 3g \cos 30 = 3g \times 5.12 \times \sin 30$ and hence giving a required speed of 6.20 m s^{-1}.

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MATHEMATICS

Paper 9709/43
Mechanics

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Such a diagram would have been particularly useful here in **Questions 4** and **5**.
- Non-exact numerical answers are required correct to 3 significant figures (or correct to 1 decimal place for angles in degrees) as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- In questions where acceleration is given as a function of time, such as **Question 7** in this paper. Calculus must be used, and it is not possible to apply the equations of constant acceleration.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1, 2, 5(a),** and **7(a)** were found to be the easiest questions whilst **Questions 3, 4** and **5(b)** proved to be the most challenging.

In **Question 6** the angle was given exactly as $\sin\theta = 0.8$. There is no need to evaluate the angle in this case and in doing so can often lead to inexact answers (and therefore a possible loss of accuracy).

One of the rubric points on the front cover of the question paper was to take g = 10 and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

(a) This part was answered extremely well with most candidates applying the principle of conservation of linear momentum twice to find both correct values of u. The most common error was those candidates who formed the equation $0.1\times 4u - 0.3\times u = 0.1\times 2 + 0.3\times(-4)$ and so implying mathematically that the two particles had somehow managed to pass through one another.

(b) Most candidates who found both correct values of u in **1(a)** used the larger of the two to correctly work out the largest possible loss of kinetic energy in the collision. No credit could be given to candidates who had only found one value of u in **1(a)**, and the final mark was dependent on having found both correct values in **1(a)**.

Question 2

This was probably the best answered question on the entire paper with almost all candidates correctly applying Newton's second law twice (on either the van, trailer or system) to find the correct values of X and T. When errors occurred they were usually sign errors or incorrectly including T in the system equation.

Question 3

(a) Most candidates managed to make a start in this part by correctly calculating the velocity at $t = 20$ and calculating the displacement of the particle at $t = 40$. However, many struggled with calculating

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the speed at $t = 50$ (normally due to issues with finding the acceleration for the second line segment) and therefore could not set up a correct equation for T.

(b) While many correct answers to this part were seen, there were examples where a negative value for the acceleration was found for the third line segment, when the graph indicated that this value would be positive.

Question 4

Candidates found both the unstructured nature and unfamiliar scenario posed by this system of forces a challenge. Many formed equations in terms of the tensions in the strings rather than the masses and many candidates were unsure which forces were acting at which points (or on which of the three blocks). Some candidates working was also difficult to follow at times as they did not use notation that was easy to understand. While many candidates correctly stated that $25g\sin 30 - mg\sin\alpha = 0$ (from resolving horizontally at O) only the most able could also derive the second required equation as $25g\cos 30 - 20g - mg\cos\alpha = 0$ (from resolving vertically at O) with many including additional incorrect forces. Of those candidates who did form two correct equations in m and α, most were able to solve them correctly.

Question 5

(a) This part was answered extremely well with almost all candidates correctly applying $P = DF \times v$ to find the given answer of $k = 0.5$. While most then correctly found the acceleration of the van at speed $25\,\text{m s}^{-1}$, some candidates mistakenly used a value of 1250 rather than 2500 for the driving force.

(b) This part was also answered extremely well with many candidates correctly applying Newton's second law twice to obtain two correct equations in a and θ, then going on to solve these equations simultaneously. When errors occurred, they were usually due to sign issues/errors, missing a term(s) in one or both equations, or assigning the two (different) accelerations incorrectly. However, due to the mention of power in this part, some candidates thought that the problem could be solved using an energy approach; the mention of acceleration should have been a hint that Newton's law of motion was the correct and only valid way of approaching this part.

Question 6

(a) The methods employed by candidates were almost equally split in this part; about half used an energy method while the other half applied Newton's second law and the equations of constant acceleration. Examiners noted that there was no noticeable difference in the success rate of the two different methods used. The most common error seen in both attempts was using an incorrect angle when working out the normal contact force between the man and the waterslide. Finally, many candidates gave an answer of 8.6 even though this was not an exact answer and the front cover of the question paper clearly indicates that non-exact numerical answers should be given correct to 3 significant figures.

(b) This part was answered extremely well with almost all candidates applying the work-energy principle with the correct number of terms to find the required work done against the resistance force acting on the man. The most common error was to use the wrong vertical height when calculating the change in gravitational potential energy.

Question 7

(a) This part was done extremely well with almost all candidates correctly using differentiation to find the correct velocity at $t = 4$ although some did attempt to use the equations of constant acceleration.

(b) (i) This part was also well done by candidates with many correctly integrating the given acceleration expression and setting up a correct equation for T. The most common error was to set up an equation in T that came from velocity expression which did not contain a constant term – many candidates assumed that the constant of integration would be zero which was not the case.

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(ii) For the last part of the examination this was also answered extremely well with many candidates correctly finding the total distance travelled by X. Many candidates correctly worked out the distance travelled between times 4 and T using integration but then either forgot or incorrectly calculated the distance travelled in the first 4 seconds, while some struggled to integrate their expression for v from **7(b)(i)** correctly.

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MATHEMATICS

Paper 9709/45
Mechanics

Key messages

- Non-exact numerical answers are required correct to 3 significant figures (or correct to 1 decimal place for angles in degrees) as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- In questions such as **Question 6** in this paper, where velocity is given as a function of time, calculus must be used, and it is not possible to apply the equations of constant acceleration.
- **Questions 3, 5(b), 6(a), 6(c)** and **7(a)** have exact answers. It should be noted that only non-exact answers should be approximated, so the answers to these questions do not need to be given to 3 significant figures.

General comments

- The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. The whole of **Question 6** was found to be the most accessible question, whilst **Questions 2(b)** and **7(b)** proved to be the most challenging.
- In **Questions 4** and **7,** the angle was given exactly. There is no need to evaluate the angle in situations like this, and it is better not to do so, as it can lead to a loss of accuracy. A significant number of candidates showed all their working in terms of sin α and cos α rather than working out $\sin\alpha = \frac{15}{17}$ and $\cos\ \alpha = \frac{8}{17}$ (for **Question 4**). If the final answer given by a candidate was incorrect then it was not clear whether they had used the correct value of sin α and so partial marks could sometimes not be awarded.

Comments on specific questions

Question 1

Most candidates produced a good solution to this question. Using an energy method was more usually seen, with most candidates who attempted this finding the two required energy terms. A few candidates failed to multiply by 12 or later multiplied both terms (incorrectly) by 12 as part of their equation. Many candidates set up the equation with the correct number of terms but used 50 instead of 50 × 12. A few candidates appeared unsure which value they required, often going on to divide a correctly derived Work Done term by 12.

For those candidates who used the Newton's second law method, most candidates found a = 0.24, but fewer candidates went on to use this correctly in an appropriate equation. Many candidates found F = 90.8 but then omitted the required multiplication by 12. On both methods, many candidates gave their answer with incorrect units, for example W or N. This perhaps indicates that candidates were uncertain about what the question was asking for.

Question 2

(a) Many good answers were seen for this question, with most candidates setting up an appropriate equation with the use of conservation of momentum with a very small number of candidates mistakenly including g with their mass terms. However, a significant number of candidates did not give their answer to 3 decimal places. An answer of 31.9 (3 significant figures, not 3 decimal places) was common.

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(b) Fewer than half of candidates scored full marks here for the correct answer from correct working. In both methods, omitting the *mg* terms was very common – which, with rounding, often led to the 'correct' value to 3 significant figures, but from incorrect working, and so could not be credited. Most candidates understood what the question was asking, and the split between an energy method and a Newton's second law method was approximately equal. A few candidates used the wrong value of v, for example 32, or attempted to find the difference in kinetic energy between the two velocities.

Question 3

Most candidates answered this question very well. The most efficient method was to realise that the velocity time graph was a trapezium (which candidates usually drew) and equate the total area under the trapezium to 4800. Many used their own methods to find T and arrived at the correct answer. The other common method was to effectively combine the displacements from the 1^{st} and 3^{rd} phases into a single triangle area.

A few candidates did not manage to get the answer correctly, often because they set up displacements for the 1^{st} and 3^{rd} phases in terms of accelerations and could not simplify their model appropriately. A few candidates did work through successfully with the two unknown accelerations. Occasionally candidates set up and solved (often using a calculator) simultaneous equations in t and T, one as a summation of the 3 times, the other as a summation of the 3 displacements.

Most candidates found the distance moved by the train while travelling at constant speed, following through their value of T if necessary.

Question 4

(a) The majority of candidates resolved correctly and found the magnitude and angle for their resolved vector. However, of these, quite a few candidates failed to uniquely indicate the direction of their vector, for example failing to specify **above** the **positive** x-axis, and so lost the final mark. Errors in resolving the forces were common, although relatively few candidates offered a fully consistent 'sin/cos mix' version. Most candidates went on to attempt the magnitude, but a minority of candidates did not attempt to compute the direction, giving a vague answer such as 'up and right'. As mentioned in the general comments, some candidates calculated the angle and then used this in resolving, often resulting in answers which were slightly out. Only a few candidates tried to resolve in the directions of the 51N and 34N forces. These often successfully found the components and the magnitude of the resultant but rarely the direction.

(b) This was fairly well answered with the majority of candidates receiving full marks. Common errors included not realising the resultant force in the y-direction was zero or using incorrect trigonometry when resolving. A considerable number of candidates did not know how to start the question.

Question 5

(a) This was reasonably well answered by most candidates. A lot of final answers were given as 0.019, which resulted in the final mark not being awarded unless the answer was also given as an exact fraction. For those who did not get full marks, many candidates found the value of the resistance force of 750N, and then made an error in their Newton's Second Law equation, for example $\frac{28000}{32} - 750 = 1500a$.

Some candidates misunderstood the formula $F = \frac{P}{v}$, thinking that P refers to the difference of the two powers. Others applied Newton's Second Law incorrectly by forgetting to subtract the resistance force.

(b) This was answered correctly by most candidates. Most candidates found v = 40 and v = –125 and correctly rejected the value –125.

Question 6

(a) This was an accessible question for most candidates and generally two marks were scored. A number of candidates forgot to include the '±' sign when taking the square root of 100 and so only

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one mark was gained for t = 2.5. This was the most popular route and more efficient than expanding and solving the 3-term quadratic.

(b) Mostly candidates answered this part very well for full marks. Many used an efficient method where they obtained the expression for acceleration, solved $a = 0$ and then substituted into the velocity to show that v is also zero, or solved $v = 0$ to obtain the same time (7.5).

Some candidates tried to find the time when velocity equals the acceleration, but to score more than one mark they then had to substitute the value back into at least one equation to confirm that $a = 0$ or $v = 0$. A few candidates differentiated wrongly, usually missing out the negative sign to get an answer of $a = 60 – 8t$.

(c) This part was also well attempted by most candidates. Correct integrands were nearly always seen although candidates who did not multiply out the expression for velocity, sometimes omitted the negative sign, or multiplied by –2 rather than dividing.

There were occasional mistakes with finding the constant of integration or with limits during calculation. There were some false finishes with the limits applied to the unexpanded integrand, where candidates did not recognise that it would take the form '0 – negative value' and incorrectly took the modulus or otherwise got a positive answer from incorrect working.

Question 7

(a) This question was fairly well answered with most candidates scoring at least 5 marks. Some candidates, perhaps thinking that friction always acts up the plane, only found the value of 0.1 kg. A few candidates failed to recognise that the system was in equilibrium when m had its maximum and minimum values and mistakenly applied the formula $F = ma$. However, many candidates successfully resolved the forces and obtained the corresponding values for both the maximum and minimum values of m. Candidates usually worked in terms of the tension, although some used the equation for the system from the start, without involving tension at all. Additionally, many candidates overlooked the key requirement of the question, which asked for a range of values rather than exact maximum and minimum and so were not awarded the final mark.

As in **Question 4** there were some errors in the use of sine and cosine ratios, and some candidates mixed these up, particularly when calculating friction and weight forces.

(b) This question proved to be challenging for most candidates, with many scoring very few marks or not attempting it at all. This part also caused the most variation in candidate approach and interpretation, with four distinct valid solution methods observed (in order of popularity):

1. Using work-energy for the whole system.
2. Using Newton's Second Law to find the tension and then using an energy equation.
3. Using work-energy but treating the particles separately.
4. Using Newton's Second Law to find the acceleration and hence find the resultant force on either particle, followed by an energy method to find the speed.

It should be noted that the question required an energy method and so candidates who found the acceleration and then used constant acceleration equations to find the velocity could only gain marks for finding the acceleration. This was a common error for those finding the acceleration.

Candidates who used method 1 were the most successful. The most common error with this method was to make an error in the potential energy. A few candidates, although using the whole system, only used the kinetic energy of either particle A or particle B instead of both.

Candidates who used method 2 were often also successful, usually finding the tension correctly. Some however then did not know how to proceed after this or included extra incorrect terms and sometimes wrongly used the kinetic energy of both particles.

Although method 3 was not seen very often, candidates who used this were usually successful, finding equations for each particle separately and then eliminating the tension to find the velocity.

Method 4 was hardly ever used successfully, with most candidates using constant acceleration equations.

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MATHEMATICS

Paper 9709/51
Probability & Statistics

Key messages

Candidates need to be aware that working and method must be shown clearly to support their answers. Values calculated must be verified by including the mathematical operations and processes that produced them. Successful solutions require both clear communication and accuracy. The necessary intermediate steps should be included especially when proving a given result.

Where a diagram is requested, candidates would be well advised to be precise in its execution. It should be clearly and accurately drawn and appropriately labelled.

Candidates should state only non-exact answers correct to 3 significant figures; exact answers should be stated exactly. To justify a final answer correct to 3 significant figures, working values correct to at least 4 significant figures should be used in candidates' calculations throughout. There is no requirement for fractions to be converted to decimals.

When considering scenarios in combinatorial questions or outcomes in probability questions, it is important to ensure that all the required values, including 0, are considered.

General comments

Most candidates used the answer spaces effectively. When extra space is required, the additional page should be used in the first instance and the question number should be made clear.

Where there is more than one attempt at a question, care should be taken to identify the intended solution.

The use of supporting diagrams, sketches and tables were often a feature of correct solutions. They help to organise multi-step processes and organise information efficiently.

Many candidates performed well in the early parts of **Questions 2**, **3**, **4** and **5** (where the context was understood by most). Frequently the latter parts of **Questions 2**, **5** and **6** were more challenging. Some candidates were unable to tackle **Questions 2(c)** and **6(c)**.

Sufficient time seems to have been available for most candidates who used efficient and appropriate methods to attempt all the paper. A few candidates did not appear to have been adequately prepared for all topic areas of the syllabus (the Normal distribution in particular).

Comments on specific questions

Question 1

Good solutions used the Normal distribution table to find the *z*-value associated with a probability of 0.78 (correct to 3 decimal places or more) and used it with the standardisation formula. Those who drew a helpful diagram and shaded the appropriate area were more likely to be consistent with the signs and get the right answer. Some candidates gave a 2 decimal place value for *z* while others subtracted their *z*-value from 1 and used that in their formula. A small minority subtracted 0.22 from 1 incorrectly and looked up a value of 0.88 instead. Weaker solutions used the probabilities associated with *z*-values of 0.22 or 0.78. A significant number of candidates were unable to tackle this question.

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Question 2

(a) Strong candidates understood that it was necessary to place the two As together first and then consider the placement of the Os.

The method adopted by most was to find the total number of arrangements with the As together, $\frac{7!}{2!}$, and then to subtract the number of arrangements with the As together and the Os together, 6!

Others considered the various options for placing the Os around the two As with some listing the options at great length. Weaker candidates did not allow for there to be repetitions of the Os or As or divided by 2! inappropriately, considering that the As and Os were distinguishable.

(b) Most solutions calculated the probability of obtaining an A on the first, second, third, fourth or fifth attempt, with a few incorrectly including the sixth attempt. Others recognised that the Geometric Distribution was an appropriate model and subtracted the probability of getting any other letter on 5 occasions from 1, with a few incorrectly including the sixth attempt. Weaker candidates used the Binomial distribution as a model. A few candidates who knew the required method used incorrect fractions for their probabilities such as $\frac{1}{8}$ or $\frac{1}{6}$.

(c) The strongest candidates realised that the sixth roll had to provide an A and found the probability of the other A being in the first, second, third, fourth or fifth place. Some candidates thought that they needed to multiply their answer to **2(b)** by $\frac{1}{4}$. Most omitted the factor of 5, not allowing for the combination of outcomes. Some good candidates lost the final mark by rounding their answers correct to 2 significant figures.

Question 3

(a) Most candidates were able to provide a stem and leaf diagram as requested. Successful solutions included a stem with the correct values in increasing order and the leaves ordered correctly and lined up vertically. Most, but not all, provided the Gulls data on the left as required. Many who selected the appropriate diagram, lost marks through inaccuracy in alignment, separating the data with commas or the inclusion of decimal points in the stem or data. Some candidates omitted the units in their key or left out a key altogether.

(b) This question was tackled well by the vast majority and many complete solutions were provided. A small number of candidates found the mean rather than the median or used the data for the Herons.

(c) A lot of good solutions were seen where candidates appreciated the need to show the sum of 175 and 252 and divide this by 50 (some good candidates omitted this stage). Weaker candidates found the mean time for the Eagles and added it to the mean time for the Swifts, dividing by 2 in most cases.

(d) This part was challenging for many. Strong solutions incorporated the given summary statistics in the correct formula and showed each of the necessary steps in its rearrangement to obtain $\sum y^2$. Some candidates used standard deviation rather than variance while others used the data for 20 or 30 runners rather than 50. A significant number of blank responses were seen.

Question 4

A minority of candidates did not seem to have been prepared for questions on the Normal distribution and were unable to make a start. Some very good candidates did not include the standardisation formulae to show their method; this is a requirement of the paper.

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(a) Strong solutions included two standardisation formulae using 31.2 and 33.6 as the boundaries for the probability area and the correct calculation for that area. Supporting diagrams helped many good candidates to identify the area correctly and avoid finding the two tails of the distribution or simply the probability of obtaining a time less than 33.6 minutes. Candidates are reminded that they must show their working; this includes providing standardisation formulae. A small number of solutions provided no evidence of standardisation to substantiate their answer and simply quoted a probability of 0.6844. Unsupported answers do not gain credit as indicated on the front cover of the question paper. As the probability found is a working value it must be given to at least 4sf before being multiplied by 600 (a good number of candidates forgot to do this) to find the expected number of competitors. The best solutions provided the answer as an integer with no reference to approximation. Some candidates who knew what to do, lost the final mark through premature rounding of the probability or giving two solutions.

(b) Most attempts to this question gained at least 1 mark by the inclusion of a correct Binomial term. The most common method was to find the probabilities that 0,1,8 or 9 competitors had times less than 36.0 minutes and subtract them from 1. In many cases the probability of 0 competitors was omitted, losing the remaining 2 marks. Other candidates found the sum of 2, 3, 4, 5, 6 and 7 competitors. This method, although less efficient, was more likely to be correct providing that the calculations for each term were carried out sufficiently accurately.

(c) The use of the Normal distribution as an approximation for the Binomial is a familiar application. The best candidates labelled the variance correctly and recognised that a continuity correction was required because the variable was discrete. In some cases, the value 49.5 was used rather than the correct value of 50.5. Weaker solutions used the mean and variance from **4(a)**. Once again, a helpful sketch enabled good candidates to appreciate that they needed to subtract the probability found in the table from 1.

Question 5

(a) This question was done well by most candidates. Most were able to identify the required scenarios for the musicians and calculate the product of the correct combinations for each one before finding their sum. Some selected 1 drummer and 3 guitarists and then considered the other possible members for the group. Again, some candidates omitted the scenario with 0 pianists. Many did not clearly identify the scenarios and thus did not get full credit. Weaker candidates found the sum of each individual choice of musicians.

(b) Most candidates got the first mark only. Many identified the correct scenarios but found their sum rather than their product, not appreciating that a band for France, a band for Italy and a band for Spain needed to be selected. A few considered the total number of ways of selecting each musician rather than selecting the musicians for each venue.

Question 6

(a) Many candidates recognised the significance of ‘without replacement’ and multiplied $\frac{6}{10}$, $\frac{5}{9}$, $\frac{4}{8}$, and $\frac{3}{7}$ together. Of the successful candidates some listed the six possible arrangements of the fractions and most listed the six possible arrangements of the colours to justify their answer. A well organised list of outcomes was a feature of strong solutions. Many candidates who knew what to do did not appreciate the rigour required in mathematical proof and scored just 1 mark.

(b) Strong solutions gave a clear table showing the values of X as 0, 1, 2, 3 and 4. A significant number who knew how to find most of the probabilities omitted the outcome where no blue marbles were selected and gave only four values for X, thus potentially losing 2 or 3 marks. Weaker candidates attempted to give a possibility space or gave a table with just the given value in place.

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(c) Very few correct solutions were seen in this part. Some statements of the formula for conditional probability were seen but often the candidates were unable to substitute the correct probabilities. In some cases, good candidates did not show where their values came from. Of those who used the correct probabilities, a significant number found the product of the probabilities of 2 blue and 3 blue marbles rather than their sum. Some candidates did not appreciate that the inclusion of at least 1 red marble meant that 4 blue ones could not be selected.

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MATHEMATICS

Paper 9709/52
Probability & Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively and is an essential tool when showing given statements are true. When errors are corrected, candidates would be well advised to cross through and replace the term. It is extremely difficult to interpret accurate terms that are overwritten.

There should be a clear understanding of how significant figures work for decimal values less than 1. It is important that candidates realised the need to work to at least 4 significant figures throughout to justify a 3 significant figure value. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent; there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Candidates should be aware that the axes should be labelled with both the variable and the units where appropriate.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. A few candidates did not appear to have prepared well for some topics, in particular when more than one technique was required within a solution. Many good solutions were seen for **Questions 1** and **2**. The context in **Questions 4** and **6** was found to be challenging for many.

Comments on specific questions

Question 1

This question was answered well by the majority of candidates. Most attempted to form an appropriate probability distribution table, although there were occasions where the omission of lines made it more challenging to link probabilities to outcomes. Common errors were the omission of $X = 0$ and swapping the value for $X = 1$ and $X = 2$. A small number of candidates calculated probabilities for all the possible scenarios but then failed to combine their answers suitably. It was encouraging to see that very few candidates converted their exact fraction to an inexact decimal value in this question.

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Question 2

This accessible question was well answered by many candidates. The best solutions provided clear calculations for the mean and variance, substituted accurately into the standardisation formula recognising that a continuity correction was required as the data was discrete and used a simple diagram of the normal distribution to clarify the probability area required. Weaker candidates either omitted the continuity correction or misinterpreted the boundary that was required.

A small number of candidates misinterpreted the question and calculated the expected number residents who owned a bicycle using the probability they had obtained.

Candidates should be aware that the correct vocabulary and notation is expected when identifying the mean and variance or standard deviation.

There were a number of candidates who attempted to use a binomial distribution, which although could be evaluated would gain no credit as this is not a suitable approximation to use.

Question 3

Tree diagrams were noted in many successful solutions as this effectively clarified the given conditions.

(a) The best solutions identified clearly the required scenarios and calculated the probabilities as fractions accurately. The common error was to ignore the condition that the red marble was not replaced if drawn first. A more surprising error was not replacing the blue marble when drawn.

(b) This conditional probability was found more challenging because of the replacement conditions. Most candidates used the same approach as in **3(a)**, although a few corrected their interpretation error. Weaker candidates failed to use the conditional probability formula appropriately. A small number of candidates simply calculated $P(B_1 \cap R_2)$.

Some good solutions recognised that $P(R_1 \cap R_2)$ had been calculated in **3(a)**, and transferred their value as required.

Question 4

This probability question based around the geometric approximation was found challenging by many candidates. A common misconception was that in **4(b)** and **4(c)**, where it was anticipated that candidates would calculate the probability of 'not turning left' as 70%, it was necessary to consider going straight on or turning right as separate outcomes.

(a) The best solutions identified that permutations should be involved because the order of the vehicle turning would produce distinct scenarios. Weaker solutions simply multiplied the given probabilities without considering the order of turning. A number of candidates found the probability that the cars would all turn in the same direction.

(b) This standard geometric approximation technique was performed well by many candidates, but it was omitted by approximately 10 per cent of candidates. Although some candidates used the more efficient $1 - 0.7^8$ approach, the majority of successful solutions simply added the probabilities of the car turning left for the first, second, etc. A common error was to interpret the success criteria 'before' as including the 9^{th} vehicle. Candidates would be well advised to focus on interpreting success criteria before the examination.

(c) This question was found challenging by many candidates and no attempt at a solution was presented by nearly 20%. Good solutions considered the problem in two stages, initially calculating the probability that only 1 vehicle turned left in the group of 6 vehicles, then multiplying by the probability that the 7^{th} vehicle turned left. Again, good solutions recognised that the order was relevant and multiplied by 6. Many solutions found the probability that any 2 of the 7 vehicles turned left and used ${}^7C_2$ as the coefficient.

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Question 5

Although this was a quite standard statistical question, many candidates found it challenging because of the accuracy expected in statistical diagrams and determining the data accurately to estimate the mean and standard deviation.

(a) Many candidates found drawing the cumulative frequency graph challenging. The most successful graphs used a scale of 1 cm = 10 minutes on the horizontal axis and either 2 cm = 40 students or 2 cm = 50 students on the vertical axis. These enabled the time coordinate to be plotted accurately and the frequency coordinate to be either clearly on or off the grid lines. Although there was a pleasing increase in well labelled axes, a high proportion of students omitted the units on the time axis or labelled the cumulative frequency as frequency or not at all. Very few candidates used line segments to join the points, which is encouraging as this is not acceptable at this level.

(b) Where a graph was present, most candidates attempted to find the value of k. Good solutions identified that 180 students took less than k minutes and marked their graph clearly to indicate use and read from their scale accurately. As in previous sessions, many candidates simply read the graph at 120, with more able candidates then subtracting their answer inappropriately from 90 minutes. As the question instructs the candidate to use the graph, there must be supporting work visible on the graph to gain credit.

(c) Many excellent solutions were seen, with the most efficient being presented in a table to minimise errors and communicating clearly the processes that are being used. The majority of candidates used the formula approach, stating the calculations required before evaluating. This was quite a standard question and was often successfully answered by candidates who found other topics challenging. The most common errors were using the class width or upper boundary as the mid-point value. Almost all candidates stated the mean and variance formulas accurately, with squaring the frequence rather than the mid-point value as the common error in the variance formula. Candidates did not always evaluate their correctly stated expressions accurately, or rounded values prematurely so that final answers were outside the acceptable range.

Question 6

This permutations and combinations question was found challenging by many. Candidates who listed logically possible scenarios or used simple diagrams to illustrate their approach often achieved good solutions. Candidates do need to ensure that they read the conditions for each part carefully and consider the different approaches that could be used rather than trying to use the same technique in each part.

(a) The subtraction method was the most common approach to this part, but was nearly always unsuccessful as candidates assumed that the success criteria would be met if just the ways that all 4 As together were removed, then no As would be together. More successful attempts started to identify the different ways that the As could be together but did not always identify all possible scenarios.

The more successful approach was to consider M ^ ^ ^ ^ M to calculate the number of ways that the letters without As could be arranged and then identifying the number of ways that the 4 As can be inserted separately into the spaces.

(b) The most successful approach to this part was considering the number of arrangements of the 8 remaining letters with the Ms fixed M ^ ^ ^ M ^ ^ ^ ^ ^ and then determining the number of places that the Ms can be positioned. The majority of candidates divided by 4! to remove the effect of the repeated As. Less successful approaches focused on how the As were positioned in relation to the Ms, with candidates not identifying all possible scenarios or not appreciating how their scenario allowed letters to be placed.

(c) Many candidates found this part challenging, and the question was not attempted by a surprising number. The best solutions listed the possible scenarios in a logical order, recognised that only the number of ways that the letters other than M and A are selected needed to be found and then evaluated the terms accurately. A small number of more able candidates recognised that the base requirement was MAA and sought to calculate the number of selections for the remaining letters without any restriction on the As which enabled repeated selections to be involved. Some candidates attempted the question without showing scenarios, but these were generally unsuccessful as terms were omitted.

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Question 7

This binominal and normal approximation question was quite standard in approach, so it was unexpected that so many candidates did not attempt **7(b)** or **7(c)**.

(a) Most candidates recognised that the normal standardisation formula needed to be used. Good solutions included a simple diagram of the normal curve to help identify the required probability area. A small number of candidates assumed that the conditions were symmetrical around the mean. An unusual, but frequent, error was finding the *z*-value correctly for $\Phi(0.4688)$ but the probability value when finding $\Phi(0.75)$. It was encouraging that more candidates found the correct area, although premature approximation created some issues with accuracy. The majority of candidates failed to complete the question and find the expected number of kestrels. When this value is found, it is anticipated at this level it will be interpreted to state an integer answer, rather than simply rounded, stated to a specific degree of accuracy (e.g. 2 significant figures) or shown as approximately equal to.

Candidates are well advised to read the question again when they finish their solution to ensure that they have met the requirements given.

(b) Over 10 per cent of candidates did not attempt this question. Good solutions used the normal standardisation formula to generate two equations from the information provided, used a clear algebraic method to solve these simultaneously and stated both the mean and standard deviation to at least 3 significant figures. Weaker solutions often equated the normal standardisation formula to a probability rather than a *z*-value or used the incorrect tail for the *z*-value.

(c) Over 15 per cent of candidates did not attempt this question. Almost all solutions recognised that the binomial approximation was appropriate and stated at least one correct term. Good solutions stated the required un-simplified terms and then evaluated with no intermediate steps, which were often the cause of errors or final answers outside the acceptable range. A small number of candidates used a *z*-value from **7(b)** for their probability. As in previous sessions, a common error was to include P(3) for the success criteria. Candidates are well advised to focus on interpreting success criteria in a variety of contexts in preparation for the exam. Very few candidates used the less efficient 1 – P(3,4, … 9, 10) approach.

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MATHEMATICS

Paper 9709/53
Probability & Statistics 1

Key messages

Candidates are to be reminded to show all necessary working clearly as no marks are given for unsupported answers from a calculator.

Candidates are to be reminded to read the question carefully to ensure that all parts have been addressed in their answer.

It is important that candidates realise the need to work to at least four significant figures throughout to justify a three significant figure value answer.

It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent; there is no requirement for probabilities to be stated as a decimal.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

It was encouraging that more candidates used a ruler to construct box-and-whisker plots and scales that enabled the five key-values to be plotted accurately. Candidates should be aware that the axes should be labelled with both the variable and the units.

Sufficient time seems to have been available for candidates to complete all the questions. Many good solutions were seen for **Questions 6** and **7**. The context of **Question 3(c)** was found to be challenging for many candidates.

Candidates should recognise efficient methods to solve the problems set to maximise their best use of time in the exam.

Comments on specific questions

Question 1

(a) Most candidates recognised the need to expand the summation to produce an equation in the variable *'k'*. This usually led to the correct exact answer. The best solutions showed clear algebraic manipulation of the linear equation needed for this.

(b) Most candidates recognised the opportunity to apply the formula given on the formulae sheet and went onto adapt this to find the variance leading to a correct value for the standard deviation. Very few candidates attempted the question by algebraic expansion and substitution of the appropriate values and of those that did, very few were accurate in applying this time-consuming approach.

Question 2

(a) Most candidates attempted one of the two efficient methods to find the correct probability. Those that attempted 1 – P(0) almost always achieved the correct final probability. Those that attempted P(1) + P(2) + P(3) occasionally forgot to apply the coefficients to each term. Candidates who

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attempted the inefficient method of listing all possibilities and adding the relevant probabilities were often not systematic and failed to identify all possibilities so did not arrive at the correct answer.

(b) Most candidates recognised this as a problem requiring the Binomial distribution and the most common approach was to add the probability of 8, 9 and 10 candidates. Those who opted for the longer calculation of subtracting all the probabilities of less than 7 candidates' completions were infrequent. Candidates should be reminded to use their calculator efficiently to achieve a final answer correct to 3 significant figures. A common error was to apply rounding to the calculated probabilities of each term leading to the most accurate sighted answer not in the acceptable range, often 0.09955.

Question 3

(a) This was a standard probability question which most candidates answered correctly and gave their answer to the appropriate degree of accuracy. This was the only question where some candidates used only 2 significant figures in their final answer and failed to show a more accurate value.

(b) Candidates again answered this question well with many choosing the most efficient $1 - \left(\frac{5}{6}\right)^8$ approach. Others who chose the full expansion of P(1), P(2), …. P(8) were also generally successful showing good calculator skills. A few candidates misinterpreted the success criteria and included the 9^{th} trial in their solution.

(c) This was the most challenging question on this paper with most candidates not able to fully grasp the context of the question. Successful candidates used a diagram to illustrate the possibilities and then worked out the probabilities. Not all candidates identified the total number of combinations for each of the four possible scenarios. Those who did not use a diagrammatic or listing approach to the four scenarios often found a single incorrect probability for their solution.

Question 4

This standard question on the reading and interpretation of a cumulative frequency graph was generally well done with almost all candidates attempting the box-and-whisker plot. A misconception by some candidates saw them attempting to draw the box-and-whisker plot based on the cumulative frequency and so gained no marks.

(a) There were no major problems for most candidates in calculating the estimate required here. A small minority of candidates indicated the two values required from reading the graph but then failed to find the difference successfully.

(b) In general candidates used an appropriate scale for their box-and-whisker plots. It was encouraging to see that a ruler was used in most plots. The constructing of a plot freehand is not appropriate at this level as an accurate representation of the data is anticipated. Most candidates were aware that a linear scale was required but often did not label this with both the variable and units (time and minutes). It was pleasing to see that very few candidates extended their whiskers through the box. The plotting of the three key values, median and the two quartiles was generally well done.

Question 5

This was a well answered question with most candidates able to construct a tree diagram for **5(a)** and attempt to find the three probabilities needed for the solution to **5(b)**. The final part of the question proved only slightly more difficult, and it was pleasing to see that many candidates showed a good theoretical understanding of conditional probability, even if some found it difficult to apply this in the context here.

(a) This question part was answered well, and most candidates produced a clear two-stage tree diagram with the outcomes and probabilities clearly labelled on each branch. The candidates who struggled with the question were often unable to establish a two-stage process and gave two separate trees for Bag A and Bag B. Others introduced an initial stage with probabilities of $\left(\frac{1}{2}\right)$

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leading to outcomes Bag A and Bag B. A common error was to omit the Green branch of the diagram for Bag B even when it had a non-zero probability.

(b) Most candidates realised that they needed to sum the probabilities of two Reds, Greens and Yellows. Those whose tree diagram was incorrectly structured were unlikely to be able to answer this question correctly.

(c) Most candidates who attempted this part used the conditional probability formula appropriately. Candidates often recalculated their P(BB) as their numerator with a few recognising that they had the value already used in **5(b)**. It was pleasing to see a high degree of accuracy in the use of calculators working in fraction mode. There is no requirement to convert an exact fractional value to an inexact decimal equivalent.

Question 6

(a) This standard normal distribution question was attempted well by most candidates, with over 90 per cent scoring full marks. A few candidates obtained the complement of the answer, and these solutions rarely had a sketch of the normal distribution curve to check if the value was reasonable. Using a sketch to help support finding the probability required would be recommended when using the normal distribution.

(b) The best solutions clearly stated the standardisation formula with the appropriate values substituted in or recognised that for a distribution looking for a probability that is 1.65 standard deviations above the mean suggests that $Z > 1.65$ directly. These candidates showed a clear understanding of the nature of the Normal distribution. A frequent error was to consider this problem as a '2-tail test' rather than just $P(Z > 1.65)$ thus getting a probability of 0.099. As in previous years a significant number of candidates did not attempt the last part of the question which asked them to take their probability and attempt to find the number of bags of pasta, and it is recommended that candidates do re-read the question to confirm that they have completed the task set before moving on. Candidates should be aware that they are interpreting the value found, rather than simply approximating or stating to a given number of significant figures.

(c) Most candidates used the tables correctly to find the z-values required and then linked them successfully to the appropriate standardisation formula. They then produced two equations in μ and σ and attempted to solve them simultaneously using a clear method.

Question 7

This permutations and combinations question was attempted well by most candidates by using one or more of the range of methods possible leading to the correct answers.

(a) The most common method was to calculate $7! - 6! \times 2$ although omitting the $\times 2$ was noted surprisingly frequently. The alternative multiplication method using 5! was also a popular approach. Stronger candidates who used the alternative $5! \times 6 \times 5$ (or $5! \times {}^6P_2$) were usually successful.

(b) This question was answered well by most candidates. A few candidates failed to multiply by 2 (required for swapping the blocks of men and women).

(c) This question was answered well by most candidates. A common error was to add the terms rather than multiplying, perhaps suggesting that knowledge of the and/or construct in such questions is not well embedded in some candidates' knowledge.

(d) Most candidates were able to find the number of arrangements for groups of 6 and 3 but some then failed to complete the problem. Good candidates expressed their total arrangements as a fraction of their answer to **7(c)** (although some re-calculated this) to find the required probability. Again, a significant number stopped before finding the probability and so did not actually answer the question. Very few candidates worked with probabilities throughout this question.

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MATHEMATICS

Paper 9709/55
Probability & Statistics 1

Key messages

Candidates must make their method clear when answering complex questions such as **Question 6c** but also always show their working throughout the paper. Candidates should be reminded to represent information on statistical diagrams in a visual way that is easy to understand and should include labels or units where appropriate.

General comments

Premature approximation resulted in inaccurate answers on many occasions in this paper. When finding final answers correct to at least 3 significant figures then the input numbers for further calculations must be accurate to 4 or more significant figures.

Comments on specific questions

Question 1

(a) The majority of candidates understood what was required and answered the question well. Probability distribution tables were well presented with most candidates giving the probabilities in fraction form, appreciating that working with decimals can be less accurate. The most successful approach was to set out a sample space diagram before calculating the probabilities and this helped to ensure that their probabilities had the correct denominator of 36. A significant number of candidates who did not use a sample space diagram thought that the denominator was 21, the sum of the numbers on the dice.

A few candidates misunderstood the question, the most commonly seen error being to ignore the last four words of the second bullet point and make X the ‘larger score’ if the two scores were not equal rather than ‘the larger score minus the smaller score’.

The value in checking that the probabilities sum to 1 in this type of question, cannot be underestimated.

(b) This question was answered well with most candidates appreciating the need to show their working in full. Almost all knew the formulae for E(X) and Var(X) and only a few mistakenly squared the probabilities in the Variance calculation or forgot to subtract the square of E(X). Those who prematurely approximated the E(X) as 1.94 and used that value in their calculation of Var(X) produced an inaccurate final answer and dropped marks.

Question 2

(a) Most candidates successfully applied the Normal standardisation formula with the mean equal to 20 and the standard deviation equal to 5, correctly identified the z-value as –1.2 and used tables to find the probability of a tree being classified as small. Careful candidates used a sketch of the Normal distribution and shaded the area to the left of –1.2, clearly demonstrating that they were looking for a probability less than 0.5.

A significant number forgot that they had been asked how many of 150 randomly chosen trees would expect to be classified as small and did not attempt the final part of the question. Of those

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who did try to finish the question, only those who read it carefully appreciated that their final answer needed to be an integer and not a decimal or an approximation.

(b) This question was answered well with most candidates knowing that they were dealing with a probability of 0.75, which is a critical value for the normal distribution, and we needed to see the z-value of 0.674. Only a few candidates used the tables the wrong way round and equated their standardisation formula to a probability instead of a *z*-value. As in the previous question, a sketch helped to achieve the correct answer as it demonstrated that the value of '*h*' was going to be greater than the mean value of 20.

Question 3

(a) This was a familiar style of question which required candidates to use the normal approximation to a Binomial distribution, and it was answered well. Most candidates remembered to apply a continuity correction, and almost all applied it in the correct direction. A few used the variance instead of the standard deviation in their standardisation formula and those who prematurely approximated the standard deviation to 4.38 forfeited accuracy in their final answer. As in **Question 2**, a sketch of the normal distribution with mean 32 and 27 marked to the left of the mean helped to make sure that their final answer was in the correct area and less than 0.5.

(b) The majority of candidates recognised the need to apply the binomial theorem with $n = 10$ and $p = \frac{2}{5}$. The words 'no more than 2' caused confusion for some but most realised that they needed to add the probabilities of there being 0, 1 or 2 candidates having music lessons.

A few incorrectly tried to apply a normal approximation in this part of the question as well as in **3(a)** and did not recognise that the condition for an approximation was not satisfied here as the number of candidates is too small ($10 \times \frac{2}{5}$ is less than 5).

Those who prematurely approximated the three probabilities before adding them, once again sacrificed accuracy in their final answer.

Question 4

(a) A surprising number of candidates did not seem to think about what they were representing in their diagram. If a candidate is accepted or rejected, he does not take the test again. Many candidates assumed that each branch of their tree had three branches at each of the three stages and ended up with 27 pathways which usually led to incorrect answers in the following parts of the question. Those who put more thought into the described situation realised that only the third option of 'required to take another test' produced further branches in the tree.

Although many candidates struggled to produce the correctly structured tree diagram, almost all understood how to attach the probabilities correctly to the appropriate branches.

(b) Candidates who had drawn tree diagrams with many extra branches usually struggled to produce the correct answer to this question. They needed to recognise that there were three ways that a candidate could achieve acceptance at the first, second and third attempt. We needed to see the sum of $0.3 + 0.5 \times 0.3 + 0.5 \times 0.5 \times 0.25$. This sum produces the exact answer of $\frac{41}{80}$ or 0.5125 and those who rounded the answer to 0.513 without showing the exact answer lost marks. The instruction on the front page asks for 'non-exact numerical answers' to be given to 3 significant figures.

(c) For this part of the question, candidates needed to recognise the need for the conditional probability formula. The numerator needed to be the probability that a candidate was accepted after 2 or 3 tests which many candidates appreciated would be their answer to **4(b)** minus 0.3, the probability of being accepted at the first attempt. Those who understood how to calculate a conditional probability usually recognised that the denominator would be the probability of the candidate being accepted, i.e. their answer to **4(b)**. Premature approximation of the answer to **4(b)** usually resulted in an inaccurate final answer.

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(d) Candidates who had made a good attempt at the previous two parts of the question usually realised that the probability of a candidate being rejected was 1 minus the probability of the candidate being accepted. However, a number of candidates started again and added the chance of being rejected at each of the three stages (0.2 + 0.5 × 0.2 + 0.5 × 0.5 × 0.75) which was an equally valid method. Some stopped at this stage and gave this probability of rejection as their final answer. Others incorrectly multiplied this answer by 3, applying the Addition Rule of probability rather than the Multiplication Rule and correctly cubing the probability of being rejected.

Question 5

(a) This question was answered well with most candidates understanding what was required in drawing a back-to back stem-and- leaf diagram. The instruction to put the Smarts on the left-hand side was followed by most and almost all were careful to line up their leaves even if they had to make a correction when writing out the leaves. Very few mistakenly inserted commas between the leaves and almost all remembered to label the two sides with Smarts and Teasers. A few struggled with how to deal with the single digit value in the data and either omitted the 0 in the stem or occasionally put the 9 in the stem rather than as a leaf. The most challenging part of the question was providing a key with the correct units. A significant number of candidates did not attempt a key, forgetting that a statistical diagram needs to tell a story to its readers. A few candidates repeated the same digit for Smarts and Teasers in their key (e.g. 1/2/1) which introduced unnecessary ambiguity.

(b) Careful candidates realised that they needed to identify the median and quartiles for the Smarts team before drawing the box-and-whisker plots and listed them (9, 13 and 22) before starting the diagram. Although most candidates understood what was required in this question, disappointingly few produced two accurate plots labelled ‘Smarts’ and ‘Teasers’ above or below a scaled line with at least 3 values stated and the labels ‘time’ and ‘minutes’.

(c) This proved to be one of the least well answered questions on the paper. Candidates were asked to make comparisons between ‘the times for the two teams’. Comparisons of statistical measurements such as medians, interquartile ranges or ranges was not enough to answer the question. There needed to be a reference to ‘time’ in both comparisons and one needed to be about which team was faster or took more or less time and the other needed to be about consistency of time.

As the median for the Smarts team (22) was lower than the median for the Teasers (25) it was clear that the Smarts team were faster or took less time or that the Teasers team were slower or took less time.

Comparing consistency was a little more complicated as while the Smarts team had a lower interquartile range, the Teasers team had a lower range. We normally expect candidates to compare the interquartile ranges for their comment on consistency and so we hoped for comments such as ‘Smarts’ times are more consistent’ or ‘Teasers’ times are less consistent’. If they compared ranges and produced the opposite statement, we needed to see the justification for the comment such as ‘Smarts times have a larger range and so are less consistent’. We insisted on the inclusion of a reference to time and so ‘Smarts are more consistent’ was not accepted as a valid answer.

Question 6

(a) The information in this question was well understood and generally well answered. The most commonly seen approach was to use combinations with only a few ignoring the request for a probability and giving their final answer as 7C1 × 5C3 = 70. Those who used a breakdown of probabilities as their method usually appreciated the ‘without replacement’ aspect and produced a product of 4 probabilities with denominators 12, 11, 10 and 9. However, they often forgot that there were 4 ways that this selection could be made and that they needed to multiply their product by 4.

(b) The information that ‘each team enters a competition in a different town’ caused some confusion, particularly in **part (c)**. In this part of the question, the majority took the meaning as intended and successfully found the number of possible combinations sent to the three towns. Common errors seen were multiplying the correct answer by 3! or only dealing with the first town and multiplying

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12C4 by 3. A few candidates found the correct combinations for the towns and added them rather than multiplying.

(c) Despite there being only 3 marks available for this question, it proved to be the most challenging on the paper. Strong candidates had a clear strategy and knew to explain their working in full. The most successful approach was to list the 2 possible distributions of men and women in the three towns, multiply together the 6 combinations (2 for each of the 3 towns) for each distribution and then multiply each total by 3!/2! before adding.

Those who chose to subtract the number of ways the teams could be chosen where at least one team had all men or all women from the total number of ways were less successful. They usually ignored the overlap case where the distribution contained a team of all men and a team of all women.

The most commonly seen incorrect response was to consider the ways the teams could be chosen for one town only (3M 1W, 2M 2W, 1M 3W) by summing three 2-term products of combinations.

(d) This question was well answered by the majority with the most common error being to forget to deal with the switch around of Tom and Harry.

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MATHEMATICS

Paper 9709/61
Probability & Statistics 2

Key messages

- In all questions, sufficient method must be shown to justify answers; unsupported correct answers will not gain full credit.
- It is important that candidates read the question carefully and extract the relevant information.
- Candidates need to work to the required level of accuracy of 3 significant figures. To maintain 3 significant figures accuracy in a final answer, all intermediate working must be to at least 4 significant figures.
- All working should be done in the correct question space of the answer booklet. If answers need to be continued on the additional page, all working must be clearly labelled with the correct question number.
- Clear presentation of work is of vital importance; in particular digits must be clear and unambiguous.

General comments

Candidates were not always fully prepared for the demands of this paper. Questions where candidates performed well were **Questions 1** and **3(a)** and questions that candidates found more demanding were **Questions 2(b)**, **6(b)** and **7(b)**.

On the whole candidates presented their answers well and with sufficient working; it is important that sufficient method is shown, for example if a probability is being calculated using a Posson distribution then the term or terms in the Poisson expression must be seen.

Candidates must note that conclusions to a Hypothesis test must be written in context and with a level of uncertainty in the language used.

Timing did not appear to be an issue, and presentation was generally acceptable.

Comments on individual questions follow which identify common errors, though it should be noted that there were some good and fully correct solutions seen as well.

Comments on specific questions

Question 1

This was a reasonably well attempted question. Many candidates used a suitable approximating distribution Po(4) and reached the correct probability. It is important for candidates to note here that the full Poisson expression should be shown. Errors included omission of the final term in the Poisson expression, use of a Normal distribution instead of Poisson, and some candidates used a Binomial distribution.

Question 2

(a) For this question $N(6, \frac{1.5}{100})$ was required. Many candidates found $\mu = 6$, but the main error seen was to use N(6,1.5) and fail to divide by 100. Other errors included finding the wrong area for $\overline{X} > 6.2$, and there were some candidates who did not know how to approach the question.

(b) There was a general lack of understanding of the Central Limit Theorem shown here, with few candidates able to give the required answer.

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Question 3

(a) This was a particularly well attempted question. Many candidates correctly found the unbiased estimates of both the population mean and the population variance. There were a few candidates who confused the two different formulae for the unbiased estimate for the variance and there were a few candidates who found the biased estimate.

(b) This part was not quite so well attempted. The hypotheses were not always correctly stated using μ or 'population mean'. When standardising, $\sqrt{60}$ was often omitted and a valid comparison with 1.645 (or an area comparison with 0.05) was not always stated. The conclusion to the test should be written in context and in non-definite terms.

Question 4

(a) In this part, many candidates found the correct mean (6) but when calculating the variance, a common error was to give an answer of 3.6 (0.04×9) rather than 1.44 ($0.04^2 \times 9$).

(b) Again, many candidates found the correct mean but made errors when finding the variance.

Question 5

(a) (i) This part was reasonably well attempted. Many candidates used Po(3.5) and found P(3,4,5). Some candidates found P(4), and others unsuccessfully attempted calculations using Po(1.2) and Po(2.3).

(ii) Many candidates realised that N(580,580) was the approximating distribution required. Errors included omitting or using an incorrect continuity correction when standardising and finding the wrong probability area. Some candidates did not know how to approach the question.

(b) Some candidates successfully stated the correct equation using Poisson probabilities (though incorrect use of brackets was often seen). Some candidates successfully simplified their equation and found a quadratic equation in λ which could be solved and one solution rejected.

Question 6

(a) Many candidates stated correct hypotheses, but some used μ or x rather than p, or 5 instead of 0.25.

(b) Finding the probability of a Type II error caused problems for many candidates. Use of B(30,0.1) was required to calculate $P(X \geqslant 5)$. Many candidates omitted the question entirely, and of those who made an attempt, the main error was to use an incorrect value for p (0.25 rather than 0.1).

(c) This part proved easier for many candidates. The correct z value of 1.96 was often used, and there were many correct attempts seen to set up the confidence interval. Candidates should note that the final answer should be an interval.

Question 7

(a) Many candidates realised that they needed to integrate $f(x)$ and many stated correct limits. However, few candidates knew how to integrate $\cos(\pi x)$ correctly.

(b) Again, many candidates realised that they needed to integrate $xf(x)$ but were unable to integrate successfully.

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Paper 9709/62
Probability & Statistics 2

Key messages

- Candidates should be encouraged to read questions carefully and extract the relevant information.
- Candidates should be encouraged to reread the question when they have completed their solution to ensure that all the requirements have been fulfilled.
- Candidates should be reminded that in all questions, sufficient method must be shown to justify answers.
- When the question requires an answer to be shown, it is particularly important that all relevant working is presented with no steps in the solution omitted.
- Candidates need to keep to at least 4 significant figures accuracy during working in order to ensure a final answer is correct to 3 significant figures.
- If questions are continued on additional pages, the question number must be clearly identified.

General comments

This was a reasonably well attempted paper. There were some very good and well-presented scripts, but there were also some candidates who were not fully prepared for the demands of the paper. Questions that were well attempted were **Questions 2, 3(a), 6** and **7** whilst **Questions 4, 5** and **8** were found to be more demanding. On the whole adequate working was shown, but there were occasions where candidates did not fully justify their answers.

It should be noted that the conclusion to a hypothesis test must not be written in definite terms (i.e. a phrase such as "There is sufficient evidence to suggest…." should be used rather than "The test proves that ….")

Many candidates attempted all the questions; there did not appear to be any issues with timing.

Comments on individual questions follow, but it should be noted that there were some good fully correct solutions offered as well as the common errors highlighted below.

Comments on specific questions

Question 1

(a) Many candidates did not answer this question in enough detail. For example, merely saying that each candidate was allocated two numbers from the dice was not sufficient as it did not exclude the possibility that numbers could be repeated leaving omissions i.e. a side or sides of the dice not allocated to a candidate. Candidates who gave an example of how the sides of the dice could be allocated were generally more successful. Some candidates gave a method which required more than a single throw, which was not acceptable.

(b) This was a reasonably well attempted question. Most candidates found the correct mean and variance and standardised correctly to reach 0.115. Many candidates then left this as their final answer.

Question 2

This was particularly well attempted. There was the usual confusion between the different formulae for the unbiased estimate of the population variance, but on the whole candidates were successful in finding Σh^2. Use of the formula for the biased estimate was occasionally seen.

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Question 3

(a) This was well attempted. A correct expression was stated by most candidates.

(b) Candidates were, in general, able to write down the correct equation in *n*. However, finding the value of *n* involved simplifying factorials and powers, and many candidates were unsuccessful in doing this. Errors such as $\frac{n!}{(n+1)!} = \frac{n}{(n+1)}$ were commonly seen.

Question 4

There was some confusion on this question between the number of times the spinner landed on red, and the proportion of times it landed on red (defined by '*a*' in the question). Consequently, many candidates incorrectly gave their final answer as $a = 60$ or $a = 140$ rather than correctly stating the proportion which was $a = 0.300$ or $a = 0.700$. Many candidates used a correct equation for the width of the confidence interval, usually with a correct value for *z* and were able to reach a quadratic equation in *a* (though algebraic errors were sometimes made when rearranging their equation). Errors were also made in solving the quadratic equation, and some candidates did not keep to the required level of accuracy.

Question 5

On this question, although candidates were requested to state a necessary assumption very few did; candidates either did not read the question carefully enough and did not realise that an assumption was required, or they did not know the assumption to state. Many candidates used the correct value for *z* and standardised correctly to find the required value. However, errors included omission of $\sqrt{10}$ when standardising and standard deviation/variance confusion.

Question 6

(a) (i) Many candidates realised that Po(3.5) was a suitable approximating distribution, and many gained full marks. However, some candidates did not find an approximating distribution and used B(700, 0.005), and some attempted to use a Normal distribution. It is important that candidates know which approximating distribution is suitable.

(ii) Again, candidates using Po(7) were usually successful on this part of the question (though inclusion of an extra term in their Poisson expression was occasionally seen). Binomial and Normal distribution attempts were seen on this part as well. Some candidates attempted a combination method using Po(3.5), which could lead to the correct answer, but was time-consuming and candidates did not always identify all the combinations; this is not a method to be recommended.

(b) This was well attempted. The correct approximating distribution here, N(200, 200), was used by many candidates; a common error noted was an incorrect or missing continuity correction.

Question 7

(a) Many candidates attempted to integrate f(*x*) and equate to 1. It is important when the answer is given that candidates show enough working to convincingly reach the required answer. On the whole this was done well. Candidates should be reminded that once an expression has been integrated, the integration sign should no longer be used and should be replaced by brackets.

(b) Many candidates realised they needed to integrate *x*f(*x*) and many reached the correct answer. Errors when rearranging their equation were seen, and once again there was misuse of integration signs.

(c) This part proved to be slightly more challenging for some candidates. Incorrect limits, and errors when calculating $\frac{k}{a^2}$ were often seen.

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Question 8

(a) The significance test was performed reasonably well by candidates. It was important that in the Binomial expression for the probability of $\leqslant 2$, all terms were seen. A common error was to merely calculate a single term (i.e. probability of exactly 2). A valid comparison with 0.05 should be clearly shown, and a conclusion to the test should be written in context and in non-definite terms. Not all candidates were aware of how the conclusion to a significance test should be expressed.

(b) Most candidates used $B(30, \frac{1}{6})$ but not all found $P(\leqslant 1)$. Some candidates did not give an answer to the required degree of accuracy.

(c) Again, most candidates used $B(30,0.02)$ but not all candidates found $1 - P(\leqslant 1)$. Errors included use of $B(30, \frac{1}{6})$, or calculating $1 - P(\leqslant 2)$.

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MATHEMATICS

Paper 9709/63
Probability & Statistics 2

Key messages

Throughout the paper it was necessary to show complete methods and full working.

General comments

For questions involving a significance test, the conclusion does need to follow certain guidelines. In particular it should not be expressed in definite form. Many candidates do use the recommended wording such as 'there is evidence to suggest that …'. Wording such as 'has been proved' is undesirable as this would imply a definite result.

Comments on specific questions

Question 1

(a) The new Poisson parameter for a time period of 1-minute was required (0.39) and then the probability that at least 2 customers arrive (1 – P(0 or 1). Many candidates found these correctly and showed the necessary terms. A few candidates incorrectly included the extra term P(2).

(b) (i) As the Poisson parameter value was 23.4 (> 15) the suitable approximating distribution was a normal distribution. The required parameters were the mean and variance, each of which had the value 23.4.

(ii) For the requested change from the Poisson distribution to the normal distribution, a continuity correction was required (20.5 and 29.5). From the two standardisations the two probability values had to be combined suitably to find the area between the given *X* values (for example by finding $p_1 + p_2 - 1$). A number of candidates did this correctly. Other candidates used incorrect continuity corrections and some candidates combined their probabilities incorrectly.

Question 2

(a) (i) In this question the value of sigma was not given, but the value of the test statistic was given as 1.995. Correct standardisation was required, involving $\sqrt{50}$. The rearrangement of the terms could then give the value of sigma. Many candidates did this correctly. Other candidates made errors with the variance or with the rearrangement.

(ii) The basic steps of the significance test were required here and the two hypotheses were needed. If these had already been given in **2(a)(i)** the mark was allowed here. The comparison of the test statistic (*z*) and the relevant critical value (1.995 > 1.96) was required, or the comparison of the relevant probabilities (0.023 < 0.025). Some candidates re-calculated the standardisation to obtain $z = 2.00$ and used this value in the comparison. This was allowed. Then the conclusion was needed, and this had to be given in a suitable form as indicated in the mark scheme.

(b) An answer of 'no' and a relevant reason were required.

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Question 3

(a) (i) Correct formulae had to be shown to be used to calculate the unbiased estimates. Many candidates did this correctly. A few candidates tried an incorrect formula or found only the biased variance.

(ii) To find the 95 per cent confidence interval required, use of the mean of samples with variance $\frac{595}{150}$ and the relevant *z* value of 1.96 was required. Many candidates used these correctly and found the required interval. Some candidates omitted the 150 or the square root. Other candidates used an incorrect *z* value.

(b) A complete reason was necessary here. This could be that the new sample size ($100 < n < 120$) was smaller than the previous size of 150 leading to the value of $\frac{595}{n}$ being larger and hence that the confidence interval width would be larger or wider. Some candidates gave this reasoning completely. Other candidates stated only that '*n* was smaller', which was not sufficient.

Question 4

(a) With one throw of two fair coins the probabilities of 0 heads, 1 head, or 2 heads were $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ respectively. As these probabilities were not equal this method was not random. To score the marks it was necessary to calculate all of these probabilities correctly.

(b) With one throw of a fair six-sided dice, it was necessary to indicate the relevant people pairings such as PQ, QR, RP and to state how these would be connected to the scores on the dice. For example, the connections could be dice 1 or 4 choose group P and Q; dice 2 or 5 choose group Q and R; dice 3 or 6 choose group R and P.

Some candidates described this process accurately and completely so that the choices were fair and equally likely. Other candidates did not give a complete process. Some candidates paired the dice scores with only a single person as their choice, not the required two people. Other candidates referred to several throws of the dice which was not as requested.

Question 5

The necessary assumption was that the men on the bridge were a random sample.

From the given critical probability (0.01) it was necessary to find the corresponding *z* value (2.326). It was then necessary to consider the normal distribution of the *n* men, namely $N(70.3n, 5.9^2 n)$. Standardisation could then lead to a quadratic equation in $\sqrt{n}$ or equivalent. Solving this equation could then lead to the relevant value for *n*. To find the final value it was necessary to consider progressing from the decimal value obtained from the equation to finding the maximum value for *n*. Correct working was essential. Many candidates made errors in trying to find the distribution for the *n* men.

Question 6

(a) To answer this question each of the two methods involved the use of the property that the area under the probability density function graph is equal to 1. The majority of candidates used integration of f(*x*) between the limits 0 and b to find this area. Some candidates drew a sketch and found the area of the triangle in terms of a and b. For either method rearrangement then produced the required expression. Many candidates followed their method correctly.

(b) There were two stages to answering this question. The first stage required the expectation to be found. This required the integration of *xf*(*x*) between the limits 0 and b. Many candidates did this correctly. The second stage required the integration of f(*x*) between the limits 0 and E(*X*) or the use of a sketch to find the relevant area. Some candidates did this correctly. Other candidates used the wrong limits. A few candidates instead tried to find the variance which was not relevant.

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Question 7

(a) To establish the largest value of *r* required candidates to find the two relevant probability sums, P($\leqslant$3) and P($\leqslant$4) using the binomial distribution B(35, 0.25). It was also necessary to show their relevance by comparing with 0.04. The final mark depended on calculating all of these correctly.

(b) Describing the meaning of a Type I error here required the form of a faulty conclusion and a true hypothesis. These needed to be expressed in context. Correct work carried out in **7(a)** would have established that the critical region was $X \leqslant 3$.

(c) Given that the new probability was 0.05, this work required the use of the Binomial distribution B(35, 0.05). For a Type II error the director would be accepting the new null hypothesis and using P(X > 3) by calculating 1 – P($X \leqslant 3$).

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MATHEMATICS

Paper 9709/65
Probability & Statistics 2

Key messages

- In all questions, sufficient method must be shown to justify answers.
- It is important that candidates read the question carefully and refer back to it when they have completed the question to ensure they have answered it in full.
- Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- For answers that are required 'in context', quoting general textbook statements will not be sufficient.
- All working should be done in the correct question space of the answer booklet. If answers need to be continued on the Additional page, it must be clearly labelled with the correct question number.
- Candidates should make corrections by crossing through and replacing the work, not by over-writing their answer.
- Only one solution should be offered.

General comments

The paper reflected a generally competent effort, with candidates showing a sound understanding of statistical methods and their practical use. Candidates performed better in **Questions 1**, **4**, and **5**, whereas they found **Questions 2(b)**, **3(c)**, **6(b)** and **7(c)** more challenging.

The working was adequate, especially in answering **Question 6(a)**. However, a significant proportion of candidates omitted to write all terms in the expressions used to work out the required probabilities in **Questions 1(a)** and **4(a)**. Also, candidates must note that if a question asks to show a given result, then they are required to show enough steps to convince the examiner that they are able to obtain the result, like in **Question 1(b)**.

Comments on specific questions

Question 1

(a) This part question was answered well by most candidates. A common error was to use the Poisson distribution with parameter 1.5 instead of 4.5. Some candidates did not write all terms in the Poisson distribution.

(b) Many candidates had no problems in writing the expressions for the probabilities $P(S \leq 1)$ and $P(S \leq 2)$. The best answers clearly showed how to obtain the given answer. Candidates must note that if a question asks for an exact answer, then they should not express an irrational number like $e^{-4.5}$ as a decimal, as the decimal representation of an irrational number is not exact, even if one writes all the digits of their calculator's display.

Question 2

(a) This part question required the candidates to obtain unbiased estimates for the mean and the variance of x, and to use them to find the required confidence interval. Many candidates were able to do that, often in a very efficient way. A few candidates calculated the variance from first principles, showing good algebraic manipulative skills, but using a rather inefficient method. Common errors considered the biased estimate for the variance, or an incorrect z, or even t value. Another common error was to write the estimate for the mean of x as 0.3 instead of 2.3. A few

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candidates calculated the confidence interval for $x - 2$ and then added 2 to both extremes to obtain the correct final answer.

(b) Many candidates found this part question very challenging. They often stated the conditions for the applicability of the Central Limit Theorem or mentioned the distribution of X.

Question 3

(a) The question required to round up the result of the multiplication, and the most common error was to not do this.

(b) Many candidates answered this question correctly, including both random numbers. A common error was to consider only 0.798 and not 0.799 too.

(c) The candidates who answered **3(b)** correctly, usually also answered this part well. A common error was to round up 850 ÷ 851, thus obtaining 0.999 instead of the correct 0.998.

Question 4

(a) Most candidates answered this question correctly, showing their working well. The most common error was to add the expressions $1 - e^{-4}\left(1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!}\right)$ and $1 - e^{-2.5}(1 + 2.5)$ instead of multiplying them.

(b) This part question was well attempted by most candidates. The most common error was to incorrectly apply the continuity correction, or not to apply it at all.

Question 5

This question was well attempted by many candidates. The most common error was in the calculation of the variance of F, where some candidates did not multiply 3.8 by 2^2. Another error was to omit the division of their variance by 25 in the standardisation. Some candidates rounded their answer prematurely. Candidates should be reminded that when providing answers to 3 significant figures, the working should be of at least 4 significant figure accuracy to ensure their final answers are to the required 3 significant figures accuracy.

Question 6

(a) This part question was well attempted by most candidates, who correctly wrote the hypotheses, showed all the calculations in full, performed a suitable comparison, and finally drew the appropriate conclusions. A typical error was to write a conclusion that was either definite ('the percentage is less than, …' instead of e.g., 'there is evidence that the percentage is less than …), or not in context ('there is evidence that the probability has decreased', instead of e.g., 'there is evidence that the percentage who support Forward Now is less than 28%').

(b) This part question proved challenging for most of the candidates, who could not make the link between the fact that a Type I error requires H_0 to be rejected, and that this is exactly what happened in **6(a)**. Many candidates either stated the definition of a Type I error or said that in **6(a)** H_0 was rejected but omitted to say whether this meant that a Type I error was possible. Other candidates answered yes, but without providing a valid reason for their answer. Some answers were excellent, using accurate language and correct logical arguments, and showed a good understanding of the topic.

(c) Many candidates realised that, to answer this part question in full, it was not enough to just write the probability of a Type I error, but that they also had to check that $P(X < 5) = 0.116 > 0.1$ in order to fully justify their answer.

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Question 7

(a) This part question was answered correctly by most candidates. The most common error was to use of an incorrect parameter (typically X or $\bar{X}$ instead of μ), or to state 'mean' instead of 'population mean'.

(b) This question was generally well attempted, with most candidates performing a correct standardisation and comparison. Most of the candidates who wrote the conclusion (which was not required), did not incur contradictions.

(c) This question was not particularly well attempted, even though a good proportion of candidates realised that the answer involved $1 - \phi(1.845) = 0.0325 = 3.25\%$. The most common errors were to consider a single value instead of an interval ($\alpha = 3.25$ instead of $\alpha > 3.25$), or to consider $\alpha\%$ instead of α ($\alpha > 0.0325$ instead of $\alpha > 3.25$).

Question 8

(a) This part question was generally well attempted, using a variety of approaches (area of a triangle, integration, slope of the graph of the probability density function). The most common error was to consider the median as the midpoint between the origin and a, and to incorrectly deduce that $a = 2\sqrt{2}$.

(b) The candidates who answered **8(a)** correctly generally did well on this part too. Many candidates easily found the correct value of a and used it to find the value of $E(X)$. The candidates who assumed that $a = 2\sqrt{2}$ were able to gain some credit for a correct method.

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