FURTHER MATHEMATICS

Paper 9231/11
Further Pure Mathematics 1

Key messages

Candidates should read each question carefully so that they use all the information given and answer all aspects in adequate depth.

When asked to prove a given result, candidates should take particular care to give every step in the process to reach the result.

Proper use of brackets helps avoid errors with signs and can simplify working in both algebra and arithmetic.

When working with inequalities, candidates must ensure that they consider the sign when multiplying by an expression which could be negative.

All sketch graphs need to be fully labelled and carefully drawn to show significant points and behaviour at limits.

General comments

The majority of candidates demonstrated good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. It seemed that almost all were able to complete the paper in the time allowed.

Comments on specific questions

Question 1

(a) This was mostly done well. A few did not use 10n, and others made mistakes in simplification. The best solutions contained clear step-by-step working with appropriate use of brackets and awareness of signs.

(b) Almost all candidates found the correct partial fractions.

When writing out the terms, the best solutions simplified to deal with the minus signs. Then they used the space to write successive terms beneath each other to show the terms that cancel. A small number simply assumed their first and last terms were left without checking for the appropriate cancellation.

(c) This followed correctly from **part (b)**.

Question 2

(a) This was mostly done well. There were two main methods used. Substitution of a surd followed by simplification, or rearrangement to a form including x^3 followed by substitution, were equally successful. Most demonstrated understanding of the need to isolate the surd term prior to cubing.

A few tried using the roots of the original equations to create the roots of the new equation but were largely unsuccessful.

(b) The result usually followed correctly from their answer for **part (a)**.

(c) Most candidates used $S_3 + 6S_2 + 20S_1 + 16S_0 = 0$ accurately.

A few tried to use formulae but these formulae were often wrongly remembered, or incorrect values were used.

Question 3

(a) Most candidates demonstrated understanding of the basic structure of a proof by induction. Successful candidates showed the base case, several went on to check the result for $n = 2$ but this is not needed.

The inductive hypothesis needs to state the result that is to be proved in terms of the formula, and to assume that it is true for some particular value, such as k. A few assumed the result for all integers.

Weaker candidates tried to prove the given recursion formula, rather than the form of the kth term. Most candidates did the appropriate algebra and wrote the term in the form $u_{k+1} = 6^{k+1} + 1$.

The best solutions gave a final conclusion showing the formula that has been proven, and stated that it is true for all positive integers.

(b) Those who recognised u_{2n} as the difference of two squares and factorised to give $u_{2n} = (6^n + 1)u_n$ proved the result very quickly. Many failed to recognise the factorisation.

There was also a successful version of the solution using the difference between u_{2n} and u_n, and then showing that division must work.

Question 4

(a) The types of rotation were usually correct, and they were given in the right order. Some candidates went on to give full descriptions of the transformations. This was not required by the wording of the question.

(b) Most candidates appeared to not read this question carefully and looked for invariant lines rather than the line of invariant points that was needed.

Almost all could calculate the matrix M and apply it to a general point.

To find a line of invariant points candidates needed to write down $M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ to give two equations connecting x and y in terms of θ. Then they could eliminate either x or y to give an equation in θ. There were some excellent solutions which reached the condition $\sin\theta + \cos\theta = 1$ and solved it to give $\theta = \frac{\pi}{2}$.

Question 5

(a) Largely done quite well; there were a very few closed or incomplete curves but most sketched a spiral, clearly showing the pole and the initial line. The best sketches made it clear that r is strictly increasing with θ.

(b) The correct integral was usually written down, only a small number omitted the factor of $\frac{1}{2}$ or the limits, or got wrong limits.

The integral needed integration by parts to be applied twice and the process is clearly well understood.

The best solutions wrote out their u, v, u' and v' explicitly and this enabled them to see that their constant terms were correct.

For example $u = \theta^2 \quad \frac{dv}{d\theta} = e^{\frac{1}{4}\theta}$

$$\frac{du}{d\theta} = 2\theta \quad v = 4e^{\frac{1}{4}\theta}$$

When the first integration of parts was correct, errors often appeared in the second application and in the final substitution because of poor use of brackets.

(c) Most candidates used the correct formula for y and the differentiation was usually shown correctly. Candidates were proving a given result, and need to show all steps to gain full marks. Only the best solutions remembered to state that they could divide by $e^{\frac{1}{8}\theta}$ because it is non-zero.

Most calculated correct numerical values for $\theta\cos\theta + (\frac{1}{8}\theta + 1)\sin\theta$ at $\theta = 5$ and $\theta = 5.05$ to show the sign change to locate the root.

Question 6

The best solutions of vector questions label the vectors clearly to avoid confusion. Careful checking of arithmetic helps to minimise errors which can make later working not only inaccurate but also more difficult.

(a) This was usually done by the cross-product and the majority produced a correct equation for the plane. A few candidates made errors in calculating two vectors lying in the plane, and this affected the whole of the question.

(b) The normal to plane *ABD* was usually found in the form $\begin{pmatrix} 2t+4 \\ 0 \\ 0 \end{pmatrix}$ and the best solutions realised that it could be rewritten as $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ because it was only the direction that was needed. Those who worked with the factor $(2t + 4)$ in place managed to find the numerical answer with rather more work.

(c) This part needed particular care over accuracy. Most candidates knew the formula and many applied it properly and found a quadratic equation for t. Problems arose when candidates used the wrong vectors; they would find it helpful to label the ones they are using. This would make it easier for them to decide whether it is just the direction that is important or whether both the magnitude and direction are needed.

Question 7

(a) All three asymptotes were usually correctly written as equations.

(b) Many used the quotient rule correctly to differentiate the fractional form. Then there were sometimes algebraic mistakes which led to wrong stationary points. It can be useful to check that the x coordinates of the stationary points satisfy the unsimplified $\frac{dy}{dx} = 0$ equation before proceeding.

Those who divided through to get an expression in the form $y = 1 + \frac{2x+4}{2x^2 - 7x - 4}$ before differentiating made fewer errors.

(c) When drawing a graph candidates should use a ruler for straight lines, and clearly label the axes and the asymptotes. For this graph the left branch needed to cross the asymptote, take the minimum value found in **part (b)** and then approach the asymptote from below.

Some candidates missed that the question asked for the coordinates of intersections with axes. When drawing a graph it is helpful to mark on the intersections and any maximum or minimum to help to decide the shape and position.

(d) The idea of reflecting the part of the graph where $y < 0$ in the x axis was well applied and the points (0, 0) and (2.5, 0) showed the correct sharp turn.

(e) The position of the vertical asymptotes showed that $2x^2 - 7x - 4$ took positive, zero and negative values. It was common for candidates to multiply an inequality by this quadratic without looking at the effect on the inequality sign. Good solutions worked with equations, found the critical values for x and then looked at the graph to give the final inequalities. While it was sensible to find decimal equivalents for the critical values to position them in the correct order, the question asked for exact values for the final inequalities.

FURTHER MATHEMATICS

Paper 9231/12
Further Pure Mathematics 1

Key messages

Candidates should read each question carefully so that they use all the information given and answer all aspects in adequate depth.

When asked to prove a given result, candidates should take particular care to give every step in the process to reach the result.

Proper use of brackets helps avoid errors with signs and can simplify working in both algebra and arithmetic.

When working with inequalities, candidates must ensure that they consider the sign when multiplying by an expression which could be negative.

All sketch graphs need to be fully labelled and carefully drawn to show significant points and behaviour at limits.

General comments

The majority of candidates demonstrated good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. It seemed that almost all were able to complete the paper in the time allowed.

Comments on specific questions

Question 1

(a) This was mostly done well. A few did not use 10n, and others made mistakes in simplification. The best solutions contained clear step-by-step working with appropriate use of brackets and awareness of signs.

(b) Almost all candidates found the correct partial fractions.

When writing out the terms, the best solutions simplified to deal with the minus signs. Then they used the space to write successive terms beneath each other to show the terms that cancel. A small number simply assumed their first and last terms were left without checking for the appropriate cancellation.

(c) This followed correctly from **part (b)**.

Question 2

(a) This was mostly done well. There were two main methods used. Substitution of a surd followed by simplification, or rearrangement to a form including x^3 followed by substitution, were equally successful. Most demonstrated understanding of the need to isolate the surd term prior to cubing.

A few tried using the roots of the original equations to create the roots of the new equation but were largely unsuccessful.

(b) The result usually followed correctly from their answer for **part (a)**.

(c) Most candidates used $S_3 + 6S_2 + 20S_1 + 16S_0 = 0$ accurately.

A few tried to use formulae but these formulae were often wrongly remembered, or incorrect values were used.

Question 3

(a) Most candidates demonstrated understanding of the basic structure of a proof by induction. Successful candidates showed the base case, several went on to check the result for $n = 2$ but this is not needed.

The inductive hypothesis needs to state the result that is to be proved in terms of the formula, and to assume that it is true for some particular value, such as k. A few assumed the result for all integers.

Weaker candidates tried to prove the given recursion formula, rather than the form of the kth term. Most candidates did the appropriate algebra and wrote the term in the form $u_{k+1} = 6^{k+1} + 1$.

The best solutions gave a final conclusion showing the formula that has been proven, and stated that it is true for all positive integers.

(b) Those who recognised u_{2n} as the difference of two squares and factorised to give $u_{2n} = (6^n + 1)u_n$ proved the result very quickly. Many failed to recognise the factorisation.

There was also a successful version of the solution using the difference between u_{2n} and u_n, and then showing that division must work.

Question 4

(a) The types of rotation were usually correct, and they were given in the right order. Some candidates went on to give full descriptions of the transformations. This was not required by the wording of the question.

(b) Most candidates appeared to not read this question carefully and looked for invariant lines rather than the line of invariant points that was needed.

Almost all could calculate the matrix M and apply it to a general point.

To find a line of invariant points candidates needed to write down $M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ to give two equations connecting x and y in terms of θ. Then they could eliminate either x or y to give an equation in θ. There were some excellent solutions which reached the condition $\sin\theta + \cos\theta = 1$ and solved it to give $\theta = \frac{\pi}{2}$.

Question 5

(a) Largely done quite well; there were a very few closed or incomplete curves but most sketched a spiral, clearly showing the pole and the initial line. The best sketches made it clear that r is strictly increasing with θ.

(b) The correct integral was usually written down, only a small number omitted the factor of $\frac{1}{2}$ or the limits, or got wrong limits.

The integral needed integration by parts to be applied twice and the process is clearly well understood.

The best solutions wrote out their u, v, u' and v' explicitly and this enabled them to see that their constant terms were correct.

For example

$$u = \theta^2 \quad \frac{dv}{d\theta} = e^{\frac{1}{4}\theta}$$
$$\frac{du}{d\theta} = 2\theta \quad v = 4e^{\frac{1}{4}\theta}$$

When the first integration of parts was correct, errors often appeared in the second application and in the final substitution because of poor use of brackets.

(c) Most candidates used the correct formula for y and the differentiation was usually shown correctly. Candidates were proving a given result, and need to show all steps to gain full marks. Only the best solutions remembered to state that they could divide by $e^{\frac{1}{8}\theta}$ because it is non-zero.

Most calculated correct numerical values for $\theta\cos\theta + (\frac{1}{8}\theta + 1)\sin\theta$ at $\theta = 5$ and $\theta = 5.05$ to show the sign change to locate the root.

Question 6

The best solutions of vector questions label the vectors clearly to avoid confusion. Careful checking of arithmetic helps to minimise errors which can make later working not only inaccurate but also more difficult.

(a) This was usually done by the cross-product and the majority produced a correct equation for the plane. A few candidates made errors in calculating two vectors lying in the plane, and this affected the whole of the question.

(b) The normal to plane *ABD* was usually found in the form $\begin{pmatrix} 2t+4 \\ 0 \\ 0 \end{pmatrix}$ and the best solutions realised that it could be rewritten as $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ because it was only the direction that was needed. Those who worked with the factor $(2t + 4)$ in place managed to find the numerical answer with rather more work.

(c) This part needed particular care over accuracy. Most candidates knew the formula and many applied it properly and found a quadratic equation for t. Problems arose when candidates used the wrong vectors; they would find it helpful to label the ones they are using. This would make it easier for them to decide whether it is just the direction that is important or whether both the magnitude and direction are needed.

Question 7

(a) All three asymptotes were usually correctly written as equations.

(b) Many used the quotient rule correctly to differentiate the fractional form. Then there were sometimes algebraic mistakes which led to wrong stationary points. It can be useful to check that the x coordinates of the stationary points satisfy the unsimplified $\frac{dy}{dx} = 0$ equation before proceeding.

Those who divided through to get an expression in the form $y = 1 + \frac{2x+4}{2x^2 - 7x - 4}$ before differentiating made fewer errors.

(c) When drawing a graph candidates should use a ruler for straight lines, and clearly label the axes and the asymptotes. For this graph the left branch needed to cross the asymptote, take the minimum value found in **part (b)** and then approach the asymptote from below.

Some candidates missed that the question asked for the coordinates of intersections with axes. When drawing a graph it is helpful to mark on the intersections and any maximum or minimum to help to decide the shape and position.

(d) The idea of reflecting the part of the graph where $y < 0$ in the x axis was well applied and the points (0, 0) and (2.5, 0) showed the correct sharp turn.

(e) The position of the vertical asymptotes showed that $2x^2 - 7x - 4$ took positive, zero and negative values. It was common for candidates to multiply an inequality by this quadratic without looking at the effect on the inequality sign. Good solutions worked with equations, found the critical values for x and then looked at the graph to give the final inequalities. While it was sensible to find decimal equivalents for the critical values to position them in the correct order, the question asked for exact values for the final inequalities.

FURTHER MATHEMATICS

Paper 9231/13
Further Pure Mathematics 1

Key messages

- Candidates should read every question very carefully so that they can answer all aspects in adequate depth.
- Candidates should show all steps that lead to a solution, particularly when proving a given result.
- Candidates need to be familiar with the formula sheet (MF 19) so that they can use the results it contains.
- Candidates should ensure that all sketched graphs are labelled fully and draw carefully to show significant points and behaviour at limits.

General comments

Most candidates demonstrated good knowledge across the whole syllabus. Candidates showed their working clearly and were accurate in their handling of algebra and calculus. They also showed a good understanding of linear algebra. It seemed that all were able to complete the paper in the time allowed.

Comments on specific questions

Question 1

(a) This was done well, with a few candidates getting the order of matrix multiplication the incorrect way round.

(b) This was done well, with a few candidates considering invariant points rather than invariant lines. Some candidates used 2**M** rather than **M**.

(c) This was also done well, with a few candidates finding the inverse for 2**M** and others used the determinant from 2**M** with the entries from **M**.

Question 2

The majority of candidates demonstrated understanding of the structure of an induction proof and the base case was usually proved correctly.

The wording of the hypothesis was often incomplete. Candidates needed to assume that $2025^k + 47^K - 2$ is divisible by 46 for some positive integer k, rather than just assume that $2025^k + 47^K - 2$ is true.

Candidates who found the difference between terms are reminded that they need to explain how this being divisible by 31, and the hypothesis for the kth term, together lead to the divisibility of the (k + 1)th term.

In most cases, the summing up was correct.

Question 3

(a) This was correctly answered by nearly all candidates, using $(\alpha+\beta+\gamma+\delta)^2 = \alpha^2+\beta^2+\gamma^2+\delta^2+2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$. A few candidates made mistakes with their coefficients as they did not realise that there was no x^3 term.

(b) Most candidates used $S_4 + 7S_2 + 3S_1 + 88 = 0$, and were successful. A few candidates attempted to use a substitution method but without complete success.

(c) Most candidates answered this correctly. Some candidates who had made errors in other parts of the question followed their incorrect answers with correct working. A few candidates did not use the correct formulae for the summation of series from MF 19.

Question 4

(a) Most candidates were able to provide a convincing proof for the given result.

(b) The result from part a helped in answering this question, and candidates who recognised this often obtained correct expressions. Many candidates were unable to manipulate these to a correct expression in *n.*

(c) This part proved challenging to candidates, with few fully correct solutions were seen. Some incorrect responses seen included working that incorrectly factorised $x^{W}n+1 – x^{W1}$ to $x(x^{Wn} – 1)$. Another common omission was the sum to infinity for $x = \pm 1$. Very few candidates correctly considered the different values of x that lead to a convergent sum.

Question 5

(a) Candidates were very successful in this part, with most achieving full credit. The few errors that were made were usually with arithmetic.

(b) There were many good solutions to this part, and a variety of successful approaches were seen. Most candidates made clear what method they were using.

(c) This was usually correct with most candidates using the formula correctly.

Question 6

(a) Most candidates were fully successful in this part. The few errors seen were in simplifying the result of the division.

(b) This part was well done. The few errors were slips in arithmetic and not fully simplifying their final answer in *a*.

(c) There were many good graphs here. Candidates had usually labelled the asymptotes, and both branches of the graph approached them correctly. A few candidates did not label the point of the intersection of the graph with the *y*-axis.

(d) This part was very well answered with most candidates understanding the effect of the modulus on their graph.

(e) Most candidates found the critical points and associated quadratic equation. Few candidates realised that *a* needed to be positive, or that the discriminant needed to be strictly greater than zero, and therefore few candidates achieved full credit in this part.

Question 7

(a) Candidates who realised that differentiation was required for this part were usually successful, with many candidates achieving full credit. A variety of approaches were seen with the most straightforward being implicit differentiation. A few candidates made errors in their differentiation, such as presenting the polar coordinates in reverse order, and some candidates did not provide their final answer to required degree of accuracy which was stated in the question.

(b) This part was successfully completed by many candidates. Those that selected the correct expression to differentiate were usually successful. A few candidates had errors in their differentiation, presenting the polar coordinates in reverse order or presenting them as (x, θ). Some candidates failed to provide their final answer to required degree of accuracy that was stated in the question.

(c) There were a few very good sketches shown. Many candidates drew a closed loop for the graph and had a positive gradient at the intersection (1, 0) as required. The curvature of the graph approaching $\theta = \frac{\pi}{2}$ proved more challenging for candidates to sketch successfully.

(d) Most candidates were able to quote the correct integral and limits. A variety of approaches were seen for the integration. Many candidates used substitution successfully. Candidates who used integration by parts usually did not have much success.

FURTHER MATHEMATICS

Paper 9231/14
Further Pure Mathematics 1

Key messages

Candidates should read each question carefully so that they use all the information given and answer all aspects in adequate depth.

When asked to prove a given result candidates should take particular care to give every step in the process to reach the result.

Proper use of brackets helps avoid errors with signs, and can simplify working in both algebra and arithmetic.

When working with inequalities candidates must ensure that they consider the sign when multiplying by an expression which could be negative.

All sketch graphs need to be fully labelled and carefully drawn to show significant points and behaviour at limits.

General comments

The majority of candidates demonstrated good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. It seemed that almost all were able to complete the paper in the time allowed.

Comments on specific questions

Question 1

(a) This was very well answered with most candidates earning both marks.

(b) The most usual correct method was to take the right-hand side, put the fractions over a common denominator and simplify.

Many candidates attempted to use partial fractions. The denominators are quadratic, and so the numerators must be taken to be linear. A small number realised this and solved the resultant equations in four unknowns to find the partial fractions as the required answer. Most split the given fraction up into two partial fractions with constant numerators, which is not a valid method.

(c) This was generally well done, the method of differences was well understood.

Candidates should ensure that they write down enough terms to show how the cancellation works, and check that the complete terms for $r = 1$ and $r = n$ are shown. The best solutions used the space to write successive terms beneath each other to show the terms that cancel.

(d) The majority of the candidates got the correct answer.

Question 2

Candidates should recognise that proof by induction is proving a given result and so all steps of the working must be carefully shown. The overall plan is well known but candidates need to be sure that they express the steps clearly.

For the base case the best solutions showed that $\frac{dy}{dx} = 1 + \ln x$ leading to $\frac{d^2y}{dx^2} = \frac{1}{x}$ and then showed the substitution of $n = 2$ in all three terms of the given formula to compare.

The hypothesis should include both the algebraic formula being assumed true and that it is true for some particular value of n, usually k.

The differentiation was usually correct and complete solutions rearranged this into the form where n was replaced by $(k + 1)$ without simplification. This is needed to justify the inductive step.

The best solutions gave a final conclusion showing the formula that has been proved, and stating that it is true for all positive integers $n \geqslant 2$.

Question 3

All parts were well answered. The basic methods for vector questions are clearly understood and many of the errors were numerical. Candidates are advised to check they have written down the numbers accurately and that any cross-products are correct, as errors here are common.

(a) Most candidates found the direction of the normal using the cross-product of two vectors lying in the plane. When an equation has been found it can be helpful to check the accuracy by substituting in the coordinates of the given points.

(b) This usually followed correctly from the answer for **(a)**.

(c) Most candidates used the cosine and the correct scalar product. Only a few forgot to subtract this answer from 90° to give the required answer.

Question 4

(a) Most candidates wrote down the determinant correctly and equated it to zero. The answer was given and to fully prove the result candidates needed to explain why $\alpha\beta\gamma = 1$ with reference to the coefficients of the given equation with roots α, β, γ. This was often done successfully.

(b) There were many efficient solutions for this question. The neatest method was to write down expressions for $\alpha^2 + \beta^2 + \gamma^2$ and $\alpha^3 + \beta^3 + \gamma^3$ in terms of b and c and solve the resulting equations.

Question 5

(a) This was straightforward and most wrote down the two correct matrices in the right order. The most frequent error was to forget to multiply to find the matrix M.

(b) The question asked for a full description of the transformation M^{-1}. This required an identification of the two inverse transformations and the order in which they are applied. Many candidates just found the matrix M^{-1}.

(c) Most responses used the pre-multiply by M^{-1} method. The alternative method using simultaneous equations was often successful.

(d) Finding the invariant lines was well done. Very few candidates made the mistake of writing $M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ which shows that they are looking for invariant points.

Question 6

(a) Good sketches showed the initial line and the pole, with the curve starting at the pole and spiralling out to meet the initial line. Tan is an increasing function and for full marks the curve needed to increase with θ. Most realised that the maximum distance was a . For the final mark the maximum distance needed to be clearly stated.

(b) This was well done, candidates used the correct area formula and limits. The most efficient solutions replaced $\tan^2\left(\frac{\theta}{8}\right)$ with $\sec^2\left(\frac{\theta}{8}\right)-1$ and integrated directly. Most candidates dealt with the factor of 8 correctly and achieved the final answer.

(c) Most candidates used the correct form $y = a\ \tan\frac{\theta}{8}\sin\theta$, differentiated accurately and used a correct trigonometric formula to show the given result.

Question 7

(a) This was usually correct.

(b) This question required a proof of the range of values for *y*. Many candidates simply found the critical values which were given in the question, rather than checking the inequalities.

The most popular route was to form a 3-term quadratic equation in *x* and use the discriminant to obtain a condition for real roots. Having obtained a correct quadratic, the next step was to demonstrate clearly which of the regions defined by the given critical values gave the required value for the discriminant. The most efficient way was to write the condition in the form $(y-1)(7y+17) \leqslant 0$. To complete the solution a convincing explanation of why $y \neq 1$ was needed. There were some thorough and completely correct solutions using this approach.

Another method was to arrange the function in the form $y = 1 - \frac{6}{x^2+x+2}$. The most elegant and effective use of this method came from writing $x^2+x+2 = \left(x+\frac{1}{2}\right)^2 + \frac{7}{4}$ and using inequalities.

(c) This was usually fully correct. The most efficient solutions spotted that $(2x+1)$ is a common factor and simplified the working.

(d) There were many well-drawn graphs, with axes and asymptote properly labelled and a single smooth curve. The graph needed to be symmetrical in the line $x = -\frac{1}{2}$ and show a smooth minimum.

(e) It was clear that most understood that the new graph should be symmetrical in the *y* axis, and the most accurate ones showed the sharp turn at (0, –2). The most elegant solutions were those that calculated when $y = -\frac{1}{2}$ for the right-hand half of the graph and used the symmetry to give the correct final inequality.

FURTHER MATHEMATICS

Paper 9231/21
Further Pure Mathematics 2

There were too few candidates for a meaningful report to be produced.

FURTHER MATHEMATICS

Paper 9231/22
Further Pure Mathematics 2

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth, particularly when an answer is required in a certain form or in terms of a given variable.
- Candidates should make use of results derived or given in earlier parts of a question or given in the List of formulae (MF19).

General comments

Most candidates demonstrated very good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of linear algebra. It seemed that all were able to complete the paper in the time allowed. Sometimes candidates did not fully justify their answers and reached conclusions without justification, particularly where answers were given within the question. Gaps in knowledge were evident in some scripts.

Comments on specific questions

Question 1

Good candidates showed clear working, starting from z^3 with the correct argument, and listed all three roots in the required form. A significant number of candidates used $\frac{1}{4}\pi$ as the argument of z^3 instead of $-\frac{1}{4}\pi$. This led to incorrect answers when they attempted to take the cube root. A simple mental diagram would aid in visualising that the argument is in the fourth quadrant, not the first. A common error was giving the third cube root outside the specified argument range given in the question. A few candidates gave a decimal modulus instead of the exact value and should be reminded of the importance of providing exact answers whenever possible.

Question 2

(a) Most candidates made a correct choice when selecting parts for integration by parts. However, many missed the negative sign when differentiating $(1-x)^n$, which often led to the correct final answer but through incorrect working.

(b) This part was generally well done. Most candidates successfully applied the reduction formula to establish the relationship between I_0 and I_2. A few did not evaluate $\cosh(0)=1$, which led to an incorrect final answer.

Question 3

Most candidates were able to apply the binomial expansion formula correctly. However, some did not show the necessary grouping of the expression $z^n - z^{-n}$. A common error was the omission of the imaginary unit i in the identity $z - z^{-1} = 2\mathrm{i}\sin\theta$, which led to incorrect simplifications.

Question 4

(a) Most candidates correctly wrote the sum of the areas of the appropriate rectangles and compared it with the definite integral using the correct limits. Those who attempted to evaluate the integral using integration by parts, which is not suitable in this case, failed to make further progress. Candidates who instead used a substitution approach were often successful in justifying the required upper bound.

(b) Candidates who completed **part (a)** successfully were usually able to carry their reasoning through to **part (b)** and determine a suitable lower bound for the given series.

Question 5

This question was generally well done. Almost all were able to differentiate their general solution correctly using the product rule and successfully determine the integration constants by applying the given initial conditions. However, it is important to note that candidates must not change the variable names arbitrarily. The independent and dependent variables should be clearly stated and consistently used throughout the solution.

Question 6

(a) Most candidates achieved full marks on this question, demonstrating a clear understanding of the required identity. However, for those who lost marks, it is important to highlight that, when proving an identity, candidates should avoid working simultaneously from both sides of the equation with equal signs in between. This approach assumes the result is true before it has been proven. Good solutions started from one side and manipulated it logically and clearly until it matched the other side. This method ensures mathematical rigour and avoids invalid reasoning.

(b) This question proved challenging for many candidates. A common error was the incorrect assumption that $\text{sech}^{-1} t$ is the reciprocal of $\cosh^{-1} t$, or the reciprocal of $\text{sech}\, t$. Candidates are reminded that inverse hyperbolic functions have specific definitions and should not be confused with reciprocals. The strongest solutions used implicit differentiation or the logarithmic form of $\text{sech}^{-1} t$ to obtain the given answer.

(c) The majority of candidates found the first derivative correctly using parametric differentiation, although sign errors occasionally occurred when deriving the given answer.

(d) The attempts to find the second derivative varied in length, with strong candidates showing the required level of algebraic fluency and remembering to divide by $\frac{dx}{dt}$ after differentiating with respect to t. Candidates should be reminded that for the second derivative in parametric form, the application of the chain rule is essential.

Question 7

While many candidates successfully solved the differential equation, a significant number failed to recognise the importance of the negative sign when determining the integrating factor. This seemingly small oversight often led to an incorrect integrating factor, which in turn changed the entire integration process. Among those who obtained the correct answer, some chose to integrate the expression from first principles rather than using the standard result provided. While not incorrect, this approach was often inefficient and sometimes led to unnecessary algebraic errors. This highlights the importance of being familiar with and effectively using the List of formulae (MF19) during the exam.

Question 8

(a) Many incorrectly attempted to ‘cancel’ or divide eigenvectors from both sides of the equation $Ae = \lambda e$, which is mathematically invalid since eigenvectors are vectors, not scalars. Only the strongest candidates manipulated the equation correctly, multiplying the left-hand side by A^2.

(b) This part was well done overall. The majority of candidates were able to correctly form the characteristic equation to find the eigenvalues. Some unnecessarily expanded the equation from its factorised form, which added extra steps.

(c) This part was also generally well done. Most candidates correctly used the cross product of two distinct rows of $A - \lambda I$ to find the corresponding eigenvectors. However, some candidates mistakenly used the same row twice when forming their vector product. This resulted in the zero vector, which is not a valid eigenvector, and reflects a misunderstanding of the process.

(d) A common misconception was to substitute $\mathbf{A} - 2\mathbf{I}$ into the characteristic equation of matrix $\mathbf{A}$. Only the strongest candidates knew to expand $(\mathbf{A} - 2\mathbf{I})^3$ or use the eigenvalues from $\mathbf{D}$ to find the characteristic equation of $\mathbf{A} - 2\mathbf{I}$.

FURTHER MATHEMATICS

Paper 9231/23
Further Pure Mathematics 2

Key messages

Candidates should show all the steps that lead to a solution, and justify solutions completely, particularly when proving a given result.
Candidates must be confident in the use of chain rule when differentiating f(g(x)), and the subtleties when working with parametric equations.
Candidates must follow instructions carefully, especially in the case where the answer has a required form.
Algebraic accuracy led to a significant number of marks being dropped despite the candidate often understanding the demand of the question.

General comments

A high level of understanding was demonstrated across the whole syllabus. The majority of candidates understood the demand of each question and make good progress towards an answer. The level of rigour shown in questions where the answer was given was occasionally insufficient. Candidates appeared able to complete the paper in the time allowed. Gaps in knowledge were evident in some scripts.

Question 1

Most candidates were familiar with the demands of the question to find a Maclaurin series and were able to produce a fully accurate response. Whilst the first derivative did not present many problems, the need for product rule (and some sign errors) occasionally lead to an incorrect second derivative. Minor errors were observed when assembling the final series, including sign errors, the absence of the $e^{0.5}$ or forgetting to divide the second derivative by 2. On rare occasions, candidates made no progress with the question because they had erroneously tried to use the standard series for e^x.

Question 2

(a) A small number of candidates did not follow the instruction in the question to use exponentials and gained zero marks. Most candidates could quote correct formulae in terms of exponentials and manipulated them successfully to obtain the given result. Occasionally there was a slip in notation along the way, which loses the accuracy mark in a proof question. Many candidates used mixed variables (x and t) in their proof, which also incurred an accuracy penalty. Candidates are advised of the need for more rigour when presented with proof and answer given.

(b) The majority of candidates understood the demands of using the parametric form of the arc length integral. However a significant number of candidates were not able to use chain rule effectively to find the correct numerator in $\frac{dy}{dt}$. This led to limited progress. For those who found the correct derivatives, the link with **Question 2a** was regularly established and the integral completed without any problem. A small number of attempts used the incorrect formula $\int\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dt$, once more forgetting the need for chain rule and the extra $\frac{dx}{dt}$ required in this formula. It was surprising that several candidates did not spot the link with **part (a)**, writing everything in terms of $\sinh t$ and $\cosh t$ rather than using the given identity.

Question 3

(a) The majority of candidates identified the demands of implicit differentiation in **part 1** of the question, although once more the absence of the correct numerator when differentiating $\sinh^{-1}(xy)$, through the use of chain rule, was seen on occasion. Most candidates attained full marks on this question.

(b) The correct concepts were clearly applied in the question, although the need for algebraic accuracy was the greatest barrier to achieving full marks. A number of different approaches were seen, often dependent on their approach in 3a and how the expression for the first derivative has been manipulated. Some arrangements were simpler to deal with than others and candidates who separated terms clearly prior to differentiating were generally more successful. In general however, only a small minority achieved full marks.

Question 4

This was a standard and straightforward question for most candidates, with the most common outcome being full marks. Candidates knew how to find both the complimentary function and particular integral. Where credit was not earned, this was often through numerical errors or algebraic slips, although this was rare.

Question 5

(a) Most candidates chose to start with an expansion of $(c+\mathrm{i}s)^5$, although a significant minority approached the question from an expansion of $\left(z+\frac{1}{z}\right)^5$ to obtain $32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$. Success was seen with both approaches, although more often with the former. The most common omissions in this question part were not explaining the step of dividing the numerator and denominator of the fraction by $\cos^5\theta$. Candidates should be reminded that when a question has a given answer to be proved, full justification for each step must be given. The order in which the Pythagorean identity and dividing both numerator and denominator by $\cos^5\theta$ was different many between candidates, with both leading to successful outcomes. Of the candidates who employed the expansion of $\left(z+\frac{1}{z}\right)^5$, the required repeated use of compound angle formulae represented a barrier to further progress.

(b) Most candidates successfully made the link with **part (a)** and proceeded to solve $\cos 5\theta = \frac{\sqrt{3}}{2}$. To gain the last two marks, candidates needed to find five distinct solutions. Sometimes, duplicates were given, meaning the last mark was lost. Effective calculator use is advised to check for duplicate roots. Candidates are also reminded to read the rubric of the question, with a significant minority just giving values of θ, rather than writing the roots in the form $\sec(q\pi)$ as required.

Question 6

(a) Most candidates followed the instruction to write out the areas of the required rectangles. Once more the need for more rigour when working towards a given answer is enhanced, in this case establishing the terms of the sum through algebraic manipulation was required. The connection with the integral was established with ease and hence the majority of candidates gained full marks on this question.

(b) This question required candidates to take care to assemble the correct series of rectangle areas. Most candidates identified they were looking at the rectangles about the curve. Stronger responses understood the manipulation to go between $\sum_0^{n-1}$ and $\sum_0^{n}$ through addition and subtraction of the 0th and nth rectangles and hence found the required result. Sign errors were often seen.

(c) This question represented little difficulty for candidates who had been successful in in **parts (a)** and **(b)**. 0 was a common incorrect answer

Question 7

This again was a straightforward question for most candidates. The majority achieved full marks. The most common error was failing to included the minus sign in the integrating factor and consequently finding x^2+6x+5 as the integrating factor rather than $\frac{1}{x^2+6x+5}$. Once the integrating factor was found, occasionally candidates struggled to deal with the integral of $\frac{4}{x^2+6x+5}$, not spotting the simple split into partial fractions. Of those who completed the square, some used the correct expression from the formula book whereas others incorrectly obtained expressions involving $\cosh^{-1}$. Those who completed the integration correctly usually went on to achieve full marks.

Question 8

(a) Most candidates were able to evaluate the determinant correctly. For those who did not, the most common error was to make a sign error on the middle term. A significant minority of candidates thought that the determinant had to be non-zero rather than zero for the system not to have a unique solution. However the majority of candidates earned full marks.

(b) The characteristic equation of A was usually obtained correctly. Candidates generally understood the Cayley-Hamilton theorem and applied it successfully. Having multiplied once by the inverse of A, some candidates went onto square both sides of the equation in an attempt to find an expression for B^2. This was usually unsuccessful, although some correct solutions were seen using this method (substituting for powers of A). A small number of candidates obtained the correct expression for A^{-2}, but failed to make the equivalence with B^2. Candidates are also reminded that a constant term 'k' in the characteristic equation must be replaced with 'kI' when applying the C-H theorem.

(c) Most candidates took the expected approach of working with matrix A, but there were a significant number who worked from other matrices, for example A^{-1} or even B^2. Errors were often made with the latter approaches, with common factors extracted and then not dealt with correctly when finding the eigenvectors. Most candidates were aware of the need to match up columns of matrix P with the entries in matrix D. Some obtained zero eigenvectors and did not appear to realise that this could not be correct. A few candidates neglected to take reciprocals when forming matrix D.

FURTHER MATHEMATICS

Paper 9231/24
Further Pure Mathematics 2

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth and note which algebraic form answers are required to take.
- Candidates should make use of results derived in earlier parts of a question or given in the List of formulae (MF19).

General comments

The majority of candidates demonstrated very good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of linear algebra. It seemed that all were able to complete the paper in the time allowed. Sometimes candidates did not fully justify their answers and jumped to conclusions without justification, particularly where answers were given within the question. There were many scripts of a very high standard.

Comments on specific questions

Question 1

(a) This was successfully answered by the majority of candidates. Common errors included incorrect algebra when simplifying to arrive at a three term quadratic or stating that $k \neq 1, 4$.

(b) Most candidates were able to correctly state that the three planes meet a common line or formed a sheaf. The main errors were from incorrect or unclear working when trying to use the three equations to form two simultaneous equations. Good candidates labelled the equations, used all three equations and clearly stated the operations. Many reduced to row echelon form with clear row operations.

Question 2

Most candidates completed this question successfully, making their use of the logarithmic of $\sinh^{-1}$ explicit. The main errors were completing the square incorrectly or applying the wrong formula when integrating.

Question 3

Almost all knew how to approach this question and completed it to a high standard. There were some inaccuracies when comparing coefficients to find the particular solution and some problems with notation. A few candidates gave expressions instead of equations as their answer.

Question 4

(a) This was answered well with the majority of candidates factorising the cubic as $(t-1)(t^2+1)=0$ to show that there was only one real root.

(b) The majority of candidates found the first derivative correctly using parametric differentiation, although sign errors occasionally occurred when deriving the given answer.

(c) The attempts to find the second derivative varied in length, with strong candidates showing the required level of algebraic fluency and remembering to divide by $\frac{dx}{dt}$ after differentiating with respect to t. Candidates should be reminded that for the second derivative in parametric form, the application of the chain rule is essential. For the Maclaurin's series, a few candidates incorrectly substituted $t = 0$.

Question 5

(a) This was very well answered. The main errors were slips in algebraic manipulation or in the expansion of higher power terms when converting to an expression in terms of $\sin\theta$ to reach the given answer.

(b) Almost all rearranged the polynomial equation correctly, linking with the previous part of the question by substituting $x = \sin\theta$. Good candidates excluded $\sin\theta = 0$ from their solutions. Solving $\sin 7\theta = 0$, most candidates obtained $x = \sin\left(\frac{1}{7}\pi\right)$. A few candidates gave the values for θ rather than actual solutions, highlighting the need to read the question carefully. The five other solutions were often given in different ways, and some candidates gave repeated solutions.

Question 6

While many candidates successfully solved the differential equation, a significant number did not recognise the importance of the negative sign when determining the integrating factor. This seemingly small oversight often led to an incorrect integrating factor. Those who obtained the correct integrating factor usually also integrated $\tan^{-1} x$ by parts to arrive at the correct general solution. After substituting the initial conditions and dividing through by the integrating factor, there was some poor notation where $x\ln 2$ was written as $\ln 2x$.

Question 7

(a) Candidates who used the vector product method to find the eigenvectors tended to be most successful, although sign errors were common. Candidates should be encouraged to check that their proposed eigenvector does have the required property by performing matrix multiplication. Almost all candidates showed an awareness of how to find the matrices **P** and **D**. A small number of candidates neglected to take the 6th power of the eigenvalues of **A** or used a zero vector in their final answer.

(b) Most candidates had the correct characteristic equation and arrived at the answer by making $\mathbf{A}^3$ the subject and squaring both sides. A few candidates used lengthier methods, multiplying through by **A** to obtain $\mathbf{A}^4$ and $\mathbf{A}^5$.

Question 8

(a) Most candidates drew the correct shape of $y = \tanh x$. Graphs from good candidates were not truncated and got closer to (but did not touch) the asymptote as $x \to \infty$.

(b) The majority of candidates showed enough detail to justify the given answer, writing $\ln(\cosh x)$ before substituting in the limits 0 and N. There was some poor notation, where r was used both in the sum and integral, and a few candidates used 1 as the lower limit for their integration.

(c) (i) Most candidates stated the correct formula for the surface area. There were some missing limits and incorrect powers of $\operatorname{sech} x$ and $\tanh x$. Most candidates also successfully found $\frac{du}{dx}$. There were a few errors arising from incorrect powers or signs. Only the strongest candidates fully justified the given limits using the given substitution, showing enough working when substituting $x = \frac{1}{2}\ln 3$.

(ii) There were many fully complete and correct answers seen when finding the given integral. A few candidates tried to integrate using $\tanh^{-1} u$, which is invalid due to the restricted domain.

FURTHER MATHEMATICS

Paper 9231/31
Further Mechanics 31

Key messages

In all questions, candidates are advised to show all their working, as credit is given for method as well as accuracy. When a result is given in a question, candidates must take care to give sufficient detail in their working, so that the examiner is in no doubt that the offered solution is clear and complete.

A diagram is often an invaluable tool in helping a candidate to make good progress. This is especially the case when forces or velocities are involved. If a diagram is given on the question paper, then it may be sufficient to annotate that diagram, although a candidate is always free to draw their own diagram as well.

General comments

Candidates who drew a suitable diagram or, in the case a diagram is provided, annotated it, tended to be more successful in understanding the problem and modelling it correctly.

A common reason for candidates not receiving full credit was that their equations were not dimensionally consistent. This is particularly important when writing moments and conservation of energy equations. When applying Newton's second law, for example, to set up a differential equation, or in questions involving collisions, candidates must ensure that they explicitly include the mass, or masses, involved.

When the answer is given in the question, it is essential that candidates show their working in full, even if it involves the use of elementary algebra.

Comments on specific questions

Question 1

(a) Many candidates were successful in finding the correct expression for v. Algebraic slips or omitting the constant of integration often resulted in an expression for v^2 that could not be factorised to give an expression for v in the correct form. Candidates who attempted to solve the problem using *suvat* equations could not be awarded any credit as the acceleration is not constant in this problem.

(b) Successful candidates showed full working to support the integration of their expression for v from **part (a)**. A common error in this part was the omission of the constant of integration. Use of the initial condition leads to this constant being zero, but candidates are expected to show full working to justify this.

Question 2

(a) The majority of candidates successfully applied the information given in the question in the equation of the trajectory of a projectile. Errors typically arose from rearranging the equation into a three-term quadratic equation.

(b) The key to solving this part of the question is to realise that the minimum distance from O to where P lands corresponds to the larger of the two values of the angle of projection found in **part (a)**. Several successful methods were seen, including using $y = 0$ in the equation of the trajectory or using the horizontal motion to find the time of flight followed by the range with this time of flight.

Question 3

(a) Most candidates successfully used Hooke's law to find the extension corresponding to a tension of 8 N. As is the case for all 'show that' questions, full working must be shown to ensure that full credit can be awarded.

(b) Most candidates attempted to resolve horizontally and vertically. Some candidates were not awarded full credit because they made sign errors or omitted terms. For equations to be credited, they must be dimensionally correct, with the correct number of terms and correct signs.

Question 4

(a) Many candidates were successful in writing an accurate and dimensionally correct moments equation. Weaker responses often did not subtract the moment for the hemisphere from the moment for the cone.

(b) The majority of candidates were able to provide a correct expression for the tangent of the required angle. Some responses used the reciprocal of the required fraction for $\tan\theta$. Drawing a sketch of the toppling situation with the object on an inclined plane could help avoid such errors and help weaker candidates make further progress.

Question 5

Successful candidates recognised that the particle reaches its maximum velocity when its acceleration is zero. The most common misconception seen was that the maximum velocity is achieved when the string reaches the point *A*. Since the particle still has velocity at *A*, it continues to move down the plane until the resultant force on it is zero. At the point when the tension in the string balances the component of the weight of the particle acting down the plane, the object will reach its maximum velocity. Candidates were generally more successful when considering conservation of energy for the whole motion, rather than splitting the problem into two (or more) parts.

Question 6

(a) Most candidates answered this part well by writing down an equation for conservation of momentum and an equation relating the kinetic energies of the two particles. Candidates who took the square root of the kinetic energy equation often achieved a simpler solution. Candidates who did not square root the kinetic energy equation typically reached a quadratic equation for which one possible solution had to be rejected.

(b) Many candidates successfully wrote down equations of motion both parallel and perpendicular to the wall. Only the strongest candidates were able to make further progress by solving the two trigonometric equations, typically squaring and adding the equations.

Question 7

(a) Most candidates successfully applied Newton's second law at *A* and at *B* and formed an energy equation for the motion from *A* to *B*. A common error was to have the direction of the reaction of the sphere on the particle at *A* passing through the point on *BC* directly above *O* instead of passing through *O*. Weaker responses gave equations that were not dimensionally correct.

(b) Successful candidates recognised that the velocity at *B* will be zero.

(c) Some candidates were able to make progress with this problem by finding the speed of the particle when it reaches *C*. A common error was using the equation of the trajectory with $x = 2a$ instead of $x = BC = 2a\sin\theta$. Only the strongest candidates went on to use the energy equation to find an expression for *u*.

FURTHER MATHEMATICS

Paper 9231/32
Further Mechanics 32

Key messages

In all questions, candidates are advised to show all their working, as credit is given for method as well as accuracy. When a result is given in a question, candidates must take care to give sufficient detail in their working, so that the examiner is in no doubt that the offered solution is clear and complete.

A diagram is often an invaluable tool in helping a candidate to make good progress. This is especially the case when forces or velocities are involved. If a diagram is given on the question paper, then it may be sufficient to annotate that diagram, although a candidate is always free to draw their own diagram as well.

General comments

Candidates who drew a suitable diagram or, in the case a diagram is provided, annotated it, tended to be more successful in understanding the problem and modelling it correctly.

A common reason for candidates not receiving full credit was that their equations were not dimensionally consistent. This is particularly important when writing moments and conservation of energy equations. When applying Newton's second law, for example, to set up a differential equation, or in questions involving collisions, candidates must ensure that they explicitly include the mass, or masses, involved.

When the answer is given in the question, it is essential that candidates show their working in full, even if it involves the use of elementary algebra.

Comments on specific questions

Question 1

(a) Many candidates were successful in finding the correct expression for v. Algebraic slips or omitting the constant of integration often resulted in an expression for v^2 that could not be factorised to give an expression for v in the correct form. Candidates who attempted to solve the problem using *suvat* equations could not be awarded any credit as the acceleration is not constant in this problem.

(b) Successful candidates showed full working to support the integration of their expression for v from **part (a)**. A common error in this part was the omission of the constant of integration. Use of the initial condition leads to this constant being zero, but candidates are expected to show full working to justify this.

Question 2

(a) The majority of candidates successfully applied the information given in the question in the equation of the trajectory of a projectile. Errors typically arose from rearranging the equation into a three-term quadratic equation.

(b) The key to solving this part of the question is to realise that the minimum distance from O to where P lands corresponds to the larger of the two values of the angle of projection found in **part (a)**. Several successful methods were seen, including using $y = 0$ in the equation of the trajectory or using the horizontal motion to find the time of flight followed by the range with this time of flight.

Question 3

(a) Most candidates successfully used Hooke's law to find the extension corresponding to a tension of 8 N. As is the case for all 'show that' questions, full working must be shown to ensure that full credit can be awarded.

(b) Most candidates attempted to resolve horizontally and vertically. Some candidates were not awarded full credit because they made sign errors or omitted terms. For equations to be credited, they must be dimensionally correct, with the correct number of terms and correct signs.

Question 4

(a) Many candidates were successful in writing an accurate and dimensionally correct moments equation. Weaker responses often did not subtract the moment for the hemisphere from the moment for the cone.

(b) The majority of candidates were able to provide a correct expression for the tangent of the required angle. Some responses used the reciprocal of the required fraction for $\tan\theta$. Drawing a sketch of the toppling situation with the object on an inclined plane could help avoid such errors and help weaker candidates make further progress.

Question 5

Successful candidates recognised that the particle reaches its maximum velocity when its acceleration is zero. The most common misconception seen was that the maximum velocity is achieved when the string reaches the point *A*. Since the particle still has velocity at *A*, it continues to move down the plane until the resultant force on it is zero. At the point when the tension in the string balances the component of the weight of the particle acting down the plane, the object will reach its maximum velocity. Candidates were generally more successful when considering conservation of energy for the whole motion, rather than splitting the problem into two (or more) parts.

Question 6

(a) Most candidates answered this part well by writing down an equation for conservation of momentum and an equation relating the kinetic energies of the two particles. Candidates who took the square root of the kinetic energy equation often achieved a simpler solution. Candidates who did not square root the kinetic energy equation typically reached a quadratic equation for which one possible solution had to be rejected.

(b) Many candidates successfully wrote down equations of motion both parallel and perpendicular to the wall. Only the strongest candidates were able to make further progress by solving the two trigonometric equations, typically squaring and adding the equations.

Question 7

(a) Most candidates successfully applied Newton's second law at *A* and at *B* and formed an energy equation for the motion from *A* to *B*. A common error was to have the direction of the reaction of the sphere on the particle at *A* passing through the point on *BC* directly above *O* instead of passing through *O*. Weaker responses gave equations that were not dimensionally correct.

(b) Successful candidates recognised that the velocity at *B* will be zero.

(c) Some candidates were able to make progress with this problem by finding the speed of the particle when it reaches *C*. A common error was using the equation of the trajectory with $x = 2a$ instead of $x = BC = 2a\sin\theta$. Only the strongest candidates went on to use the energy equation to find an expression for *u*.

FURTHER MATHEMATICS

Paper 9231/33
Further Mechanics 33

Key messages

A diagram is often an invaluable tool in helping a candidate to make good progress. This is particularly the case when forces or velocities are involved. If a diagram is given on the question paper, then it may be sufficient to annotate that diagram, although candidates are always free to draw their own diagram as well.

Candidates would benefit from giving greater attention to ensuring that the equations they write are dimensionally correct and consistent.

When a result is given in a question, candidates must take care to give sufficient detail in their working, so that the examiner is in no doubt that the offered solution is clear and complete. In all questions, however, candidates are advised to show all their working, as credit is given for method as well as accuracy.

General comments

Candidates who drew a suitable diagram or, in the case a diagram is provided, annotated it, tended to be more successful in understanding the problem and modelling it correctly.

A common reason for candidates not receiving full credit was that their equations were not dimensionally consistent. This is particularly important when writing moments and conservation of energy equations. When applying Newton's second law, for example, to set up a differential equation, or in questions involving collisions, candidates must ensure that they explicitly include the mass, or masses, involved.

When the answer is given in the question, it is essential that candidates show their working in full, even if it involves the use of elementary algebra.

Candidates should be reminded to correctly set their calculator to degrees or radians, according to what is required in the question. It is essential that candidates are aware of the need for radian measure in problems involving calculus.

Comments on specific questions

Question 1

Many candidates showed their working clearly and demonstrated good algebraic manipulation skills. Some responses could be improved through clearer communication and presentation of work. Stronger candidates demonstrated their approach by clearly communicating the equations they are using and specifying explicitly the value of r. Weaker responses often included the incorrect assertion that $r = a$.

Question 2

(a) Candidates who were successful in this question typically either wrote the energy equation in terms of *OP* or in terms of the extension of the string. In the latter case, some candidates forgot to add a to their answer of $\frac{2}{3}a$ to obtain the correct distance. Weaker responses either considered the equilibrium of forces instead of the principle of conservation of energy or included the kinetic energy in the energy equation. A common reason for candidates not obtaining full credit was to disregard the natural length of the string when writing their conservation of energy equation.

Some candidates split the problem in two parts and considered first the conservation of energy between point O and the point at a distance a from O, and then from there to the point where the particle stopped. Candidates were generally less successful in arriving at the correct answer using this approach.

(b) Successful candidates recognised the need to apply the equilibrium of forces in the direction of the plane. Stronger responses often included a diagram, which aided understanding that friction and tension had opposite directions. A common error was to think that the tension in the string and the frictional force had the same direction.

Question 3

(a) Stronger candidates showed their working well and provided a convincing argument in arriving at the result printed in the question. Weaker responses omitted the mass of the ball, m, in the differential equation. Candidates should be reminded that, when applying Newton's second law, they must explicitly mention the mass, or masses, involved. Another common omission was the negative sign in the differential equation.

(b) Only the strongest candidates demonstrated the appropriate working required. These candidates commonly showed their working very clearly, irrespective of whether they solved the differential equation in terms of k and g or substituted their numerical values prior to integrating. A common reason for candidates not obtaining full credit was the use of degrees instead of radians.

Question 4

Candidates used a variety of strategies to answer this question. Candidates are reminded to communicate their approach clearly. One common approach seen involved finding the *x*-coordinate of the centre of mass of the lamina first, ignoring the attached particle, and then considering the system of the lamina and particle. Candidates were also often successful when considering the ratio of masses of the rectangular lamina, the triangular lamina and the particle and using them in a moments equation. Weaker responses involved equations that were not dimensionally correct. Many of those, for example, considered k proportional to a^2.

Question 5

(a) Most candidates were able to make good progress with this part question. Successful responses commonly considered the journey from point *A* to point *B* directly. Some candidates split the path in two parts and considered first the path from point *A* to the lowest point in the cylinder, and then from there to point *B*. Candidates were generally less successful using this approach. Some responses considered the change in Gravitational Potential Energy between the highest point of the cylinder and point B, resulting in a numerically correct answer from an incorrect method which could not be awarded full credit.

(b) Stronger candidates who made good progress through this question commonly omitted to add the vertical distance between points *O* and *B* to determine the greatest height above *O,* having found the correct value for the greatest height of the particle above *B*. Many candidates obtained the correct expression for the speed of the particle at point *B*. However, a common reason for candidates not obtaining full credit was the use of the speed, instead of the vertical component of the velocity, in the equation to determine the greatest height above point *B* reached by the particle.

Question 6

(a) Successful candidates made the directions that they were considering clear and used signs appropriately to match their communicated approach. Many candidates included incorrect directions for the velocities of the spheres before and/or after the collision. Weaker responses omitted the masses in their conversation of momentum equation. A common misunderstanding was considering u as the velocity of sphere B after the collision.

(b) A variety of correct approaches were seen from candidates, all given credit accordingly. Weaker candidates did not appreciate that the horizontal component of the velocity of *A* is zero after the collision, an integral element of solving this part question.

Question 7

(a) This part question was well attempted by most candidates. A commonly seen incorrect expression for the vertical component of the velocity was $U\sin 45$ instead of the correct $U\sin 45 - gT$.

(b) Stronger answers showed all working in full, even stages that involve the use of elementary algebra, and made it clear how the signs being used related to directions being considered. Many candidates often considered $\frac{v_y}{v_x} = \tan 60°$ without any comment to explain that v_y was the opposite sign of their answer to **part (a)**.

(c) Many candidates successfully expressed D and/or H in terms of U and T, or U and g. Only the strongest candidates were able to obtain a correct expression for the ratio $H : D$. A common incorrect final answer was an expression for $\frac{H}{D}$ instead of $H : D$.

(d) Successful responses were clear in their use of signs and provided fully correct working. Common errors seen in this part question included incorrect signs or values for w_y, and not using the horizontal component of *w* in the answer.

FURTHER MATHEMATICS

Paper 9231/34
Further Mechanics 34

Key messages

In all questions, candidates are advised to show all their working, as credit is given for method as well as accuracy. When a result is given in a question, candidates must take care to give sufficient detail in their working, so that the examiner is in no doubt that the offered solution is clear and complete.

A diagram is often an invaluable tool in helping a candidate to make good progress. This is especially the case when forces or velocities are involved. If a diagram is given on the question paper, then it may be sufficient to annotate that diagram, although a candidate is always free to draw their own diagram as well.

General comments

Candidates who drew a suitable diagram or, in the case a diagram is provided, annotated it, tended to be more successful in understanding the problem and modelling it correctly.

A common reason for candidates not receiving full credit was that their equations were not dimensionally consistent. This is particularly important when writing moments and conservation of energy equations. When applying Newton’s second law, for example, to set up a differential equation, or in questions involving collisions, candidates must ensure that they explicitly include the mass, or masses, involved.

When the answer is given in the question, it is essential that candidates show their working in full, even if it involves the use of elementary algebra.

Comments on specific questions

Question 1

Most candidates considered the motion from the initial position to the final position and formed an appropriate energy equation. Common errors included omitting necessary terms in the energy equation or including a kinetic energy term. A minority of candidates used an incorrect formula for elastic potential energy, often omitting the square in the numerator. This gave an energy equation that is dimensionally incorrect. Candidates should be encouraged to check that any equation they write is dimensionally correct. Candidates were generally more successful when considering conservation of energy for the whole motion, rather than splitting the problem into two (or more) parts.

Question 2

(a) Most candidates successfully set up and solved the differential equation. Common errors included sign errors, arithmetical errors, or omitting the constant of integration.

(b) Many candidates were successful in determining the displacement. Common errors were similar to those outlined in **part (a)**, in particular omitting the constant of integration.

Question 3

(a) Most candidates gave clear derivations of the equation of the trajectory, starting with consideration of the equations of motion for the horizontal and vertical displacements of the particle and then combining these to form the given expression of the trajectory equation. As is the case for all 'show that' questions, full working must be shown to ensure that full credit can be awarded.

(b) Many candidates were successful in finding both values. Slips in substitution or algebraic manipulation were the main errors seen.

Question 4

Successful candidates often included a clearly labelled diagram or added to the diagram printed in the question paper. A common error was to omit the normal reaction force of the cone on the particle at *A*. This force is essential to the mechanics of the problem, and its omission is a critical error.

Question 5

(a) Successful candidates showed clear working to derive the given result for the distance of the centre of mass of the lamina from *AD* and obtain the expression for the distance of the centre of mass of the lamina from *AB*. Candidate responses included variety of approaches for solving this problem. A minority of candidates divided shape *AFED* into a rectangle and a right-angled triangle. Many candidates opted instead for a 'subtraction' method, removing a triangle and a rectangle from the square *ABCD*. Numerous other approaches were also provided and credited accordingly. When the correct result is quoted following a solution that contains algebraic or manipulation errors, this cannot be awarded full credit.

(b) The majority of candidates were able to provide a correct expression for $\tan\theta$. Some responses gave the reciprocal of the required fraction for $\tan\theta$ or used $\tan\theta = \frac{\bar{y}}{\bar{x}}$. Drawing a sketch of the situation could help avoid such errors and help weaker candidates make further progress.

Question 6

(a) Most candidates answered this part well by writing down the equations for the conservation of momentum and Newton's experimental law. Successful candidates clearly showed their method for solving these equations to obtain the given expression for the speed of sphere *B*. Candidates are reminded that full credit can only be awarded following convincing working when an answer is given within the question.

(b) Successful responses found the speed of sphere *A* after the collision by recognising that the corresponding velocity has two components: one along the line of centres found from the equations in **part (a)**, and one perpendicular to the line of centres. Common errors in this part were either omission of the perpendicular component of the speed of *A* after the collision, or algebraic errors in manipulating the kinetic energy equation.

Question 7

Successful candidates recognised that the greatest and least tensions are at the lowest and highest points of the circular motion, respectively. These candidates clearly demonstrated their application of Newton's second law at the highest and lowest points, determined two appropriate energy equations, and made use of the ratio given in the question. This gives five equations: two from Newton's law, two from energy considerations, and one from the ratio of the tensions. Errors in algebraic manipulation or simplification often prevented candidates combining these equations to find a value for $\cos\theta$. Common errors included additional trigonometric terms in the tension equations, sign errors in energy equations, and applying the ratio incorrectly.

FURTHER MATHEMATICS

Paper 9231/41
Further Probability and Statistics 41

Key messages

- In all questions, sufficient method must be shown to justify answers.
- Care must be taken with the language used when interpreting the result of any test.
- When a result is given in a question, candidates should give sufficient detail in their working so that the offered solution is clear and complete.
- Candidates need to work to the required level of accuracy of three significant figures (3sf). To maintain 3sf accuracy in a final answer, all intermediate working must be to at least four significant figures.

General comments

The language used when interpreting results of hypothesis tests should recognise that a hypothesis test is not a proof, and it is not appropriate to use definitive statements. Concluding statements should always include some degree of uncertainty, for example, 'there is insufficient evidence to support the claim that…' rather than 'the test proves that…'.

When the answer is given in the question, it is essential that candidates show their working in full, even if it involves the use of elementary algebra.

Comments on specific questions

Question 1

Successful candidates demonstrated a sufficient level of working and used appropriate language to interpret the outcome of the test. Some candidates used biased estimates of the population variances or used the pooled sample variance.

Question 2

(a) The diagram was given to encourage candidates to consider the area under the graph to find a. Candidates who did this were almost always successful. A number of candidates formed equations in a, b, c. Some candidates then substituted the given value of a without demonstrating how the value of a is obtained.

(b) Candidates typically performed well in this part. Successful candidates clearly showed the integrals that they were using. A minority of candidates considered the integral of $\mathrm{f}(x)$ instead of $x\mathrm{f}(x)$.

(c) Most candidates identified that the median was between 0 and 5 and successfully proceeded to use the given value of a to obtain the correct answer.

(d) This part was challenging for a number of candidates. Most candidates attempted to find the CDF by integrating the PDF of f. The most common error related to the constant for the second part of the CDF, which was either omitted or calculated incorrectly. The substitution was generally done well, however a minority of candidates substituted y^2 instead of $\sqrt{y}$. A small number of candidates spoiled an otherwise correct solution by not specifying the CDF of Y for all real values of y, usually by omitting $G(y) = 0$ for $y < 0$.

Question 3

(a) Some candidates were unable to use the information given in the table to calculate the total number of broken eggs in the sample. A minority of candidates estimated the mean number of broken eggs rather than the probability that an egg is broken.

(b) Candidates who summed the expected frequencies were almost always successful. Correct solutions using the binomial distribution were also seen.

(c) Successful candidates provided hypotheses that mentioned both the model/distribution and the data. Some candidates provided hypotheses that were too brief or vague. The test statistic was commonly calculated correctly. A minority of candidates did not combine cells.

(d) Only the strongest candidates were able to provide an appropriate reason in the context of the problem. A common error was to refer to the actual data instead of the situation being modelled.

Question 4

(a) Candidates who calculated the pooled sample variance often went on to produce fully correct solutions. Common mistakes included combining the individual sample variances or choosing an incorrect t-value. A small number of candidates chose a z-value.

(b) Only the strongest candidates made the connection between the confidence interval containing zero and the hypothesis test presented in the question.

Question 5

(a) Hypotheses given in words often omitted **population** when discussing medians. A minority of candidates made errors with the inequality for the alternative hypothesis. Most candidates obtained the correct test statistic but sometimes compared this with an incorrect critical value (commonly 8).

(b) Many candidates were successful in calculating the required probability and reached the correct conclusion. A comparison with the result that had been obtained in **part (a)** was commonly omitted.

(c) Stronger candidates were able to demonstrate their understanding of the context to identify the flaw in the design of the experiment.

Question 6

(a) Many candidates successfully used the fact that $G_Y(1) = 1$ to obtain the correct expression for k.

(b) Many fully correct solutions using a series expansion were seen. Weaker candidates attempted to set $t = 2$ to calculate $P(Y = 2)$.

(c) Most candidates made successful attempts to differentiate the probability generating function with a number of fully correct responses seen. Common errors included numerical or algebraic slips in performing the differentiation.

FURTHER MATHEMATICS

Paper 9231/42
Further Probability and Statistics 42

Key messages

- In all questions, sufficient method must be shown to justify answers.
- Care must be taken with the language used when interpreting the result of any test.
- When a result is given in a question, candidates should give sufficient detail in their working so that the offered solution is clear and complete.
- Candidates need to work to the required level of accuracy of three significant figures (3sf). To maintain 3sf accuracy in a final answer, all intermediate working must be to at least four significant figures.

General comments

The language used when interpreting results of hypothesis tests should recognise that a hypothesis test is not a proof, and it is not appropriate to use definitive statements. Concluding statements should always include some degree of uncertainty, for example, ‘there is insufficient evidence to support the claim that…’ rather than ‘the test proves that…’.

When the answer is given in the question, it is essential that candidates show their working in full, even if it involves the use of elementary algebra.

Comments on specific questions

Question 1

Successful candidates demonstrated a sufficient level of working and used appropriate language to interpret the outcome of the test. Some candidates used biased estimates of the population variances or used the pooled sample variance.

Question 2

(a) The diagram was given to encourage candidates to consider the area under the graph to find a. Candidates who did this were almost always successful. A number of candidates formed equations in a, b, c. Some candidates then substituted the given value of a without demonstrating how the value of a is obtained.

(b) Candidates typically performed well in this part. Successful candidates clearly showed the integrals that they were using. A minority of candidates considered the integral of $\mathrm{f}(x)$ instead of $x\mathrm{f}(x)$.

(c) Most candidates identified that the median was between 0 and 5 and successfully proceeded to use the given value of a to obtain the correct answer.

(d) This part was challenging for a number of candidates. Most candidates attempted to find the CDF by integrating the PDF of f. The most common error related to the constant for the second part of the CDF, which was either omitted or calculated incorrectly. The substitution was generally done well, however a minority of candidates substituted y^2 instead of $\sqrt{y}$. A small number of candidates spoiled an otherwise correct solution by not specifying the CDF of Y for all real values of y, usually by omitting $G(y) = 0$ for $y < 0$.

Question 3

(a) Some candidates were unable to use the information given in the table to calculate the total number of broken eggs in the sample. A minority of candidates estimated the mean number of broken eggs rather than the probability that an egg is broken.

(b) Candidates who summed the expected frequencies were almost always successful. Correct solutions using the binomial distribution were also seen.

(c) Successful candidates provided hypotheses that mentioned both the model/distribution and the data. Some candidates provided hypotheses that were too brief or vague. The test statistic was commonly calculated correctly. A minority of candidates did not combine cells.

(d) Only the strongest candidates were able to provide an appropriate reason in the context of the problem. A common error was to refer to the actual data instead of the situation being modelled.

Question 4

(a) Candidates who calculated the pooled sample variance often went on to produce fully correct solutions. Common mistakes included combining the individual sample variances or choosing an incorrect t-value. A small number of candidates chose a z-value.

(b) Only the strongest candidates made the connection between the confidence interval containing zero and the hypothesis test presented in the question.

Question 5

(a) Hypotheses given in words often omitted **population** when discussing medians. A minority of candidates made errors with the inequality for the alternative hypothesis. Most candidates obtained the correct test statistic but sometimes compared this with an incorrect critical value (commonly 8).

(b) Many candidates were successful in calculating the required probability and reached the correct conclusion. A comparison with the result that had been obtained in **part (a)** was commonly omitted.

(c) Stronger candidates were able to demonstrate their understanding of the context to identify the flaw in the design of the experiment.

Question 6

(a) Many candidates successfully used the fact that $G_Y(1) = 1$ to obtain the correct expression for k.

(b) Many fully correct solutions using a series expansion were seen. Weaker candidates attempted to set $t = 2$ to calculate $P(Y = 2)$.

(c) Most candidates made successful attempts to differentiate the probability generating function with a number of fully correct responses seen. Common errors included numerical or algebraic slips in performing the differentiation.

FURTHER MATHEMATICS

Paper 9231/43
Further Probability and Statistics 43

Key messages

- In all questions, sufficient method must be shown to justify answers.
- When a result is given in a question, candidates should give sufficient detail in their working so that the offered solution is clear and complete.
- Candidates need to work to the required level of accuracy of three significant figures (3sf). To maintain 3sf accuracy in a final answer, all intermediate working must be to at least four significant figures.
- Care must be taken with the language used when interpreting the result of any hypothesis test.

General comments

The language used when interpreting results of hypothesis tests should recognise that a hypothesis test is not a proof, and it is not appropriate to use definitive statements. Concluding statements should always include some degree of uncertainty, for example, ‘there is insufficient evidence to support the claim that…’ rather than ‘the test proves that…’.

When the answer is given in the question, it is essential that candidates show their working in full, even if it involves the use of elementary algebra.

Comments on specific questions

Question 1

Most candidates were successful in answering this question. The hypotheses were sometimes incomplete, for example with the null hypothesis stated as ‘It is a good fit’. A complete statement should specify the distribution being considered and refer to the data or context. Conclusions were sometimes given without the required degree of uncertainty with words such as ‘prove’ being used.

Question 2

Many candidates provided excellent solutions to this part, which were both accurate and clearly organised. Some solutions rounded the s^2 values to an insufficient degree of accuracy, impacting accuracy of the final answer. Working is clearer for examiners when presented in stages rather than combined into a single formula before any evaluation; candidates using such an approach are typically more successful. Most candidates used the correct value for z in their confidence interval, but a common error was the use of 1.282 instead of 1.645.

Question 3

(a) This part was particularly well attempted, but as in all ‘show that’ questions it is important for candidates to show all stages of their working to demonstrate a convincing argument that reaches the given answer.

(b) Successful candidates realised that the median must lie in the second part of the piecewise function. A common error made was equating the area between 1 and m to 0.5.

(c) Many candidates successfully provided complete solutions. Common errors included omission of the constant term in $F(x)$, omitting or providing incorrect intervals for y, or omission of ‘0 otherwise’.

Question 4

Many successful attempts at the Wilcoxon rank-sum test were seen. The hypotheses here should refer to the 'population median' (not merely the 'median'). Some responses did not clearly state 'less than 25' and 'more than 50'. Any abbreviations should be unambiguous or clearly defined. Final conclusions were not always given in context and with the required degree of uncertainty; words such as 'prove' are not acceptable.

Question 5

(a) Candidates performed well in this part question. Typical errors included not specifying the hypotheses correctly or confusing inequality signs when finding the range of values for k.

(b) Successful candidates were precise in their language when answering this part question. Weaker responses merely stated that the differences, rather than population differences, needed to be normally distributed. Other responses did not mention 'differences' explicitly.

Question 6

(a) Many candidates were successful in this part question. Successful candidates recognised that there are either one or two different colours present in Kieran's selection, meaning Y could take the values 1 or 2. Special cases were given for candidates that interpreted the phrasing in an alternative manner.

(b) Only the strongest candidates performed well in this question. Many candidates did not realise that X and Y were not independent.

(c) Many candidates attempted to multiply their probability generating functions for X and Y, despite the clue in **part (b)** suggesting that such a method is not appropriate. Successful candidates showed full working, often making each of the relevant cases clear, to demonstrate how they arrived at their probability generating function.

(d) Many candidates presented a suitable method using their answer to **part (c)** to find $E(Z)$. Use of methods to find $E(Z)$ that did not make use of the probability generating function could not be awarded credit.

FURTHER MATHEMATICS

Paper 9231/44
Further Probability and Statistics 44

Key messages

- In all questions, sufficient method must be shown to justify answers.
- Care must be taken with the language used when interpreting the result of any test.
- When a result is given in a question, candidates should give sufficient detail in their working so that the offered solution is clear and complete.
- Candidates need to work to the required level of accuracy of three significant figures (3sf). To maintain 3sf accuracy in a final answer, all intermediate working must be to at least four significant figures.

General comments

The language used when interpreting results of hypothesis tests should recognise that a hypothesis test is not a proof, and it is not appropriate to use definitive statements. Concluding statements should always include some degree of uncertainty, for example, ‘there is insufficient evidence to support the claim that…’ rather than ‘the test proves that…’.

When the answer is given in the question, it is essential that candidates show their working in full, even if it involves the use of elementary algebra.

Comments on specific questions

Question 1

Many candidates were successful in carrying out the hypothesis test. Common errors included using a z critical value instead of a t value or forgetting to divide the estimated population variance by 12 when finding the standard error. Most candidates used the correct critical value and stated an appropriate conclusion. Conclusions were sometimes given without the required degree of uncertainty with words such as 'prove' being used.

Question 2

(a) Almost all candidates calculated the test statistic correctly, and many went on to complete the test correctly. Some candidates used the wrong critical value, usually 10. Many candidates did not state the hypotheses correctly, often omitting the words population and/or median. A majority of candidates successfully concluded that the null hypothesis should be accepted (or equivalent), but a number of candidates did not then have a fully correct explanation, either missing out the word average or using phrasing such as "prove" rather than "suggest".

(b) Successful candidates referred to it being unknown whether the population or underlying distribution was normal. Many candidates made reference to the lack of a normal distribution but inappropriately referred to the distribution of the sample/data, of the differences, or of the mean.

Question 3

Most candidates correctly found the expected values and the chi-square test statistic. The main areas for development are in the hypotheses and the conclusion in context. Some candidates did not fully define the hypotheses, often saying type of item or just item rather than type of item sold. When giving the conclusion in context, some candidates stated that there is sufficient evidence to suggest that there is independence rather than insufficient evidence to suggest dependence.

Question 4

(a) Most candidates were successful in solving this problem. As in all ‘show that’ questions, it is important for candidates to show all stages of their working to demonstrate a convincing argument that reaches the given answer.

(b) Most candidates attempted to find the CDF by integrating the PDF of f. The most common error related to the constant for the second part of the CDF, which was either omitted or calculated incorrectly.

(c) Successful candidates realised that the median must lie in the second part of the piecewise function. Some candidates could have solved this problem more efficiently by making use of their correct F(x), rather than integrating f(x) again.

(d) Many correct responses were seen. Common errors included attempts to integrate $\frac{1}{x}\mathrm{F}(x)$ or $\mathrm{f}\left(\frac{1}{x}\right)$.

Question 5

(a) Most candidates were successful in solving this problem.

(b) Many correct responses were seen. The most common errors were algebraic or arithmetic slips in multiplying the generating functions.

(c) Many fully correct responses were seen. Candidates needed to make use of their probability generating function in order to be awarded full credit.

Question 6

(a) A variety of correct approaches were seen with some candidates using symmetry to obtain $\bar{x}$ and hence Σx, with others finding Σx by solving their simultaneous equations formed using the two confidence limits. Whilst many fully correct responses were seen, a common mistake was choosing an incorrect t-value with a small number of candidates choosing a z-value.

(b) Candidates who defined their hypotheses using symbols were mostly successful, although the alternative hypotheses were sometimes incorrect due to the wrong inequality or use of a two-tailed test. Some candidates who defined their hypotheses in words omitted the word population. Most candidates performed the test correctly, but a minority of candidates used separate variances rather than the pooled variance.