ADDITIONAL MATHEMATICS

Paper 4037/11
Non-calculator

Key messages

Candidates generally performed well on differentiation and coordinate geometry, with many successfully applying the quotient and chain rules. Expansion, factorisation, and simplification were often correctly attempted, though sign errors and inaccuracies with inequality regions were common. Questions involving modulus functions, logarithmic manipulation, and trigonometric identities proved more challenging for candidates, particularly in handling domain restrictions and applying standard identities. Proof and reasoning-based questions highlighted the importance of clear, logical progression.

General comments

Many candidates approached the questions methodically and showed good understanding of core techniques such as differentiation, coordinate geometry, and algebraic manipulation. A significant number of candidates displayed an incomplete understanding of vectors, logarithmic theory and combinations particularly in **Questions 1(a)**, **3(b)**, **8(b)** and **12**. It is evident that many would benefit from further practice in these areas.

Comments on specific questions

Question 1

A significant number of candidates incorrectly multiplied by $\frac{1}{4}$ instead of $-\frac{1}{4}$. Another common error was subtracting a component of the vector when it should have been added.

Question 2

Most candidates were able to expand the brackets and simplify successfully, though some errors persisted, primarily related to sign handling. In this and other questions, some candidates, upon seeing the left-hand side of the equation in factorised form, incorrectly equated it to either the right-hand side or to zero – even when the right-hand side was an algebraic term or a constant. The most common error was that after identifying the correct critical values, candidates selected the incorrect region. Those who supported their work with a small sketch of the function were generally more successful in identifying the correct solution set.

Question 3

(a) This part was generally well answered.

(b) This part was also generally well answered. A minority of candidates confused *x* and *y* coordinates. Most candidates used the centre point effectively, though a few attempted a method involving simultaneous equations leading to a quadratic.

(c) This part was also generally well answered. Errors included calculating the gradient with the *x* and *y* values reversed. A few candidates correctly calculated the gradient of the radius but did not take the negative reciprocal when finding the gradient of the tangent.

Question 4

(a) This question proved challenging for many candidates. It required an appropriate substitution. Approaches involving logarithms, squaring individual terms, or raising each term to the sixth power were ineffective. The simplest substitution was $y = 2^x$, leading to a solvable equation after rearrangement. Candidates needed to reject one of the solutions upon solving. An alternative substitution that was also valid resulted in a more complex equation to solve.

(b) The key to this part was correctly raising both sides of the initial equation to the power of 10, leading to a solvable equation. This then required standard substitution, rearrangement, and solving a quadratic, which most candidates did well. Sign errors were again the most common cause of mistakes. If $y = 3^x$ was substituted from the start, it led to an equation for which many candidates only gave one solution, often omitting $x = 0$.

Question 5

(a) Some candidates attempted to solve the modulus equation $|2x - 1| = 3$ without first splitting it into two linear cases, leading to incorrect solutions. Errors in manipulating the modulus algebraically were common, particularly failing to consider both the positive and negative cases.

(b) Sine graphs were occasionally misdrawn, with errors in amplitude, midline shifts, or misunderstanding of the period.

Question 6

(a) Most candidates made a good attempt at this part, with many earning 4 or 5 marks. Almost all successful solutions applied the quotient rule rather than the product rule. There were two main approaches: either differentiating the algebraic fraction within the brackets and combining this with the outer differential, or directly differentiating the numerator and denominator and applying the quotient rule. Both approaches were valid and required correct use of the quotient and chain rules.

(b) (i) Fully correct responses showed that the denominator needed only to equal zero, then correctly deduced the three values of x by solving the resulting equations. Where verification was attempted, candidates often struggled to communicate clearly what they were attempting to show.

(ii) Few candidates attempted to find gradients other than at the three specified points, where the gradient was already known to be zero. Those who assessed the gradient on either side of each point were generally successful, though many candidates would benefit from further practice in explaining their conclusions.

Question 7

Many candidates found this question challenging. Some candidates simplified the problem quickly by taking note of the constant term of -3, and testing for roots such as $x = 1$, $x = -1$, $x = 3$ or $x = -3$. While some candidates used long division or synthetic division, errors were often made in algebraic manipulation when determining the quotient. A few candidates incorrectly attempted to apply the quadratic formula directly to a cubic. Successful candidates typically identified a root using substitution or the factor theorem, then reduced the cubic to a quadratic.

Question 8

(a) Most candidates recognised that the argument of a logarithm must be greater than zero. Many rearranged correctly, although some provided a non-strict inequality and therefore did not receive full marks.

(b) The majority of candidates understood the need for a change of base, often selecting the simpler base on the right-hand side. Those who worked on the expression outside of the equation before reintroducing it were more successful. After applying the change of base, most candidates knew the necessary logarithmic rules to form a quadratic equation, though some failed to convert the constant term (1) into a logarithmic form, affecting their final answer.

Question 9

Most candidates successfully identified $y = 3$, differentiated correctly, and formed a correct equation for the tangent. Those who obtained the correct equation were generally able to find the x-intercept. However, few candidates were successful in calculating the correct area of the triangle – whether by integration or by using $\frac{1}{2} \times$ base $\times$ height. Integration was generally well attempted, though some candidates mishandled the division by $\frac{3}{2}$. A small number used an incorrect power of $\frac{1}{2}$ instead of $\frac{3}{2}$ in the integral. Those applying incorrect limits did not always show substitution steps, meaning they could not be credited for their method.

Question 10

(a) Many candidates forgot or misapplied key identities such as $\sec^2\theta = 1 + \tan^2\theta$, resulting in algebraically incorrect steps. The omission of the angle symbol θ throughout the proof was common, and reduced clarity. The best responses demonstrated a careful, step-by-step application of trigonometric identities, with clear justification for each transformation.

(b) Many candidates made a correct start using $\cos x = \frac{1}{\alpha}$ however they tried to use Pythagoras or $\sin^2 x + \cos^2 x = 1$ and failed in the algebraic manipulation. Only a few candidates gained full credit for identifying it as a negative term.

Question 11

(a) This part was generally well answered. A small number of candidates failed to identify $10 - 2d$ and $10 - d$, though some correctly reached the equation $a + 2d = 10$.

(b) Candidates who correctly identified the first two terms were typically able to square the first three terms accurately. Candidates who found $a = 2$ explicitly were generally more successful than those who began with $10 - 2d$ and substituted $d = 4$ later into the sum formula for 200 terms.

Question 12

Most candidates were able to secure the initial mark. However, further manipulation of expressions proved challenging for many.

ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 1 Non-calculator

Key messages

Candidates appeared well prepared for the new syllabus and new non-calculator paper. Some candidates would benefit from further practice of simplification skills. There appeared to be no problems with timing and most candidates had sufficient room for their solutions, setting them out well in most cases.

General comments

The most successful candidates simplified numerically as soon as possible in each question. As this is a non-calculator paper, this approach reduced the risks of calculation errors being made. The same applies to algebraic simplification. **Question 7(a)** was a prime example of where this needed to be done.

Questions should be read carefully so that the demands of the question are met fully and answers given in the correct form. Careful reading will also prevent misinterpretation of a situation such as in **Question 12**.

Comments on specific questions

Question 1

This question required expressions in fully factorised form. Some candidates did not give their final answer in this form. Most candidates were able to obtain at least 2 marks, usually for correct factors and for $\pm$. The final answers were not expected to include any modulus, but credit was given for correct factors and $\frac{1}{3}$ if included.

Question 2

Most candidates recognised that a quadratic equation in $x^{\frac{1}{3}}$ could be formed by multiplying the given equation throughout by $x^{\frac{1}{3}}$. Some candidates made successful use of a substitution to help with the solution of the resulting quadratic equation. Other candidates were just a successful without using a substitution. The resulting quadratic equation was correctly solved by most, however some candidates chose to discard the solution $x^{\frac{1}{3}} = -3$, and some candidates attempted to find the cube root of -3 and 2 rather than the cube of -3 and 2.

Question 3

(a) This topic is a new addition to the syllabus. Many candidates did not read the question carefully enough and attempted to rearrange the given circle equation to the form $(x-a)^2 + (y-b)^2 = r^2$, and were unable to progress further. Successful candidates considered the intersection of the line and the circle and then made use of the discriminant of the resulting equation. Many candidates did just this, with some solving the quadratic equation and others using the discriminant correctly. Of those who solved the quadratic equation, fully correct solutions included a comment about there being only one root or point of contact.

There were plausible alternative methods, but few candidates were able to complete them fully, giving enough detail to show that the line is a tangent.

(b) Most candidates were able to obtain the correct coordinates of the point *P* provided they had a correct quadratic equation in **part (a)**.

(c) Many correct solutions were seen, showing candidates dealing with the new syllabus item correctly. Most realised that they needed to find the radius of the new circle and then use $(x-a)^2 + (y-b)^2 = r^2$. Follow through credit was available to candidates who had an incorrect answer in **part (b)** but made use of a correct method.

Question 4

(a) It is expected that candidates have a basic knowledge of the graphs of $y = \sin x$ and $y = \cos x$, together with the values that these functions have at the quadrant boundaries. Many candidates obtained $\left[-\cos\theta\right]_0^{\pi}$ or equivalent, but few recalled that $\cos\pi = -1$ or made sign errors which did not result in the correct answer of -2.

(b) Most candidates showed sufficient detail and used correct equivalent trigonometric ratios and identities to obtain the given result. The majority of candidates obtained full marks.

Question 5

(a) Candidates that did not recognise the notation $\mathrm{p}'(-1)$ were usually not successful. A large number of candidates used $\mathrm{p}(-1)$ instead. Of the candidates that did recognise and use the notation correctly, most were able to obtain 2 correct equations and consequently the correct values of *a* and *b*.

(b) Successful candidates made use of the linear factor $x-2$ to find a quadratic factor. Most attempted algebraic long division, but some used observation. Candidates with incorrect values in **part (a)** were able to obtain credit for obtaining a quadratic factor whose first 2 terms were $3x^2 - x$ as these terms were not affected by the values of *a* and *b*. Correct factorisation of a correct quadratic factor usually followed.

(c) Many candidates realised that they could make use of their solutions in **part (b)** and use the fact that *x* could be replaced by e^{2y}. Candidates with incorrect values of *a* and *b*, were again able to gain marks, in this case both marks associated with the solution of $\mathrm{e}^{2y} = 2$. Fully correct solutions relied on $\mathrm{e}^{2y} = -1$ being either rejected or not considered.

Question 6

(a) Many completely correct solutions were seen, although some candidates did not give their final answer in the correct form. Any errors were usually due to substitution of incorrect values.

(b) There were mixed responses to this question part with some candidates not equating the exponential indices. Of those that did, most were able to obtain the correct answer.

Question 7

(a) Successful responses simplified both numerical and algebraic factors as early as possible in their calculations. Dividing through by common numerical factors is an essential skill needed in a non-calculator paper. Long multiplication in this case was unnecessary. Most candidates were able to write or imply the equations $ar^3 = \frac{8k^6}{27}$ and $ar^5 = \frac{32k^{10}}{243}$. However, many candidates were unable to solve correctly these 2 equations simultaneously to obtain either the common ratio or the first term. Errors usually occurred when there was no simplification by division of common numerical factors, with candidates multiplying out unnecessarily large numbers and then not being able to find the square root of them. A common error was to find the common ratio but forget to find the value of the first term. Many candidates did not consider how they were setting out their solution, often making their work difficult to follow.

(b) Of the candidates whose answers in **part (a)** were in the correct form, most were able to obtain a mark for the correct use of the sum to infinity formula. Some candidates gave an answer of 1 rather than ±1, incorrectly discounting the negative root.

Question 8

(a) Most candidates attempted differentiation of a quotient, usually using the correct form. Some chose to use the differentiation of a product which was just as acceptable provided it was a correct product. Errors usually occurred with the incorrect differentiation of $\ln\left(3x^2+16\right)$. The form of the final answer was given, however some candidates did not simplify their correct work sufficiently and answers of $-\ln 2$ were common.

(b) Most candidates realised that multiplication of their answer to **part (a)** by *h* was needed, but a mark was awarded only if their answer to **part (a)** was a single logarithm and there was also no ambiguity in the way the answer was written.

Question 9

(a) Most candidates were able to give the correct least value of *a*.

(b) Fewer candidates recognised that the range was the set of real numbers.

(c) It was pleasing to see that many candidates were able to find the inverse function correctly, using correct final notation.

(d) Candidates were told that there were 2 roots to the equation $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$. This implies that there are 2 points of intersection needed when sketching the curves. A correctly shaped curve of $y=\mathrm{f}(x)$ was an essential start. Candidates needed to produce a sketch that tended towards an asymptote in the 4th quadrant and curved correctly in the first quadrant. Many candidates sketched curves that appeared to become horizontal or have an implied maximum point in the first quadrant. Few candidates had 2 points of intersection, although many indicated correctly identified asymptotes.

Question 10

(a) Most candidates realised that the required area was made up of a circle sector with a triangle removed. Most candidates obtained a correct sector area, however there were often subsequent simplification errors. The sine rule for the area of a triangle was used by most candidates.

(b) This question part was attempted less successfully. It was essential that the correct angles were worked out so that the arc lengths *AB* and *EF* could be calculated. The length of *BE* also needed to be calculated. Most chose to use the cosine rule, but incorrect simplification was common. Other methods were acceptable and often contained less errors.

Question 11

(a) Many completely correct solutions were seen, often with working set out that was easy to follow. The key to this question part was finding $\overrightarrow{PQ}$, which most candidates did, some taking a longer approach than others. Adding $\overrightarrow{OP}$ resulted in correct answers. Very few candidates were unable to deal with the given ratios correctly.

(b) Fewer candidates obtained full marks for this question part. Many did not equate like vectors and so were unable to obtain any marks.

Question 12

It is essential that candidates read the question carefully. Many tried to obtain the gradient by differentiating $(5x-2)^{\frac{1}{3}}$, which was already the gradient. The correct method was to find the equation of the curve in order to find the *y* coordinate of the stationary point, having worked out the *x* coordinate from the given gradient.

Some candidates, having found the equation of the curve, then differentiated it to find the x coordinate of the stationary point instead of using the given gradient, making more work, often with errors, for themselves.

ADDITIONAL MATHEMATICS

Paper 4037/21
Calculator

Key messages

- Candidates should read each question carefully and take care that they have answered each question fully.
- When a particular method is specified in the question, for example completing the square, then candidates must ensure they use that method as alternative methods may not be credited.
- Candidates should consider if two parts of a question are related and whether an earlier part of a question can be used to help them answer a later part.
- When integrating, candidates should remember to include a constant of integration.
- Candidates should take care to set out their working clearly so their method is easy to follow.

General comments

Most candidates appeared to have sufficient time to answer this paper. Candidates who performed well on this paper took care to use clear and logical steps in their method, ensuring correct use of brackets. Candidates who were disorganised in their working were more likely to make slips in their working or miss out part of the question.

Candidates should take care when working with a ‘dummy variable’ to state the substitution that they are using clearly and to return to the original variable when giving their final answers. Candidates should ensure that answers in radians are given correct to 3 significant figures and that they work with at least 4 significant figures for any interim calculations to avoid errors from premature rounding.

Comments on specific questions

Question 1

(a) In this part of the question, candidates need to complete the square in order to find the coordinates of the turning point. Most candidates were able to make a good attempt at completing the square but some sign slips were made. Once they had completed the square, some candidates did not go on to give the coordinates of the stationary point. Candidates should note that it is worth re-reading a question to check that all parts of it have been answered before moving on. A few candidates did not use the required method of completing the square and used differentiation to find the stationary point, this was not given any credit.

(b) The majority of candidates drew the correct quadratic curve and labelled the intercepts on the axis correctly. Some candidates assumed the vertex would lie on the y-axis or drew a correct curve but did not label the intercepts.

Question 2

Most candidates correctly used the discriminant to form a correct quadratic inequality in k. Candidates then correctly solved the resulting quadratic to find the correct pair of critical values. Higher performing candidates usually then made a sketch of the quadratic in k to help them find the correct set of values for k.

Common errors in this question were to give incorrect inequalities for k from either not drawing a sketch or from thinking that $b^2 - 4ac \geqslant 0$ when there are two distinct roots. Some candidates incorrectly gave their final inequality in x rather than k.

Question 3

(a) Many candidates were well-prepared for this question and correctly identified that the three groups (runners, swimmers, gymnasts) should be treated as distinct blocks. They correctly applied the formula for arranging groups (3!) and the internal permutations (5!, 4!, 3!). A few did not consider the different arrangements for the 3 groups.

Candidates who did less did not use the product rule for counting and attempted to use permutations and combinations.

(b) Many candidates handled all three cases (no runners, no swimmers, no gymnasts) correctly and logically. A few candidates applied the complement method to subtract invalid cases where one or more groups were entirely excluded, but did not take into account their over-counting.

Candidates who made little progress often did not know whether they should multiply or add the combinations and did not attempt a diagram to help them make the correct choice.

Question 4

This unstructured coordinate geometry question was well answered with many candidates drawing a helpful diagram and making a sensible plan. Many candidates correctly found the equation of the perpendicular bisector of AB and then the coordinates of the point E. A few arithmetic slips were made and candidates are advised to check their working carefully. The final step of calculating the area of the triangle CDE proved difficult for many candidates.

Candidates who made less progress with this question either used the coordinates of A to find the equation of the perpendicular bisector rather than the midpoint of AB or wrongly assumed the gradient of the perpendicular of AB would also be perpendicular to the line L. These candidates may have been helped if they had drawn a diagram of the information given in the question.

Question 5

Most candidates were familiar with the product rule for differentiation and were able to attempt the derivative. When errors occurred, they were usually in the differentiation of $\tan\left(\frac{x}{2}\right)$ and often this was given as

$\frac{1}{2}\sec\left(\frac{x}{2}\right)$ rather than $\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$.

Candidates were less well prepared for using the small changes relationship and often just calculated the derivative at the point where $x = \frac{\pi}{3}$. Examiners were looking for the correct small change relationship of

$\frac{\delta y}{h} = \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=\frac{\pi}{3}}$ to be stated and used.

Question 6

(a) This question was well answered with candidates factorising the quadratic and then correctly expanding $(3x+2)^{10}$. Some candidates did not read the question carefully and did not attempt to factorise the quadratic or else gave their answer in descending powers of x rather than ascending.

Candidates who did less well either ignored one part of the factorisation and expanded $(3x+1)^5$ or made a slip with simplifying the indices and wrongly simplified $((3x+1)^2)^5$ to $(3x+2)^7$.

(b) Many correct answers were seen, and candidates were generally well prepared for this question. Some candidates identified the correct term but made errors in evaluating it. Candidates who were less successful were unable to find the correct term – usually assuming that they needed a power of 6.

Question 7

(a) The responses to this question were extremely variable. Some candidates provided a fully correct sketch of the function and its inverse, but errors were fairly common. The most common error in sketching the original function was sketching an increasing exponential curve. Other candidates sketched a correct curve but either omitted the labels of the axes and the asymptote or labelled them incorrectly.

Many candidates knew the inverse function reflects the original function in the line $y = x$, but only a few drew a mirror line to help them. A few candidates did not seem aware of the correct relationship between the graph of a function and its inverse.

(b) The majority of candidates successfully rearranged $g(x)$ to obtain the correct expression for the inverse function. Very few candidates were able to find the domain fully, and only included $x \geqslant 1$.

Question 8

(a) This question was attempted reasonably well, with many managing to find $\tan x = \frac{3}{2}$. Many candidates only considered one of the factors and usually omitted to examine $\cos x = 0$.

A common mistake was to lose accuracy on the solution of $x = 0.9827$ and round it down to 0.98. Candidates are reminded that angles in radians should be given correct to 3 significant figures.

(b) Most candidates were well prepared for solving a trigonometric equation. The most common method was to use the substitution $x = \sin(2\theta + 1)$ and rearranged to find a quadratic equation.

Candidates who use the substitution method must ensure they state their substitution clearly and return to the original variable. Rounding errors were common in this question – candidates should ensure that intermediate steps are given to at least 4 significant figures to ensure no loss in accuracy in their final answers.

Question 9

(a) The majority of candidates gained the full marks for this part, with those who were able to write down the correct calculation, gaining a fully correct answer. A very common mistake was to use formula for the surface area of a cone with 15 for r and l.

(b) Around half of candidates were successful here. The most common mistake was to wrongly think the radius of the circular top was 15 cm and use $C = 2\pi r$

(c) The success of this question continued for many candidates into this part. Many obtained completely correct solutions, and many others made a correct Pythagoras' statement in an attempt to calculate the radius. A common error was to wrongly think that $r = 15$.

(d)(i) Very few candidates gave correct answers, often because they did not see the link from the previous part of the question. Candidates would have been aided by a diagram to show the similar triangles. The most common incorrect response was $r = \sqrt{15^2 - h^2}$.

(ii) Only a small minority of candidates gave fully correct solutions for this question. The majority of candidates who gave partially correct solutions provided a relevant chain rule. Although many candidates recognised this was a rate of change question many were working with an incorrect expression for r and so made little progress. Candidates often muddled their notation and did not always realise that the depth of the water was h.

Question 10

(a) (i) Most candidates realised they needed to integrate the expression for the acceleration in order to obtain the velocity and a reasonable number of fully correct answers were seen. Common mistakes were to omit the constant of integration or to make an algebraic slip when calculating it

(ii) Candidates were slightly less successful with this part although there were still a good number of correct responses. Most candidates realised they should integrate again but a number misapplied integration rules or again omitted the constant of integration.

(b) In this part of the question candidates needed to work with the acceleration given for $4 \leqslant t \leqslant 10$. Few fully correct answers were seen, although many candidates had a good plan, they often incorrectly integrated the exponential function – either making a sign error or differentiating instead. Again, the constant of integration proved problematic for some candidates who either omitted it altogether or miscalculated it.

Question 11

(a) Nearly all candidates were able to give two correct equations in terms of a and r for the sum of the terms. Candidates who were most successful recognised that they could divide their equations and eliminate a and a good number of fully correct answers were seen. A few candidates found the values of and r, but did not go on to find the value of the 6th term. Candidates are advised to re-read the question to check they have fulfilled all of its requirements.

Some candidates rearranged one of their equations to make a the subject and then substituted this into the other equation. This resulted in a cubic equation which candidates were often unable to solve. A few candidates worked with the formula for the sum of the terms but often forgot to subtract the first term from their expression writing $\frac{a\left(1-r^3\right)}{(1-r)} = 168$ rather than $\frac{a\left(1-r^3\right)}{(1-r)} - a = 168$.

(b) Candidates who had found the correct values of a and r in **part (a)** were usually able to find the correct sum to infinity.

ADDITIONAL MATHEMATICS

Paper 4037/22
Calculator

Key messages

Candidates should read each question carefully and take notice of any key words or phrases.

Candidates need to show all key steps in their method of solution. Candidates should not rely on calculators to solve equations or to work out the values of derivatives or integrals for particular values. Candidates who do this will be omitting necessary working. Calculators should be used as efficient checking tools in these cases. It is expected that candidates use correct algebraic methods when solving equations and candidates who apply numerical methods, such as trial and improvement, at an early stage in the solution will also, most likely, be omitting necessary working.

Candidates should check that their calculator is in the appropriate mode when working with trigonometric expressions, particularly in questions involving calculus or the solving of trigonometric equations.

When a candidate rejects a solution it cannot be credited. Candidates should, therefore, think carefully before they decide to reject values they consider to be extraneous.

Candidates should take care to write their solutions using correct mathematical form and notation, where appropriate. Brackets should be used correctly to group terms when needed. When writing derivatives, the notation used should be appropriate to the derivative. For example, if the initial relationship is $S = 4\pi r$, the derivative should be $\frac{dS}{dr}$ not $\frac{dy}{dx}$.

General comments

Many candidates were able to analyse the problems in this examination and identify suitable strategies to solve them, including using combinations of processes. This was evident in **Questions 8**, **9** and **10**.

A good proportion of candidates were also able to communicate methods and results in a clear and logical form. Other candidates needed to take greater care with the presentation of their work. Disordered solutions, particularly those where candidates had not been careful with mathematical form or notation, sometimes resulted in miscopying or method errors, for example. Presentation was often poor in **Questions 5(b)**, **6**, **7(a)** and **10**, for example. Some candidates made good use of extra paper to rewrite or continue their solutions. It was very helpful when these candidates indicated where their solution continued, or had been written, in their script in these cases.

Candidates should be aware of the mathematical conventions included in the syllabus. Knowledge of the information relating to sketching graphs, communicating mathematically and accuracy stated in this section was necessary for complete success in **Questions 2**, **4(b)** and **8** in this examination.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Most candidates were able to find the correct critical values, either by forming and solving a pair of linear equations/inequalities or by removing the modulus and squaring to form and solve a quadratic equation. Candidates who used a diagram such as a number line or sketch were much more likely to offer a correct pair of inequalities as their final answer. Many candidates stated a final answer of '$x \geqslant -1$, $x \geqslant \frac{1}{5}$' or stated x

$\geqslant -1$, $x \geqslant \frac{1}{5}$ then a final answer of '$x \geqslant \frac{1}{5}$'. A few candidates were unable to solve $5x \geqslant -5$ correctly, with $x \geqslant 1$ being seen on occasion. Some candidates chose to negate $5x + 2$, but only changed the sign of one of the terms. A few other candidates, choosing to form a quadratic, squared $(5x + 2)$ but omitted to square 3. Some candidates stated $5x + 2 \geqslant 0$ and $5x + 2 \leqslant 0$ as well as $5x + 2 \geqslant 3$ and $5x + 2 \leqslant -3$. This often caused confusion as some candidates attempted to incorrectly include $-\frac{2}{5}$ as part of their solution, which unfortunately destroyed the method.

Question 2

In this kinematics question, candidates needed to draw several graphs. In each case, graphs needed to be drawn for $t \geqslant 0$ to satisfy the context of the question. Sketches of cubic and quadratic functions should therefore meet the vertical axis. Continuation of the sketch to the left of the vertical axis was condoned providing it did not destroy the shape of the graph for $t \geqslant 0$. Candidates should be aware that graphs that are drawn by plotting and joining points, instead of using intercepts and key features, are rarely acceptable as smooth curves or straight, ruled lines. When sketching curves, candidates are advised to draw the curve first and mark the intercepts or key points afterwards.

(a) A good proportion of candidates earned both the marks available. Correct solutions contained a maximum point in the first quadrant, a minimum point that was tangential to the t-axis, and correct intercepts explicitly stated. Correct curves also met the negative s-axis. Some other candidates made a reasonable attempt at drawing the curve with all intercepts correctly stated.

(b) Some candidates offered fully correct answers, in a factorised form, as required. As the expression for the velocity was quadratic, the factorised form needed to be a product of simple linear factors. In the best responses, candidates gave the answer $3(t - 2)(t - 4)$.

There were two approaches to answering this part of the question.

In the more common approach, candidates multiplied out $(t - 4)^2(t - 1)$, differentiated to find $3t^2 - 18t + 24$, then factorised. A reasonable number of candidates were completely successful using this method. A few candidates made a minor slip when expanding the expression for s but differentiated correctly earning partial credit. Some candidates found the correct derivative in un-factorised form, but they divided by 3 before factorising and stated the answer $(t - 2)(t - 4)$. A few candidates offered $3(t^2 - 6t + 8)$ as their factorised form.

In the other approach, candidates applied the product rule to $(t - 4)^2(t - 1)$. Again, a reasonable number of candidates were successful using this method. A few candidates did not notice that the terms found using this method had a common factor and multiplied out before factorising. This was not incorrect but sometimes resulted in an unnecessary error. Some candidates made a reasonable attempt at the product rule, using the correct structure, but made a slip with the derivative of $t - 1$ and/or $t - 4$. For example, these terms were sometimes differentiated as t, or -1, or -4. A few candidates offered $(t - 4)(2(t - 1) + (t - 4))$ as their factorised form.

(c) Many candidates provided fully correct solutions. Attempts at drawing the curve needed to meet the positive v-axis. A few candidates earned partial credit for a reasonable attempt at drawing the curve with all intercepts correctly stated, for example, curves that were smooth curves for the minimum below the t-axis, but had clearly ruled sections above the t-axis.

In weaker responses candidates often drew:

- curves that were quadratic but did not intersect with the v-axis or
- curves that were sinusoidal, having one or more turning points at the ends or
- a single line or a series of connected or disjoint straight lines.

(d) A good proportion of candidates earned the mark for correctly differentiating their 3-term quadratic expression for v. Some candidates used the product rule again. Some of these did not simplify their answers to **part (b),** resulting in a lot of work in this part, and increasingly the likelihood of errors. A few candidates stated a correct expression for a, then divided through by 6. In the weakest responses, candidates sometimes attempted to integrate their expression for *v* or stated and used $a = \frac{v}{t}$ or similar.

(e) A good proportion of candidates were able to draw a single ruled line, with correct intercepts and which met the *a*-axis and intersected the *t*-axis.

Some candidates drew lines which met the *a*-axis and intersected the *t*-axis however:

- was ruled and had one incorrect intercept, for example 18 rather than –18, or
- was ruled and had two correct intercepts, but stopped on the *t*-axis, or
- was a good freehand and had two correct intercepts.

Question 3

Many candidates performed well on this question and most earned at least 3 of the 4 marks available. Almost all candidates chose the correct order of composition, putting g(*x*) into f(*x*). Many candidates then multiplied by $\sqrt{x+2}+4$ to form an equation free of algebraic fractions. The simplest way to proceed was to collect terms in $\sqrt{x+2}$ and then solve. Many candidates did this successfully. A few candidates stated, for example, *x* + 2 = ±4 and then went on to give the answer *x* = 2, discarding the negative solution. $\sqrt{x}+\sqrt{2}$.

Some candidates used methods that required the solution of a quadratic equation in *x* or $\sqrt{x+2}$. In these cases, a solution that needed to be discarded was also found. Not all candidates discarded the incorrect solution. Examples of these methods were:

- Squaring both sides of $3\sqrt{x+2} = \sqrt{x+2}+4$. This was sometimes successful, but not always, as many omitted $8\sqrt{x+2}$ on the right-hand side.
- Squaring both sides of $2\sqrt{x+2} = 4$. This was more successful, but some candidates omitted to square the 2 on the left-hand side when using this approach.
- Simplifying the algebraic fraction by multiplying it by the conjugate of the denominator, stating $\frac{3\sqrt{x+2}}{\sqrt{x+2}+4} \times \frac{\sqrt{x+2}-4}{\sqrt{x+2}-4} = 1$. While some candidates were successful using this more-complex approach, it often resulted in an error being made before the equation was free of algebraic fractions.

A small number of candidates used alternative approaches. Some of these candidates stated $\mathrm{g}(x) = \mathrm{f}^{-1}(1)$ and then found and used an expression for the inverse of f. This was fairly successful. A few of these candidates found the value of *x* that resulted in f(*x*) being 1 and then solved g(*x*) equal to that value. This was also reasonably successful.

In the weakest responses, candidates attempted to use the product f(*x*) × g(*x*) rather than the composite function or attempted to find fg(1).

Question 4

(a) A good proportion of candidates used the product rule to state a correct derivative and completed the answer successfully. A few candidates made an error when entering the expression into their calculator, and some candidates had their calculators in degree mode rather than radian mode. Other candidates made an error when differentiating one or both of cos2*x* and sin2*x*. Some candidates needed to take more care with their presentation as miscopying errors, for example cos2*x* becoming cos*x*, were fairly common. Candidates who stated the answer only were not credited, due to omission of necessary working.

(b) Again, many candidates offered fully correct solutions, using exact values at all stages in their solution. Some candidates found the correct exact form and then decimalised this and stated it as their final answer, therefore not giving an exact answer as required. Some candidates stated the *y*-intercept as the *y*-coordinate. Most candidates used the gradient they had found in **part (a)** to find the gradient of the normal. Most candidates were able to state the correct *y*-coordinate for the point with *x*-coordinate $\frac{\pi}{6}$. A few candidates did not evaluate their trigonometric functions. Some candidates used decimal values from the point where they set $y = 0$ in the equation of the normal, which was not condoned.

In the weakest responses, candidates did not use a correct method to find the perpendicular gradient or made no use of calculus, for example, some candidates tried to solve $4\sin 2x\cos 2x = 0$.

Question 5

(a) (i) A good number of correct answers were seen. Common errors when seen were to offer: ${}^6P_4 = 360$, $5 \times 5! = 600$, ${}^5P_3 = 60$, ${}^5P_4 = 120$ or $5 \times {}^5C_3 = 50$.

(ii) Candidates found this part of the question more challenging. The simplest methods were to find $192 + 60$ or $300 - 48$. More successful candidates separated the problem into two sets such as 'the cases that ended 2, 4, 6, 8' plus 'the cases that ended with 0' or 'all possible cases' subtract 'the cases that ended with 5'.

Candidates who attempted to list all 21 cases often made an error of omission. Common method errors were to evaluate $4 \times {}^5P_3$ or $4 \times {}^4P_2 \times 5$ or $\frac{5}{6}\times 300$ or to sum values that should have been multiplied.

(iii) Candidates found this part of the question slightly less challenging than the previous part and more fully correct solutions were seen. More successful candidates separated the problem into two sets such as 'the cases that ended with 0' plus 'the cases that ended with 5' or 'the case that starts with 5 and ends with 0' plus 'the cases that do not start with 5'.Candidates who attempted to list all 9 cases were more successful than in the previous part as there were fewer cases to consider.

Common method errors were to evaluate ${}^2P_1 \times {}^5P_3 = 120$ or $2 \times {}^6P_3 = 240$ or to sum values that should have been multiplied.

(b) This part of the question, required the simplification of algebraic factorials in order to solve an equation involving combinations. Candidates needed to write the equation as a quadratic in a solvable form, such as $(n + 1)^2 = 4356$ and then solve it, discarding the extraneous solution. A reasonable number of candidates were able to do this successfully. Candidates who wrote $(n - 10)!$ as $(n - 10)(n - 11)!$, and then cancelled appropriately, had the simplest and neatest solutions. A small number of candidates omitted to discard the extraneous solution.

Some candidates were able to correctly simplify the numerical elements of the equation only. In these cases, candidates often wrote $(n - 11)!$ as $(n - 11)(n - 10)!$ and so had extra factors of $(n - 10)$ and/or $(n - 11)$ in their solution.

Some candidates were not able to not recall the general pattern for ${}^nC_r$; this is present on the List of formulas on page 2 of the examination paper.

Question 6

Candidates found this connected rates of change problem to be somewhat challenging. A reasonable number of candidates offered fully correct solutions. There were various different approaches, and some were far more complicated than others.

Methods which involved, for example, differentiating an expression for V in terms of S or S in terms of V were generally unsuccessful due to their complexity.

Some candidates made little progress beyond stating the value of r and sometimes a correct chain rule that could have been used to find the correct rate. Other candidates found a correct expression for $\frac{dS}{dt}$ in terms of r but made no attempt to evaluate it or find the value of r at this instant in time.

Candidates needed to use the correct notation to be credited for work that was partially correct. Writing, for example, $\frac{\mathrm{d}y}{\mathrm{d}x}$ was incorrect in this question, meaning it was not clear what candidates were referring to. Misreading the volume of the sphere as 36 cm^3 was fairly common.

The question required candidates to find an instantaneous rate of change, and they therefore needed to use calculus. A few candidates incorrectly attempted to find the average rate of change.

In some cases, candidates did not recall the correct formulae for the volume or surface area of a sphere; these are present on the List of formulas on page 2 of the examination paper.

Question 7

(a) A few candidates offered the most straightforward method of solution, rewriting the given terms as $6\ln x$, $10\ln x$ and $14\ln x$. The common difference, $4\ln x$, was then clear to see. They wrote the sum to n terms in an equation such as $\frac{n}{2}(12\ln x+(n-1)\times 4\ln x)=1032\ln x$, which then easily reduced to $\frac{n}{2}(12+(n-1)\times 4)=1032$ leading to $2n^2+4n-1032=0$. This often resulted in a neat, efficient and accurate solution. Candidates who found the common difference using the given terms and formed an equation such as $\frac{n}{2}\left(4\ln x^3+(n-1)\ln x^4\right)=43\ln x^{24}$ and then proceeded to manipulate this, were much more likely to make a sign, arithmetic or miscopying error. Candidates who manipulated the terms using laws of logarithms to raise the powers and combine to a single logarithm on each side of the equation were fairly successful, although the increased number of steps needed sometimes resulted in an error.

In weaker responses, candidates sometimes used the expression for the nth term instead of the sum to n terms when forming their equation. This did not result in a quadratic equation. A small number of candidates were confused by the structure of the given sequence and used the formulae for a geometric progression.

(b) Candidates found this part of the question to be simpler. It was not dependent on the previous part of the question and many candidates were able to state a correct expression for the 25th term and use it to form and solve an equation. Some candidates wrote the correct 25th term but were not able to simplify it correctly. In these cases, candidates usually misapplied a law of logarithms. For example $\ln x^6 + \ln x^{96}$ became $\ln x^{576}$.

Question 8

This question involved both differential and integral calculus. In the first steps, candidates needed to find the coordinates of A and B. Many candidates found A correctly. The simplest method of finding B was to find the first derivative, equate to 0 and solve to find the x-coordinate, and then the y-coordinate. A good number of candidates did this successfully. The simplest way to differentiate was to do so term by term. Some candidates created a more complex problem by applying the quotient rule to each term, or by combining into a single fraction and then either apply the quotient rule or rewriting again as a product and applying the product rule. Candidates who rearranged in this way were much more likely to make an error in differentiating one or both terms. Some candidates used a substitution such as $a = \frac{1}{x}$, rewrote the equation of the curve as a quadratic expression and completed the square, or similar. These approaches were sometimes successful, but candidates seemed to make more slips when working in this way.

Following this, candidates needed to formulate a plan which involved a subtraction of relevant areas. Many candidates chose the simple plan of subtracting the area of the triangle from the area between the curve and the x-axis for $x = \frac{1}{3}$ to $\frac{2}{3}$. This was often successful. Other candidates attempted to find the area of the triangle by finding the equation of the line AB and integrating to find the area between the line and the x-axis for $x = \frac{1}{3}$ to $\frac{2}{3}$. These were not quite as successful. A few candidates found the area under the curve but omitted to subtract the area of the triangle. A good number of candidates were able to integrate the equation of the curve correctly. Some candidates did not state $15\ln x + \frac{5}{x}$ but simply wrote the decimal value of the integral they were evaluating. Candidates should be aware that stating the expression in terms of the appropriate variable is a necessary step in the method of a question of this type.

Question 9

(a) Many candidates rewrote the equation in terms of $\cos 3x$ and $\sin 3x$ and rearranged to form $\tan 3x = \frac{\sqrt{3}}{3}$. A good proportion of these candidates went on to offer a fully correct answer. A few candidates squared both sides of the equation and used trigonometric identities to rewrite the equation in terms of $\sin^2 3x$ or $\cos^2 3x$ or rewrote it in terms of $\tan^2 3x$. These candidates were much more likely to have extraneous solutions in the range $-120° \leqslant x \leqslant 120°$ and therefore this approach is not advisable. The set of values 10, 70, –10,–70 was fairly common. Some candidates did not consider all the possible solutions and these usually stated the answer '$x = 10$' or '$x = 10$, $x = 70$' only. Some candidates worked in radians then converted their answer to degrees. A few candidates rewrote the equation in terms of $\cos 3x$ and $\sin 3x$ but made no further progress.

(b) Candidates found this to be more challenging. Some candidates understood the need to bring all the terms to one side and factorise so that no solutions were lost. These candidates often earned all the marks available. Many candidates divided by $\sin\left(y + \frac{\pi}{3}\right)$ but were able to state a correct set of solutions from $\cos\left(y + \frac{\pi}{3}\right) = \frac{1}{2}$. A fairly common error from candidates who divided by $\sin\left(y + \frac{\pi}{3}\right)$ was to try to solve $2\cos\left(y + \frac{\pi}{3}\right) = 0$. Some candidates seemed to misinterpret the range required, $0 \leqslant y < 2\pi$. These candidates rejected 0 and included 2π in their answer.

Question 10

A few candidates offered fully correct solutions to this question. Candidates who saw that $n = 6$ and worked with that from the beginning were the most successful.

Some candidates earned a good proportion of the marks available for finding n, finding the second term of the expansion, either in terms of n or using $n = 6$, and expanding and simplifying $\left(1 + \frac{1}{x^2}\right)^2$. Some of these candidates went onto find the correct value of a, although some omitted one of the terms needed from the two that were required. Some candidates also found the third term of the expansion but made an error when finding b, commonly missing one or two terms from the three that were required. Candidates whose presentation was poor often made miscopying errors or other slips in working. A few candidates used 'a' rather than '$-a$'.

In weak responses, candidates simplified $(3x^2)^n$ as $3x^{2n}$, for example, or incorrectly expanded $\left(1 + \frac{1}{x^2}\right)^2$ to $1 + \frac{2}{x^2} + x^4$ or similar.