MATHEMATICS SYLLABUS D

Paper 4024/12
Non-Calculator

Key messages

To do well in this paper, candidates need to:

- be familiar with the whole syllabus including all the new content,
- know which formulas are given in the question paper and which they are required to recall,
- be able to carry out basic calculations without a calculator,
- understand and use correct mathematical notation,
- draw accurate graphs and diagrams,
- be able to give clear descriptions using correct mathematical terminology,
- set out their work in clear, logical steps.

General comments

In general, candidates found the paper accessible and most attempted all the questions. They were well prepared and demonstrated a sound knowledge of most of the topics covered, including the new syllabus content such as surds.

Candidates need to be able to understand and use mathematical notation required by the syllabus, such as vector notation and recurring decimal notation. They should read the question carefully and ensure that they follow instructions such as giving an answer as a fraction in its simplest form, in terms of π, or as a surd in its simplest form.

Candidates generally demonstrated a solid grasp of basic arithmetic, although arithmetic errors were common when dealing with fractions and negative numbers. When calculating with fractions, candidates are advised to cancel first to avoid the need for complex multiplications.

Most candidates were competent with basic algebra and were able to solve simultaneous equations confidently. Some candidates made arithmetic errors or sign errors when rearranging equations or made errors when simplifying algebraic fractions due to cancelling of individual terms rather than common factors. Some candidates were unable to insert brackets appropriately into expressions, for example using $\frac{1}{2}\times 10\times T-100$ in place of $\frac{1}{2}\times 10\times(T-100)$.

Most candidates showed clear and methodical working so that partial marks could be awarded if their final answer was incorrect. Steps were often presented in a logical order. Candidates are advised to start working at the top of the available space so that they have sufficient space to show their complete method. Any working that is continued elsewhere should be clearly identified with a question number. Candidates are reminded of the importance of reviewing their work carefully to minimise avoidable mistakes, such as arithmetic errors, sign errors and transcription errors.

Diagrams were often clearly drawn, however candidates are advised to use a ruler when drawing lines and to label points if required. Candidates should use a sharp pencil so that lines and points are positioned accurately. They should take care to write their figures clearly, for example it was sometimes difficult to distinguish between the digits 1, 4 and 7. When several attempts have been made at a solution, the rejected methods should be clearly crossed out to make the intended solution clear. Candidates should not work in pencil and then overwrite in pen as this often leads to illegible answers.

Comments on specific questions

Question 1

(a) This question was generally well answered. Some candidates used an incorrect order of operations leading to –16, the result of doing the subtraction 6 – 2 before multiplying by –4. Other common errors were the result of using incorrect signs when working with negative numbers, such as the answer –2 after evaluating 2 × –4 as 8 followed by 6 – 8 rather than 6 – –8.

(b) Most candidates answered this part correctly. Most errors were due to incorrect use of place value, for example answers of 1.6, 160 and 0.16.

(c) The majority of candidates gave the correct answer of either $\frac{12}{45}$ or the simplified fraction of $\frac{4}{15}$.

Common errors were multiplying the given fractions or taking the reciprocal of $\frac{2}{9}$ rather than $\frac{5}{6}$.

Some wrote the correct calculation of $\frac{2}{9} \times \frac{6}{5}$ but then attempted to introduce a common denominator and made errors in this redundant work. In some cases, candidates found the correct result but made cancelling errors when trying to give the answer as a simplified fraction.

Question 2

(a) Almost all candidates gave the correct answer. Some candidates used an incorrect denominator such as 10 and some gave an answer of 4 yellow balls rather than the probability of taking a yellow ball. Candidates should appreciate that a probability question requires an answer between 0 and 1.

(b) Again, most candidates gave the correct answer. The most common error was $\frac{7}{11}$, perhaps from subtracting the answer from **part (a)** from 1. Some candidates multiplied $\frac{4}{11}$ and $\frac{2}{11}$ instead of adding, indicating a misunderstanding of probability rules.

Question 3

Some candidates produced an accurate drawing of a correct net; however a significant proportion either left the question blank or produced an incorrect drawing. Common errors included sketching a three-dimensional cube instead of a net**,** drawing the net of an open cube, sketching a net with incorrect side lengths or omitting the internal lines necessary to demonstrate the structure of the net of a cube. These issues suggest that some candidates lacked familiarity with the properties of the net of a cube or the visualisation of three-dimensional shapes in two dimensions. Few candidates showed working such as 3 × 3 = 9 to indicate their understanding that the cube had edge length 3 cm.

Question 4

Candidates who interpreted the diagram correctly and applied the rules for angles in a triangle and vertically opposite angles appropriately usually reached the correct answer. Those candidates who showed the sizes of the angles they had calculated on the diagram were often more successful than those who just showed calculations in the working space as they did not always clearly identify the angle that had been calculated. A common error was to assume that lines *AC* and *BD* crossed at 90° although the question did not state this fact: candidates should be reminded that any right angles will either be indicated on the diagram or referred to in the question. Other common errors were to assume that lines *AB* and *DC* were parallel and use alternate angles, even though there were no parallel line indicators on the diagram; or to apply the circle theorem angles in the same segment, even though the diagram did not involve a circle.

Question 5

Many candidates solved this equation correctly. Most multiplied out the brackets correctly to reach $20 - 5x = 35$. Errors that followed that step were usually related to incorrect signs, such as rearranging to $5x = 35 - 20$ or evaluating $-15 \div 5$ as 3 rather than -3. Very few candidates divided by 5 as the first step to give $4 - x = 7$. A minority of candidates made an error in expanding the brackets, such as $20 - x = 35$. It should be noted that when an answer is an integer, such as -3 here, candidates are expected to fully process their answer; an answer of $\frac{-3}{1}$ can be further processed to -3, for example.

Question 6

(a) Most candidates measured the line accurately and applied the scale appropriately to find the correct distance. In cases where the final answer was incorrect, the measured length of 6 cm was usually written down. Those candidates who did not measure the line within the acceptable tolerance often gained partial credit for applying the scale correctly to their measured distance.

(b) Many candidates lacked confidence in working with bearings. The most successful approach was to draw a line from *A* measured 60° clockwise from North and a line from *B* measured 40° anticlockwise from North and label the intersection *C*. A common error was to measure the angles from the line *AB*, either above or below the line, rather than measuring from the North line. A minority of candidates drew the bearings from the top of the North line rather than from the bottom and a significant number attempted to use arcs from compass constructions. Many candidates did not label their chosen answer with a *C* as required and had several points indicated on their diagram which made their answer ambiguous. Some candidates omitted the question, possibly because they did not have access to a protractor.

Question 7

(a) Almost all candidates gave the correct answer. The most common incorrect answers were either 5^3 or $5 \times 5 \times 5$.

(b) This part was found to be more of a challenge, although many answers were correct. Many candidates wrote $4^{-2} = \left(\frac{1}{4}\right)^2$ but, instead of evaluating this as $\frac{1}{16}$, some wrote 4 as 2^2 and attempted to cancel the 2s in the indices leading to an answer of $\frac{1}{2}$. Some used a similar incorrect approach using $4^{-2} = \left(2^2\right)^{-2}$. Other common errors were to use $4^{-2} = -4^2$, $4^{-2} = 4^{\frac{1}{2}}$ or $4^{-2} = \frac{1}{\sqrt{4}}$.
Some candidates attempted to write a correct expression that was spoilt by the omission of brackets, such as $\frac{1^2}{4}$ rather than $\left(\frac{1}{4}\right)^2$ or $\frac{1}{4^2}$.

Question 8

(a) Most candidates plotted all points correctly. A common error was to plot (9.0, 116) incorrectly as (9.0, 106). Most candidates used crosses to show their points clearly, but a few used blunt pencils or plotted a dot with a circle around it, which sometimes made it difficult to identify the intended position of their point. A small minority of candidates did not plot any points, suggesting that they may not have read the question.

(b) The given points were very close to a straight line, and candidates were expected to draw a good line of best fit covering the full range of the data with all crosses close to their ruled line. Some very good lines were drawn, but many candidates drew an inaccurate line that did not have crosses distributed equally on either side of the line. Some lines were drawn through (3, 30), the origin of the given axes, suggesting a misunderstanding about how to draw a line of best fit. Very few candidates drew a freehand line or used individual line segments to join from point to point.

(c) Most candidates who had drawn a straight line of best fit were able to read the required value from their graph. Some misinterpreted the scale by reading at 6.6 instead of 6.8 or misread the scale on the time axis.

Question 9

Many candidates understood the requirements of this question and showed the rounded values 90 and 20 and multiplied them correctly to give an answer of 1800. Some did not understand how to use significant figure accuracy and wrote values such as 90.0, 20.0 and 1800.00. Many candidates did not read the question carefully and so showed an approximate calculation but did not use 1 significant figure accuracy, such as 87 × 24, or attempted to multiply the two given values leading to an unnecessary long multiplication. Some of these candidates then rounded their result to 1 significant figure. Some candidates showed lack of understanding of the principles of rounding to 1 significant figure by rounding 87.1 to 9 and 23.6 to 2. A small proportion found the perimeter of the rectangle or used the formula for the area of a triangle.

Question 10

(a) Candidates were generally familiar with the process to write a number as a product of prime factors and most used the ladder method to reach the correct answer. Most answers were written efficiently using index form. A common partially correct answer was $2^2 \times 57$ from candidates who thought 57 was a prime number. Some candidates made arithmetic errors when dividing but gained partial credit if their method was shown clearly.

(b) Some candidates realised that they simply needed to square each value in their answer to **part (a)** and wrote down the correct answer directly. Many started again and used a second ladder to factorise 51 984 which seldom led to a correct answer. This approach usually involved arithmetic errors or the identification of numbers such as 361 as prime leading to the common incorrect answer of $2^4 \times 3^2 \times 361$.

Question 11

(a) Many candidates used the given information to write down $4x + 6y = 30$ and simplified correctly to reach the given equation. Methods were often correct but unnecessarily long such as rearranging the starting equation to give $4x + 6y - 30 = 0$, dividing by 2, and finally rearranging to the required form. Some methods involved errors such as multiplying by 2 rather than dividing by 2 or only dividing one side of the equation by 2. A few started with $2x + 3y = 15$ and tried to work backwards which is not appropriate in a 'show that' question. Some candidates did not understand what was required and instead attempted to substitute the values 4 and 6 into the equation $2x + 3y = 15$.

(b) Most candidates wrote a correct equation. A few tried to rearrange their equation, often to the form $y = mx + c$, which was then used as their substitution in **part (c)**. Some answers included kilograms and $1y$ was commonly seen. Incorrect answers often involved attempts to solve an equation or to use the values from the previous question.

(c) Most candidates knew a method to solve simultaneous equations, and many showed clear working leading to the correct solutions. Both elimination and substitution methods were seen. Some candidates who used substitution did not realise that they could easily rearrange $6x + y = 13$ to give $y = 13 - 6x$, instead choosing to make x the subject or to rearrange the other equation. This resulted in a more complex expression involving a fraction and made simplification more challenging. When using elimination, some candidates multiplied the x and y terms correctly but made errors with the constant terms or did not add or subtract the terms consistently.

Question 12

(a) Most candidates correctly identified at least one component of the required single transformation. Most identified it as a rotation. Some did not state the correct angle of rotation or omitted its direction. Common errors were to give the angle as 90° anticlockwise or 180°. Many were unable to find the correct centre of rotation, and some found the correct position but wrote it as the vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ rather than a coordinate point. Some candidates stated several transformations, for example rotation followed by translation rather than a single transformation as requested.

(b) Many candidates were able to correctly reflect triangle *A* in the line $x = -1$. Some drew a triangle in the correct orientation but translated to the right, often the result of reflection in $x = 0$. Some drew the reflection in the line $y = -1$, demonstrating confusion between the equations of vertical and horizontal lines.

(c) (i) This question required candidates to interpret information given about an enlargement that gave an image outside the grid. Candidates found this challenging, and a significant proportion made no attempt to answer the question. Many candidates were unable to use the given centre of (0, 0) to compare the coordinates of triangle *A* with the given vertex of (12, 3) and calculate the scale factor as 3. A common error was to use the given coordinates (12, 3) in the division 12 ÷ 3 leading to the answer 4.

(ii) Most candidates who had identified the correct scale factor in **part (c)(i)** gave the correct coordinates in this part although some reversed the *x*- and *y*-coordinates. Many candidates made no attempt at this part and some who had found the correct scale factor were unable to apply it correctly to find the coordinates, sometimes giving the coordinates for a 4 times enlargement.

Question 13

(a) This question requiring candidates to reduce \$85 by 20 per cent to find the sale price, was answered well by most candidates. Some candidates gave the answer 17, which is 20 per cent of \$85, rather than 80 per cent of \$85. Some candidates made arithmetical errors in an otherwise correct method, often when attempting to multiply 85 by 8.

(b) This reverse percentage question to find the original cost before a price reduction was more challenging than **part (a)** although many correct answers were seen. The most common error was to find 20 per cent of the sale price and then add this to \$40 to give the answer \$48 or to subtract to give \$32. Other candidates misinterpreted the statement that the price was reduced by 20 per cent to mean the sale price was 20 per cent of the original price and calculated 40 ÷ 0.2 instead of 40 ÷ 0.8.

Question 14

(a) Some candidates were able to find the column vector $\overrightarrow{AB}$ given the coordinates of point *A* and point *B*. The most common incorrect answers were $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$, from $\overrightarrow{OA} + \overrightarrow{OB}$, or $\begin{pmatrix} -10 \\ 4 \end{pmatrix}$, from $\overrightarrow{OA} - \overrightarrow{OB}$. Some candidates made arithmetic errors with signs after showing a correct method, for example $\begin{pmatrix} 6 \\ 1 \end{pmatrix} - \begin{pmatrix} -4 \\ 5 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$. Candidates did not appear to have made sufficient use of the given diagram: labelling *A* as (–4, 5) and *B* as (6, 1) would have made the question more accessible.

(b) Some candidates used $\overrightarrow{OB} + \overrightarrow{BC}$ to find $\overrightarrow{OC} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$ and hence gave the correct coordinates for point *C.* As in **part (a)**, many candidates were unable to form a correct vector sum leading to the position vector for *C*. The most common incorrect answers resulted from either $\overrightarrow{BC} = \overrightarrow{OC} + \overrightarrow{OB}$ leading to $\overrightarrow{OC} = \begin{pmatrix} -9 \\ -5 \end{pmatrix}$, or $\overrightarrow{BC} = \overrightarrow{OB} - \overrightarrow{OC}$ leading to $\overrightarrow{OC} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$. Again, few candidates added point *C* and $\overrightarrow{BC}$ to the given diagram which would have helped them to find the correct coordinates for point *C*.

(c) (i) Candidates found this part challenging. Those who understood that $\overrightarrow{DC}$ is half of $\overrightarrow{AB}$ often used their answers from **part (a)** and **part (b)** to find the correct coordinates for point *D*. A common error was to misinterpret the statement $AB = 2DC$ and to double $\overrightarrow{AB}$ rather than halve it. Some candidates found $\overrightarrow{DC} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ correctly, but gave the answer (5, –2) rather than using this to find the coordinates of *D*. Many candidates did not understand the importance of direction when adding vectors: $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{DC}$ and $\overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{OC}$ were both common. It is essential for candidates to understand that the middle two letters should be the same in a vector addition statement.

(ii) Some candidates understood that they needed to apply the distance formula with the coordinates of point *A* and point *D* to find the length of the line. To achieve full marks, candidates were required to use the correct coordinates (–2, –1) for *D* and the given coordinates (–4, 5) for *A* in the distance formula, and then simplify the result of $\sqrt{40}$ to $2\sqrt{10}$ as the question required a surd in its simplest form. Most candidates who reached $\sqrt{40}$ simplified it correctly, although some errors were seen and some candidates did not attempt to simplify. Candidates who had found incorrect coordinates for *D* often showed a correct method using their values. It was common to see errors when using negative values, for example writing –2 – 4 in place of –2 – – 4. Some candidates used an incorrect formula such as $\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$ or $\sqrt{(x_1 + x_2)^2 - (y_1 + y_2)^2}$. A significant number of candidates omitted this part.

Question 15

(a) Some candidates showed a good understanding of how to simplify surds and reached the correct answer of $3\sqrt{7}$. The first step was to identify the square numbers 25 and 4 as factors of 175 and 28 respectively, leading to $5\sqrt{7}$ and $2\sqrt{7}$ with many candidates simplifying at least one term correctly. It is important that candidates use the correct notation and understand the difference between $3\sqrt{7}$ and $\sqrt[3]{7}$. Some candidates did not understand how to work with surds and used the incorrect method $\sqrt{175} - \sqrt{28} = \sqrt{175 - 28} = \sqrt{147}$.

(b) Some candidates understood the process required to rationalise the denominator and gave the correct answer. Common errors were to introduce unnecessary negative signs leading to answers such as $\frac{-\sqrt{5}}{5}$ or $\frac{1-\sqrt{5}}{5}$, perhaps because they were more familiar with rationalising the denominator for an expression of the form $\frac{1}{a+\sqrt{b}}$. Another common error was to square both the numerator and denominator or to give the answer $\frac{\sqrt{5}}{25}$.

Question 16

(a) (i) Many candidates read the median correctly from the cumulative frequency diagram. The most common errors seen were answers of 35, the middle value on the time axis, or 30, from 60 ÷ 2.

(ii) Many candidates found the interquartile range correctly. Some knew that the quartiles were at 60 and 20 but used 60 – 20 = 40 to read at the cumulative frequency 40 giving an incorrect answer of 32.

(iii) Most candidates were familiar with using the cumulative frequency curve to answer this type of question and gave the correct answer. Some candidates gave the number of people who had a journey time of less than 40 minutes, rather than subtracting their reading from the total number of people in the group. A minority of candidates misread the cumulative frequency scale and calculated 80 – 71 instead of 80 – 72.

(b) Only a minority of candidates gave an appropriate explanation in this part. Candidates were expected to understand that the interquartile range (IQR) was the relevant measurement to use in a comment on the consistency of the given data and to reference this explicitly in their answer. Many candidates mentioned both IQR and median, which indicated a lack of understanding of this point. Candidates were not expected to speculate on why the journey times may or may not be more consistent. Comments relating to traffic or tiredness at the end of a day, for example, were not relevant and usually related to the median rather than the IQR.

Question 17

(a) Almost all candidates were able to rearrange the equation correctly. It is important to present a fraction clearly and most answers were either given in the form $y = 2 - \frac{3}{5}x$ or $y = \frac{10 - 3x}{5}$ rather than the ambiguous form $y = 10 - 3x / 5$. Some rearranged correctly but then simplified the fraction incorrectly leading to the answer $y = 2 - 3x$. A minority of candidates made an error in the first step of rearrangement, either $5y = 10 + 3x$ or $y + 3x = 2$, rather than $5y = 10 - 3x$.

(b) Candidates found this part more challenging, although many were able to extract the gradient of line *L* from their rearrangement in **part (a)** and use this to find the correct perpendicular gradient to use here. Common errors in finding the gradient of line *P* were to use the reciprocal of the gradient of line *L*, rather than the negative reciprocal, or to assume the gradient of line *L* was 3 or –3 leading to a gradient for *P* of $-\frac{1}{3}$ or $\frac{1}{3}$. Many candidates then substituted the given coordinates (6, 7) with their gradient to find the equation of the line. Some candidates did not use the information that the lines were perpendicular and either used the gradient of line *L* or a gradient of $\frac{7}{6}$ in their attempt to find the required equation.

Question 18

(a) Many candidates interpreted the graph correctly to identify that the speed was constant for the given time interval. Some mentioned that the acceleration was zero which was also acceptable, however it was not sufficient to state that the acceleration was constant. The most common error was to give an answer that was too vague, such as ‘constant motion’ or just ‘constant’. A small number of candidates confused a speed–time graph with a distance–time graph and stated that the cyclist was at rest.

(b) Many candidates gave sufficient working to show the required result convincingly, often starting with an expression for the gradient of the first section of the graph. The key step was to write $v = 0.25 \times 40$ leading to $v = 10$. It was not sufficient to show $\frac{v - 0}{40 - 0} = 0.25$ followed by $v = 10$ as this did not make it clear that the correct operation had been performed to reach the result of 10. It should be noted that in a ‘show that’ question, it is not acceptable for candidates to start with a calculation using the value they are required to show, such as $\frac{10}{40} = 0.25$.

(c) The majority of candidates demonstrated their understanding of distance travelled as the area under a speed–time graph but only the more able reached $T = 220$. The most successful answers resulted from use of the formula for the area of a trapezium giving $\frac{1}{2}(T + 60) \times 10 = 1400$. It was more common for candidates to work with three separate areas, usually showing correct expressions for the first triangle and the rectangle. Common errors seen in finding the area of the second triangle were to use *T* or (100 – *T*) as its base rather than (*T* – 100). A final answer of 120 was common, the result of assuming that *T* was the length of the base of the second triangle rather than the time from the start. Some candidates did not identify that the graph used metres but the distance was given in kilometres and equated with 1.4 rather than changing units as required. Weaker candidates ignored the graph and attempted to apply the formula speed = distance ÷ time using distance 1.4 or 1400 and speed 10.

Question 19

Some candidates were able to factorise both the numerator and denominator correctly and cancel to give the required result. Factorisation of the denominator to $(2x+7)(x+2)$ was often correct. Candidates had more difficulty in factorising the numerator. Many factorised to $3(x^2-4)$ but did not recognise this as the difference of two squares, $3(x+2)(x-2)$, which gives the common factor of $(x+2)$ in the numerator and denominator. It was also common to see the numerator incorrectly factorised to $3x(x-4)$. Some candidates incorrectly cancelled $(x-2)$ in the numerator with $(x+2)$ in the denominator. A small number of candidates attempted to form and solve an equation by equating the numerator and denominator.

Question 20

Candidates who correctly interpreted the recurring decimal notation were often able to convert the recurring decimal to the fraction $\frac{17}{99}$ and then use a common denominator correctly to add the fractions. The most common approach with the recurring decimal was to use $100x - x = 99x$ and $17.17 - 0.17 = 17$. The question required a fraction in its simplest form and many correctly simplified to $\frac{8}{11}$, but some did not simplify, made errors when cancelling or gave the partially simplified answer of $\frac{24}{33}$. Common errors when converting the recurring decimal to a fraction were to carry out an incorrect subtraction such as $100x - 10x$ leading to fractions such as $\frac{16}{90}$. Many candidates were clearly unfamiliar with recurring decimal notation and used either $0.\dot{1}\dot{7} = 0.1777$ or $0.\dot{1}\dot{7} = \frac{17}{100}$ and others were unaware that fractions should be written as integer/integer rather than decimal/integer. Candidates should be encouraged to use the lowest common multiple as a common denominator as many made arithmetic errors when attempting to use 9 × 99 as a common denominator when 99 would have been more appropriate.

Question 21

(a) In any vector question, it is helpful to start by writing a correct route to find the vector required using the letters in the correct order. Many recognised that $\overrightarrow{AX}$ was a part of $\overrightarrow{AB}$ and stated that $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ or $\overrightarrow{AO} + \overrightarrow{OB} = \mathbf{b} - \mathbf{a}$. Some then used the given ratio correctly to find $\overrightarrow{AX} = \frac{3}{5}\overrightarrow{AB}$, although some used $\frac{2}{3}$ in place of $\frac{3}{5}$. Some answers were spoilt through a lack of brackets with answers such as $\frac{3}{5}\mathbf{b} - \mathbf{a}$ seen. Some candidates made errors with the directions of the vectors and used **b** + **a** or **a** – **b** in place of **b** – **a**. Some candidates were unable to make progress in this part of the question.

(b) This part was found to be challenging for most candidates. Many were able to state a correct vector route such as $\overrightarrow{XC} = \overrightarrow{XA} + \overrightarrow{AC}$ or $\overrightarrow{XC} = \overrightarrow{XO} + \overrightarrow{OA} + \overrightarrow{AC}$. Some did not recognise the difference between *XA* and *AX*, for example, in the vector route. Few candidates were able to use the given ratio *AX* : *XB* = 3 : 2 as the ratio of sides in the similar triangles to identify that $\overrightarrow{AC} = \frac{3}{2}\mathbf{b}$ and make any further progress with the solution. Some candidates identified $\overrightarrow{AC} = k\mathbf{b}$, but could not find a value for *k* although many used $\overrightarrow{AC} = \mathbf{b}$. Those candidates who did use $\frac{3}{2}\mathbf{b}$ sometimes made errors when trying to simplify $-\frac{3}{5}(\mathbf{b} - \mathbf{a}) + \frac{3}{2}\mathbf{b}$. Working in this part was often confused, with many candidates writing vectors in terms of **a** and **b** without identifying which line segment they represented.

Question 22

(a) Candidates who recognised that completed-square form was required often started by writing $(x+2)^2$. Many were unable to complete the expression correctly to $(x+2)^2 - 16$, with ±8, ±12 and +16 often seen in place of –16. Other common errors were to use $(x+4)^2$ or $(x+2x)^2$. Some candidates attempted to expand $(x+a)^2$ and compare coefficients and others simply factorised the given expression.

(b) Many candidates had difficulty in relating the format of the answer in **part (a)** to the coordinates of the turning point. Those who did recognise that, at the turning point, the value of $(x+2)^2$ would be 0 usually stated the coordinates correctly as (–2, –16). Some candidates knew that the values of *a* and *b* related to the coordinates required but were confused about which signs should be changed and combinations of ±2 and ±16 were common. It was common to see the constant term –12 from the original expression used as the *y*-coordinate of the turning point.

(c) To gain full marks, a U-shaped curve covering all four quadrants was required with a minimum in the correct quadrant and labels for the three values where the curve crossed the axes. Some candidates produced excellent, clearly labelled sketches. Many candidates drew an approximate parabola and were often able to label the intersection with the *y*-axis as –12. To find the intersections with the *x*-axis, candidates had to factorise the original expression or use the completed square expression to solve for $y = 0$, but many were unaware that this step was required. Candidates who had an incorrect turning point in **part (b)** sometimes drew a negative quadratic curve and some used the values from this point to label axis intersections. Some candidates were unfamiliar with the basic quadratic shape and various graphs including cubics, reciprocals, V-shapes and straight lines were seen. Candidates need to understand that when a sketch graph is required, they are not expected to draw a table of values and calculate a series of coordinates. The only values that should be labelled on the axes are the points of intersection with the curve.

Question 23

Many clearly set out, correct solutions to this problem were seen. Candidates often used the arc length of the minor sector with radius 9 cm to find angle *AOB* = 100. Many then found the area of the required major sector by subtracting 100 from 360 and then calculating $\frac{260}{360} \times \pi \times 6^2$. A common error at this stage was to use an angle of 100 in place of 260 in the area formula. Other common errors were to use an incorrect radius in one or other formula, either swapping 6 and 9 or combining the two and using 15, or to use an area formula to find angle *AOB*. Some arithmetic errors were seen, particularly when candidates did not simplify fractions before multiplying, for example attempting 260 × 36 rather than first cancelling 36 with 360. Some candidates did not cancel π correctly, for example reaching *AOC* = 100π which led to problems in the next step of subtracting from 360.

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Key messages

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- be able to carry out basic calculations without a calculator,
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- be able to give clear descriptions using correct mathematical terminology,
- set out their work in clear, logical steps.

All working should be shown and answers clearly written in the appropriate answer space.

General comments

In general, most candidates attempted all the questions. They were generally well prepared and demonstrated a sound knowledge of most of the topics covered.

Candidates are advised to start working at the top of the available space so that they have sufficient space to show their complete method. Any working that is continued elsewhere should be clearly identified with a question number. Candidates are reminded of the importance of reviewing their work carefully to minimise avoidable mistakes, such as arithmetic errors, sign errors and transcription errors.

Candidates need to take notice of instructions given within a question, e.g. **Question 7** instructs the candidates to write each number correct to 1 significant figure in order to estimate the cost and **Question 23(b)** instructs candidates to label the values where the graph crosses the axes.

Some candidates found the cumulative frequency and problem solving questions challenging and omitted parts of these questions.

Comments on specific questions

Question 1

(a) This was usually answered correctly. Occasionally the answer $\frac{7}{10}$ was seen.

(b) This was usually answered correctly. Incorrect answers included 0.604 and $\frac{64}{100}$.

Question 2

Many candidates answered the question correctly. Some candidates seemed unclear as to what is meant by frequency and after putting tally marks in the correct column gave probability e.g. $\frac{7}{20}$, $\frac{5}{20}$ and $\frac{8}{20}$ in the frequency column. A few placed the frequencies in the tally column.

Question 3

(a) This part was usually answered correctly. Incorrect responses included 360 – 27 = 323, 90 – 46 = 44 and 180 – 46 – 46 = 88.

(b) Answers to this part were mostly correct but occasional arithmetic errors were seen e.g. 180 – 46 = 144, 134 ÷ 2 = 66 and some wrong methods e.g. 46 ÷ 2 = 23, 360 – 46 = 134 and 46 + 2*y* = 90.

Question 4

Almost all candidates showed the correct substitution but many did not then progress to the correct answer e.g. 5(4) + 3 (–2) followed by 20 + 1 = 21 and 20 + –6 = 26.

Question 5

(a) Many correct answers were seen. Incorrect answers included $\sqrt{7}$, $\frac{7}{1}$ and multiples of 7 i.e. 7, 14, 21.

(b) Many correct answers were seen to this part.

Question 6

(a) Almost all candidate plotted (3,6) correctly. A few plotted (6,3) or (0,3).

(b) (i) Some candidates answered this part correctly. A few gave $y = 0$ as their answer while others wrote down the co-ordinates of some points on the line *BC*.

(ii) Candidates were more successful with this part than **part (i)** and many correct answers were seen. Some candidates showed a correct method by doing a calculation involving (3, 3) and (–3, 3) showing $\frac{3-(-3)}{0-3}$, but then equated this to 2 instead of –2.

Question 7

Most candidates found this part challenging and made it much more difficult than intended by not observing the instruction to write each number correct to 1 significant figure. Many candidates did 14 × 19 × 0.34 which involved lengthy calculations. A few candidates did not use area at all and calculated (14 × 0.34) + (19 × 0.34).

Question 8

(a) This was answered correctly by the majority of candidates. A few made an error in moving the 7 to the other side of the equation or in dividing 16 by 4.

(b) Many correct solutions were seen. Some candidates who substituted for either *x* or *y* made the substitution correctly but then made errors in simplifying the resulting equation. Others who tried to eliminate either *x* or *y* multiplied some of the elements of the equations correctly but made errors or forgot to multiply all the elements e.g. $5y - 8x = -5$ and $12y + 8x = 14$ (forgetting to multiply 14 by 4) or $15y - 24x = -15$ and $15y + 2x = 70$ (forgetting to multiply the 2 by 5). Some candidates, having obtained the same coefficient of *x* or y in both equations, used the wrong operation to eliminate the unknown.

Question 9

(a) This part was usually answered correctly. Incorrect answers included 8 and 0.8.

(b) This was answered correctly by few candidates. While most candidates tried to do the multiplication first, many first determined a common denominator for both fractions before a multiplication could be performed, and so $\frac{2}{4} \times \frac{3}{4}$ was common often followed by = $\frac{6}{4}$. These candidates often went on to successfully subtract their product from $\frac{4}{5}$. A few candidates did the subtraction first.

Question 10

(a) This part was usually answered correctly with candidates realising that the sum of the probabilities should be one.

(b) This part was usually answered correctly.

Question 11

Some correct answers were seen but this question proved challenging to many candidates. A few found the prime factors of 12 and those of 45 and hence the lowest common multiple 180 and answer 12:00. The most common approach was to list multiples of 12 and multiples of 45 in order to find the common multiple. Some candidates started off correctly but gave up before they got to 180. Others listed the times when the red light flashed and when the green light flashed and hence when they both flashed together but made errors in the times for the red light e.g. 9:00 9:45 10:15.

Question 12

(a) While many candidates answered this part correctly, $\begin{pmatrix} 2 \\ -7 \end{pmatrix}$ was a common incorrect answer. The vector $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$, from $\begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, was also seen quite often.

(b) Many candidates did not realise that this was a question about finding the length of the vector $\overrightarrow{PQ}$. Some tried to find the length of the answer to **part (a)** and others the length of $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ 7 \end{pmatrix}$. A few candidates made arithmetical errors in calculating $4^2 + \left(-2^2\right)$. Some candidates did not attempt this question.

Question 13

(a) Many candidates answered this correctly. Almost all were using the correct formula for the area of the trapezium but some made errors in doing the calculation e.g. $\frac{1}{2}$ × 4 × (6 + 9) sometimes became $\not{\frac{1}{2}}$ × 4 × ($\not{6}$ + 9) = 4 × (3 + 9) = 48. Some candidates split the trapezium into a rectangle and a triangle and calculated the correct area that way. A few used an incorrect formula e.g. $\frac{1}{2}$ × 4 × 6 × 9.

(b) In this question, candidates needed to calculate the length of the fourth side using Pythagoras' theorem. Some assumed that the length of the unmarked side was either 4 or 6 and answers such as 4 + 4 + 9 + 6 = 19 and 4 + 6 + 9 + 6 = 25 were seen.

Question 14

(a) This part was usually answered correctly.

(b) This part was usually answered correctly. 16 was a common incorrect answer

(c) Many candidates answered this correctly made some progress by reaching either 18 or a^2. Some candidates divided 3 by 6 instead of multiplying them and forgot that *a* was in fact a^1 and had a^{-3} in their answer instead of a^{-2}. There was also some confusion over simplifying a partly correct answer e.g. $18a^{-3} = -6a$.

Question 15

(a) Almost all candidates gave the correct expression for obtaining the answer i.e. $\frac{70}{100} \times 120$ but some had problems performing the calculation. Examples of incorrect cancelling included $\frac{70}{100} \times 120 = \frac{7 \times 12}{100} = 0.84$ and $\frac{70}{100} \times 120 = \frac{70 \times 6}{100} = 420$. Examples of incorrect arithmetic included 7 × 12 = 72 or 64. A few candidates wrote $\frac{70}{120} \times 100$.

(b) This part was usually answered correctly. A few candidates found 8 per cent of 25 or 25 per cent of 8.

(c) More able candidates were able to answer this part correctly. The most common errors were to find 12 per cent of 66 (7.92) and give the answer $79.2 or add 7.92 on to 66 giving the answer $73.92.

Question 16

(a) Candidates needed to calculate the number of candidates that fell into each category, rather than reading off the values at 55, 70, 90 and 100 as the values needed to complete the table. Some candidates completed the table correctly while a few completed two elements of the table correctly, usually the final two elements.

(b) (i) Many candidates read the value at 40 (half of 80) on the cumulative frequency axis. Many read off at 50 and obtained the wrong answer.

(ii) Most candidates calculated 30 per cent of 80 (24) and read the value at 24 on the cumulative frequency scale. Some read off at 30 and obtained the wrong answer. A few made some progress by calculating 24 but did not read the value correctly.

(iii) Candidates found this part easier than the previous two parts and many answered correctly. Some errors occurred from reading the scale incorrectly e.g. reading at 66 on the mark axis instead of at 76.

Question 17

Many candidates found this question challenging. Almost all were able to clear the fraction by multiplying through by 5 but then had difficulty in knowing what to do next. The most common error was $5ax = 3x + 2$ followed by $5ax - 2 = 3x$ and then $x = \frac{5ax - 2}{3}$.

Question 18

(a) In this question candidates needed to realise that the distance travelled on a journey is equal to the area under the speed-time graph. The majority of candidates did a calculation involving distance and time. Some candidates made some progress by attempting to obtain an expression for all or part of the area under the curve and equating it to 1010.

(b) Candidates were more successful at this part, dividing their answer to **part (i)** by 20.

Question 19

Most candidates made some progress here by describing the transformation completely correctly. The majority described it as an enlargement with the correct centre of enlargement although a few included rotation as well. Incorrect scale factors included $\frac{1}{2}$, 2 and –2.

Question 20

(a) Many candidates found this question challenging. Some candidates did not write down the formula connected with the perimeter of a sector and made little progress. Some candidates that did know the formula then had difficulty dealing with $\frac{60}{360}$, with few cancelling this fraction to $\frac{1}{6}$. Many used the formula for the area of a circle instead of that for the circumference. Some candidates, having correctly evaluated 3π and 2π, subtracted them instead of adding or did not add on the *lengths AC* and *BD.*

(b) Candidates seemed more familiar with the formula for the area of a circle so some found this easier than working with the perimeter in **part (a)**. Some candidates again made slips when simplifying their correct expressions for the areas of the sectors e.g. $\frac{60}{360}\times\pi\times9^2$ simplified to $6 \times \pi \times 81$. A few candidates added the areas instead of subtracting them.

Question 21

Many candidates omitted this question or had difficulty making any progress with it. Some candidates correctly found the gradient of the line *L* and hence the equation of the line perpendicular to *L*. Some candidates knew that the product of the gradient of line *L* and that perpendicular to it was –1 but were not able to go on to find the gradient of line *L*. A few candidates found the gradient *L*, and stated the gradient of the perpendicular line, but then went on to find the equation of the line through (1,4) rather than perpendicular to it.

Question 22

Many candidates were able to answer this question correctly. A few made slips when factorising. Incorrect cancelling was common e.g. cancelling the x^2 in the numerator with the x^2 in the denominator and dividing the 9 in the numerator and the 12 in the denominator both by 3. A few candidates incorrectly formed the equation $x^2 - 9 = 5x^2 - 11x - 12$ and went on to solve it.

Question 23

(a) Some candidates expanded the brackets and simplified their answer correctly. The majority expanded two of the brackets correctly but did not go on to multiply all of those elements by the elements in the third bracket. A few carried out all the multiplications correctly but then made slips in collecting like terms.

(b) Some good sketches were seen but some candidates omitted to label the values where the curve crossed the axes and so lost marks. Some candidates drew in *x* and *y* axes but were not able to continue further. A few drew a negative cubic curve. Many candidates omitted this part.

Question 24

(a) (i) This part was usually answered correctly.

(ii) This part was usually answered correctly. A few candidates wrote $2\overrightarrow{OB}$ instead of 2**b**.

(b) Many candidates answered this correctly or were able to follow through correctly from their answer to **part a(ii)**. Some earned a method mark by writing down a correct route for $\overrightarrow{BX}$ even if they were then unable to give a vector in terms of **a** and **b**.

Question 25

(a) Some candidates were able to answer this question correctly. A very common incorrect answer was $\sqrt{242}$. A few candidates made a correct start by converting either $\sqrt{300}$ to $\sqrt{3} \times \sqrt{100}$ and hence to $10 \times \sqrt{3}$ or by converting $\sqrt{48}$ to $4\sqrt{3}$.

(b) Few candidates were able to answer this question correctly. Some candidates multiplied the fraction by $\dfrac{\sqrt{7}-2}{\sqrt{7}-2}$ but were unable to simplify $\left(\sqrt{7}+2\right)\left(\sqrt{7-2}\right)$ to reach 3. Some got as far as $\dfrac{6\sqrt{7}-12}{3}$ but then spoiled their answer by cancelling the 12 by 3 to get a final answer of $6\sqrt{7} - 4$.

MATHEMATICS SYLLABUS D

Paper 4024/22
Calculator

Key messages

Success in this paper relies on familiarity with all aspects of the syllabus. In addition, successful candidates demonstrate that they can apply formulae as required alongside interpreting situations mathematically and problem-solving within unstructured questions.

Candidates should be reminded to present their work in a clear and concise manner. When working with intermediate values within longer methods, these should not be rounded with only the final answer rounded to the appropriate degree of accuracy.

It is essential that candidates provide full working alongside numerical answers to ensure that appropriate method marks can be considered in situations where full credit cannot be awarded.

General comments

This was the first assessment of the new syllabus. For paper 2, this introduced additional shorter questions to the assessment alongside those that are multi-part. Some very good scripts were seen in which candidates demonstrated a clear knowledge of the wide range of topics tested. The standard of presentation varied considerably. Successful candidates showed clear systematic step-by-step working leading to their answers. Weaker responses lacked clear working or involved multiple conflicting approaches that were often unclear for examiners. Where candidates make an error in their working, they should be encouraged to cross their work out and start again rather than attempting to write over their original work.

Errors arising from premature truncation or early rounding of decimals within methods were common and frequently prevented candidates from arriving at the correct final values. As the general requirement is to give answers correct to three significant figures, candidates should work with values to at least 4 significant figures throughout their working.

Candidates should be reminded about the formula page at the front of the paper. A small number of candidates misquoted formulae or used their own version such as the angle version of the cosine rule when they were finding a length, which often created unnecessary difficulty for them.

The areas that proved to be most accessible included:
- Primes.
- Simple ratio.
- Angles and parallel lines.
- Probability.
- Angles and polygons.
- Simplifying algebraic expressions.
- Simple interest.
- Functions.
- Histograms.

The more challenging areas included:
- Conversion between units of area.
- Circle theorems.
- Questions requiring candidates to show a particular result.
- Multi-stage trigonometry problems.

Comments on specific questions

Question 1

Many candidates gave a correct diagram, usually adding one diagonal line and shading the correct square. Some responses included additional lines and shaded squares than required, which in some cases still retained a pattern with rotational symmetry of order 4. A common reason for candidates not obtaining full credit was leaving the correct square unshaded or not drawing the diagonal.

Question 2

(a) Many correct answers were seen. Common incorrect answers included 8012 and 1812.

(b) Most candidates gave a correct prime number with some giving both possible values. The most common error involved giving 27 or 25. Occasionally an even number was given.

(c) Many candidates gave the reciprocal as 9. Candidates should be reminded that, as stated in the specification, answers are expected to be given in their simplest form unless the question states otherwise. This means that $\frac{9}{1}$ was not an acceptable final answer. Common errors included 9^{-1}, -9, or providing a decimal version of the original fraction to varying degrees of accuracy.

Question 3

(a) Many candidates obtained the simplest form of 7 : 100. Some candidates dealt with the conversion correctly but either retained units in their final answer, such as 7 ml : 100 ml, or made numerical errors in the cancellation. A minority of candidates made errors in the conversion of the units.

(b) This part was answered well by most candidates. Occasionally some found an amount for the wrong person. A minority of candidates attempted to find Erika's share by dividing 540 by 5.

Question 4

There were many correct answers to this question. Some candidates interchanged the values of x and y, so that the values 52 and 128 were seen in the wrong positions. A few candidates thought that angles x and y were equal, often giving either 128 or 52 as the values of both x and y.

Question 5

Conversion from m^2 to cm^2 proved challenging for many. The most common error was 610, using the linear conversion factor of 100. Other errors involved multiplying or dividing by an incorrect power of 10.

Question 6

(a) Most candidates were successful in finding the missing value, understanding that the relative frequencies sum to 1. Incorrect answers usually resulted from numerical errors. Some candidates attempted to give the answer as a percentage but without including the symbol. A few gave 0.25 in the table for 'Thriller' and then gave a different answer on the answer line, often attempting to provide a frequency instead of a relative frequency.

(b) This was well answered by most candidates. Incorrect answers usually resulted from errors with the method such as $500 \div 0.35$, $0.35 \div 500$, or using an incorrect relative frequency.

Question 7

Many candidates demonstrated a good understanding of currency exchange. The majority of candidates made a correct start by finding €322. The bank giving euros in multiples of €5 was a common cause for errors being made, including some candidates dividing €322 by 5 and giving answers of €64 or €64.40. Whilst most candidates correctly went on to give an answer of €320, a common incorrect answer was €325. When converting the €2 change into dollars, some candidates rounded to $2.2 instead of $2.17 whilst others forgot to convert the change back to dollars.

Question 8

Most candidates demonstrated a good understanding of angles in regular polygons with many arriving at the correct answer. Those candidates who made some progress often stopped either after finding an exterior angle or after finding the sum of the interior angles. Weaker candidates used an incorrect number of sides or used an incorrect formula for the sum of the interior angles.

Question 9

(a) Some candidates missed the notation for complement and instead shaded $A \cap B$. Other incorrect responses included shading the region for $(A \cup B)'$ or the region for $A \cap B'$.

(b) Many successful responses were seen that found the number of students that played both the piano and the guitar. A mix of numerical and algebraic approaches were taken. For those using an algebraic approach, the most common error was to omit the 6 from the equation.

Question 10

Most candidates were successful in simplifying the expression. Incorrect or incomplete cancelling were the most common types of error.

Question 11

(a) Most candidates were able to calculate the amount of interest and correctly determine the value of the investment. The most common omission was determining the value of the investment after calculating the interest. A small number of candidates treated the question as compound interest.

(b) Most candidates were successful in finding the value of the investment and the corresponding total amount of interest. The most common omission was forgetting to subtract the principal amount to find the total interest for the 3 years. Some candidates demonstrated a correct method, but used premature approximations of 1.021^3, such as 1.0643, resulting in final answers outside of the acceptable range. A small number of candidates used an incorrect formula, and some attempted to use the simple interest formula.

Question 12

Many candidates recognised the need to group the terms into two pairs and went on to factorise correctly. Dealing with the negative signs gave rise to most of the errors seen. Some responses omitted the negative between the partial factors, such as $7x(h - 3f)\ 2y(h - 3f)$ which usually led to $(7x + 2y)(h - 3f)$. Weaker responses often incorrectly dealt with the signs inside the brackets to establish a common factor as a repeated bracket.

Question 13

(a) Most candidates had little difficulty in completing the tree diagram. Answers were sometimes reversed for the second counter after the first counter was green. Weaker responses often included incorrect denominators for the probabilities of the second counter, most commonly 12 but sometimes 10. A minority of candidates had the correct denominators but incorrect numerators.

(b) Many candidates were successful in answering this problem. The most common error seen involved adding the two probabilities instead of multiplying. This error resulted in an answer that is greater than 1, which was overlooked by the majority of candidates.

Question 14

(a) Some candidates recognised angle *PQS* being in the alternate segment to angle *SPQ*. Candidates would benefit from further study of the alternate segment theorem. Common errors included assuming that triangle *PQS* was isosceles or that *PQ* was the diameter with angle *BPQ* = 90°.

(b) Successful responses recognised that *PQRS* is a cyclic quadrilateral. Candidates are reminded to show their full working clearly, as credit was available for those who applied the circle theorem correctly using their value for angle *QPS*.

Question 15

(a) Most candidates completed the table correctly. Weaker responses typically made errors in finding the total number of beads with the most common incorrect value being 27.

(b)(i) Many candidates successfully gave an expression for the number of white beads in Pattern *n*. Some responses correctly stated 5 + 2(*n* – 1) or equivalent but did not go on to obtain full credit due to errors in simplifying the expression. Weaker responses typically included *n* + 2 or provided a numerical value.

(ii) Successful responses commonly involved use of a standard formula that candidates had learned to find the coefficients of the required expression. Carelessness in working often led to an incorrect final answer. Other successful candidates found an expression for the number of black beads and added this to the expression for the number of white beads. Another successful approach was by observation of pairs of products that resulted in terms in the sequence leading to $\frac{(n+2)(n+3)}{2}$. Some candidates applied the method of differences, but such approaches were generally less successful.

(c) Successful candidates equated their expression for the number of white beads from **part (b)(i)** to 88. Those who successfully solved the resulting equation commonly arrived at the correct answer, with the most common errors being leaving the final answer as a decimal or rounding the answer to 43. Common incorrect approaches included substituting 88 into the expression for the number of white beads or equating 88 to the expression for the total number of beads.

Question 16

Many candidates applied the formula for population density correctly. Common errors included performing the division in reverse or using multiplication. Candidates should be reminded that the units on the answer line can provide a useful clue for the operation required. A significant number of candidates gave their answer to the nearest whole number, which was acceptable in this context. Some candidates were not awarded full credit due to premature or incorrect rounding from the calculator. Weaker responses included errors with the powers of 10, often leading to answers in standard form with incorrect powers of 10.

Question 17

(a) Most candidates were successful in finding the value of f(–8).

(b) Many candidates gave the correct inverse function. Common errors included leaving the answer in terms of *y*, incorrect signs when rearranging, or providing a reciprocal answer such as $\frac{1}{4x+3}$.

Question 18

(a) Many candidates were successful in estimating the mean score. A few candidates made one or more slips, often when finding the midpoint values of the intervals. Incorrect methods seen often involved the use of the class widths instead of midpoint values, using the upper bounds of the intervals instead of the midpoints, or dividing the total score by the number of intervals instead of the total frequency.

(b) The majority of candidates drew accurate histograms. A common reason for candidates not obtaining full credit was incorrect calculation for the frequency density, often dividing the frequencies by 10 as was the case for the bar already provided. Some candidates drew bars with incorrect widths, commonly the last two bars. Candidates are advised to use a ruler carefully when drawing the bars as freehand drawings sometimes resulted in inaccurate vertical and horizontal lines on the histogram.

Question 19

(a) Many candidates completed the table correctly. Sign errors were the most common cause for candidates not obtaining full credit.

(b) Successful candidates plotted the points correctly and drew a smooth curve through their points. Weaker responses joined the coordinates with straight-line segments. Candidates should take care not to draw curves that are too thick or have multiple lines, since this makes it unclear to the examiner which coordinates they are intending for their curve to pass through.

(c) Stronger candidates successfully identified the correct line, $y = 7$, and drew it accurately to find the three correct solutions. Incorrect attempts at finding the equation of the line often resulted from algebraic slips in rearranging the equation.

Question 20

(a) (i) Successful candidates showed their working clearly in arriving at the required result. Less successful responses commonly involved sign errors or attempted to form an equation.

(ii) Stronger candidates were able to provide the anticipated level of working to arrive at the printed result. As the question requires candidates to 'show' a particular result, examiners require the full method to be shown with no errors or omissions. For example, omission of essential brackets, even if recovered within later working, was a common reason for candidates not obtaining full credit. Many candidates made successful attempts to find expressions for the area of rectangles *EFGD* and *ABCD*. Attempts at setting up an equation with areas was often done successfully but some candidates applied $\frac{1}{5}$ to the wrong area. Weaker candidates did not consider areas and tried manipulating their own algebraic terms to give the result required.

(b) (i) Most candidates quoted the quadratic formula correctly from the formula page, with a minority of candidates quoting the formula incorrectly. Many candidates used the formula correctly to find the two correct solutions. Some candidates made numerical or sign slips when substituting, especially with the value of $b = –6$. Candidates should be reminded to follow the rubric carefully as some responses gave answers to 1 decimal place, whilst other candidates made errors in rounding their final answers. A minority of candidates had incorrect working but used their calculator to find the correct solutions – credit cannot be given in this case.

(ii) Successful candidates substituted an appropriate value of x from **part (b)(i)** into an expression for the shaded area. Not all candidates appreciated that they needed to perform a calculation and instead found and simplified an algebraic expression for the shaded area.

Question 21

(a) Many candidates correctly determined the volume of the hemisphere, however a significant number gave the volume of a sphere instead. Other common errors include the use of 14 cm for the radius of the sphere and squaring the radius instead of cubing. Calculating the volume of the cone depended on first identifying the height of the cone. The strongest candidates correctly used 17 cm, however values including 24 cm, 12 cm and 10 cm were commonly seen.

(b) Many candidates recognised the need to find the external surface area of the cone and the hemisphere. Most candidates successfully found the curved surface area of the hemisphere. Some responses incorrectly included the total surface of the hemisphere whilst others used the diameter for the radius. Stronger candidates showed clear working making use of Pythagoras' theorem to find the slant height of the cone, often going on find the correct surface area. A small number of candidates used the total area of 712 to work backwards to find the slant height, but this was not an acceptable method as it used the given result. Some candidates with a correct method gave the total area as 712 without showing a more accurate value to demonstrate that it rounds to 712. A few candidates used inaccurate values for π or used premature rounding, both of which resulted in the final answer being out of range. Some candidates quoted area formulae followed by numerical values. As this was a 'show that' question, it is important to show the substitution of numerical values into the formula to demonstrate how the numerical results are achieved.

(c) Stronger candidates demonstrated a good understanding of the relationship between area scale factor and linear scale factor, which was the key to success in this question. Those that started by equating equivalent scale factors were generally successful, although in some cases premature rounding of some values resulted in answers that were out of range. The equation $\left(\frac{h}{24}\right)^2 = \frac{242}{712}$ was the most common starting point with $\frac{h}{24} = \frac{242}{712}$ the most common incorrect starting point.

Question 22

(a) Many candidates recognised the need to calculate the length of the missing side *AC* using the cosine rule and used the formula successfully. Some candidates chose to use the 'angle' version of the cosine rule despite the 'side' version being given on the formula page. This commonly led to difficulties with rearranging to correctly determine *AC*. Successful candidates went on to find the perimeter and divided by 20 to find the number of rolls needed. Some candidates did not give the integer answer necessary to show how many rolls were needed. A minority of candidates either used the wrong formula or did not apply the cosine rule at all, attempting instead to use the sine rule or Pythagoras' theorem. Some candidates rounded intermediate values too soon, particularly after calculating *AC*, which led to minor inaccuracies in the perimeter and the final answer.

(b) Successful candidates recognised that the shortest distance is perpendicular to *AC*. The most common approach involved applying the sine rule to find either angle *A* or angle *C* and then using right-angled trigonometry to find the shortest distance. Approaches involving areas were less common but generally successful. Having drawn the perpendicular, it was common to see candidates assuming that this line bisected the angle of 112, from which right-angled trigonometry involving an angle of 56 was often seen. Some candidates assumed that the perpendicular bisected *AC* and Pythagoras' theorem was often applied to one of the right-angled triangles incorrectly. A number of responses prematurely rounded or truncated values within the method resulting in final answers falling outside of the accepted range.

Question 23

The majority of candidates demonstrated an understanding of the need to find bounds for both the distance and the time as a first step. It was common to see candidates providing both the upper and lower bounds for both the distance and the time. Weaker responses demonstrated uncertainty around which bounds to work with. The most common mistake was to use both lower bounds. A minority of candidates did not apply the concept of bounds at all and instead divided the given values.

Question 24

Most candidates were familiar with the method required to write the given expression as a single fraction. Errors when dealing with the numerator commonly involved slips with signs when expanding the brackets or confusing the order of the terms. A significant number of candidates expanded the brackets in the denominator, which is not required. For some candidates, this introduced errors unnecessarily. Weaker responses often provided the correct denominator for the single fraction, whilst omitting the appropriate multiplication in the numerator. For example, 5 – 2 = 3 was commonly seen in the numerator.

MATHEMATICS SYLLABUS D

Paper 4024/23
Calculator

Key messages

To be successful on this paper, candidates need to ensure that they are familiar with the content of the syllabus and be able to select and apply the appropriate mathematical techniques to answer the questions set. Candidates need to ensure they read the questions carefully, set their work out in clear, logical steps and then transfer their final answer accurately onto the answer line. They also need to be able to understand how to use a calculator correctly and efficiently to enable them to answer questions accurately.

General comments

There were many good responses from the first half of the paper with candidates performing well on the straightforward number, algebra and geometry questions, demonstrating good basic arithmetic skills. Some candidates made sign errors when manipulating algebraic expressions and solving equations. The questions that candidates found the most difficult were those assessing probability, graphical inequalities, trigonometry, 3D geometry, functions and set notation.

There are still instances where candidates give inaccurate final answers due to premature rounding of intermediate results. It is important that candidates retain sufficient significant figures in their working and only round their final answer to 3 significant figures if their answer is not exact. Candidates should be encouraged to not round answers unnecessarily e.g. in **Question 10(b)** where the correct solution is $-\frac{8}{3}$ then an answer of –2.67 is not appropriate.

Candidates should ensure that they are familiar with the mathematical formulae provided on page 2 of the question paper. In some cases, candidates did not refer to this page and misquoted formulae, such as the one used for solving a quadratic equation.

If errors are made in their working, then candidates should be encouraged to carefully cross out their incorrect work and write the correct work next to it rather than overwrite work as this makes it difficult to see what is written. If a candidate makes a second attempt at a question, then they should cross through the first attempt, so it is clear which attempt they want to be considered for marking.

Comments on specific questions

Question 1

(a) Most candidates were able to round 4.2358 correct to 2 decimal places. Common incorrect answers include 423.58, 4.23, 4.236 and 4.2400

(b) Most candidates were able to write 34 159 correct to 2 significant figures. Common incorrect answers include 34, 341.59, 35000, 30000 and 34200.

Question 2

Most candidates were able to answer this question correctly, although there were a number of candidates who added the scale factor to the dimensions of the rectangle rather than multiplied by it. Some candidates incorrectly calculated the area or perimeter of the rectangle first and used that with the scale factor.

Question 3

Most candidates were able to calculate the correct amounts received by Ang and Bou. Common errors were dividing the original amount by 2, 6 or 7 rather than by 13.

Question 4

(a) Most candidates were able to identify the correct square needed to produce a diagram with rotational symmetry of order 2.

(b) Most candidates were able to identify the correct square needed to produce a diagram with 1 line of symmetry.

Question 5

(a) Many candidates were able to convert 6300 cm to metres although there were some candidates that divided by 10 or 1000.

(b) Many candidates found this question difficult as they did not know how many cm^3 made up 1 litre, or whether they needed to multiply or divide by various powers of 10. Common wrong answers came from dividing by 10 or 100 or from cubing 450.

Question 6

(a) Many candidates were able to correctly simplify the algebraic expression, however there were some candidates that changed the signs of the individual terms when rearranging the expression before simplification resulting in incorrect answers such as $3a + 10b$.

(b) Most candidates were able to answer this question correctly. Common wrong answers seen include c^3 and c^{48}.

Question 7

There were many correct responses to this question. Many candidates gave partially correct answers, omitting –3 or 0 from the list or including 1. Some candidates attempted to 'solve' the inequality giving an answer of $x \leqslant 4$.

Question 8

(a) This question proved challenging for many candidates. The most common slip was that candidates worked on the basis that the first tile was not replaced before the second tile was chosen. Consequently, the probabilities on the second set of branches were given as $\frac{3}{9}$ and $\frac{6}{9}$ for one branch and $\frac{4}{9}$ and $\frac{5}{9}$ for the other. Some candidates kept the denominator as 10 but still reduced the numerators by 1. Some candidates were not familiar with tree diagrams and put whole numbers on the branches rather than probabilities.

(b) Many candidates were able to go on and calculate the probability of getting 2 white tiles from their probabilities on the tree diagram although there were some candidates that added rather than multiplied the two probabilities.

Question 9

(a) Most candidates were able to calculate the amount of simple interest earned on the investment, although many then did not continue one step further to calculate the value of the investment. It was therefore common to see a final answer of \$86.40 rather than \$566.40. Occasionally, a candidate would subtract the interest from the starting amount. There were some candidates that started with an incorrect calculation, usually $480 \times 3.6 \times 5$ or $\frac{480 \times 3.6\% \times 5}{100}$. Some candidates divided by 5, or used 365 days, 12 months or 60 months in their calculations. A minority of candidates tried to use the formula for calculating compound interest rather than simple interest.

(b) Most candidates were able to calculate the value of the investment after 4 years using the compound interest formula correctly. Many candidates did not go one step further to calculate the amount of interest earned by Ben. Some candidates evaluated 1.027^4 and then used a prematurely rounded off value resulting in an inaccurate final answer. Other candidates recognised that they were looking for a smaller amount than the investment but calculated $600 \times (1 - 0.027)^4$ instead.

Question 10

(a) The majority of candidates were able to answer this question correctly.

(b) Many candidates were able to set up the correct equation to solve and went on to get the correct value for *x*. There were candidates that gave an inaccurate decimal equivalent rather than the exact fraction or omitted the negative sign when transferring their final answer to the answer line. The most common errors that candidates made were neglecting the negative sign of the *x* term after the constant terms were collected and adding 2 to 18 rather than subtracting it. There were a number of candidates that substituted 18 into g(*x*) resulting in an incorrect answer of –106.

(c) Many candidates were able to answer this question correctly. Some candidates calculated f(4) and g(4) and then added or multiplied the resulting values. Another common error was calculating gf(4).

(d) Some candidates correctly answered this question fully, but it proved challenging for many. There were a number of candidates who reached an answer of 50 but did not include the correct inequality/notation. Common wrong answers were 44 from evaluating g(–7), solving $2 - 6x > -8$ to get $\frac{5}{3}$ or finding $g^{-1}(x)$. There were some candidates that did not attempt to answer the question.

Question 11

(a) Many candidates were able to score at least one mark for completing the Venn diagram. The most common mistakes made were leaving (A ∩ B)' blank or omitting 2 from the intersection of the sets.

(b) Not many candidates answered this part correctly. Responses often did not include the intersection of the 2 sets and/or the numbers outside of the 2 sets in the list of elements of A U B'.

(c) Many candidates knew which part of the Venn diagram the question was referring to, but they gave the list of elements rather than the number of elements. Some candidates added 5, 7 and 11 to give an incorrect answer of 23.

Question 12

(a) The majority of candidates were able to answer this question correctly.

(b) Many candidates recognised the common difference of –3 between terms and attempted to use the formula $a + (n - 1)d$ to find the expression for the *n*th term. Common errors included writing the formula incorrectly, e.g. $16(n - 1) -3$, using 3 rather than –3 for the value of *d* leading to an answer of $13 + 3n$, or omitting the multiplication bracket when substituting $d = -3$ leading to $16 + (n - 1) - 3$ which was simplified incorrectly. In some cases, the common difference was used incorrectly leading to answers such as $n - 3$, $n + 3$ or $3 + (n - 1)$.

Question 13

(a) Many candidates were able to answer this question correctly. Common wrong answers seen were 123000, –123000 and 000123.

(b) This question was answered well by many candidates. The most common error was from candidates that did not recognise that the 2 numbers were not of the same order of magnitude and consequently did not adjust one of them, giving a final answer of 0.9 for *x* and 1 for *y*. Some candidates showed the correct value for *x* in their working but then approximated it to 2 significant figures on the answer line.

Question 14

(a) This part was answered well by the majority of the candidates. Occasionally the correct answer was shown in the working but then unnecessary further working spoilt their answer, or the expression was only partially factorised.

(b) Most candidates knew how to approach this question with many being able to successfully factorise the expression into double brackets. Some candidates stopped at a partially factorised expression. The most common error was by candidates who did not manipulate the negative expressions correctly, resulting in a final answer of $2x(a + 2b) \pm 3y(a - 2b)$.

Question 15

There were a number of correct responses seen to this question, but it did prove challenging for many candidates. While many candidates were able to get some or all of the equations of the 3 lines correct, they made errors with getting the inequality signs wrong. Many candidates had difficulty in working out the equations of the 2 sloping lines, particularly the line $x + y = 2$. Another common error made was giving the equation of the horizontal line as $x < 2$ rather than $y < 2$. There were many candidates that gave the coordinates of the 3 intersections of the lines rather than the equations of the lines.

Question 16

(a) Many good responses were seen to this part with many candidates being able to calculate an estimate of the mean using the midpoint values of the intervals. Of those who used the correct method there was the occasional error with one of the mid points or an arithmetic slip. Candidates should be encouraged to not give their final answer to 2 significant figures or as an interval unless they have shown a more accurate or exact answer in their working. The most common error was to use the class widths instead of the midpoint values to calculate the estimate for the mean. A small number of candidates used the lower/upper bounds of the intervals for their calculations, or they assumed that the total frequency was 100.

(b) This part was answered well by the majority of candidates. A common error was not including those people in the 40 to 100 age group in the total number of people over 24 years old.

Question 17

(a) This question proved challenging for most candidates. Many were able to identify the angle as 50° but did not give the correct reason of the alternate segment theorem. The most common incorrect answer given was 80° and alternate angles given as the reason, with the candidates having assumed that the *EF* and *BC* are parallel.

(b) This question also proved challenging for most candidates with many making limited progress, although there were some fully correct responses seen with good clear reasons stated. Many candidates were able to identify angle *EFB* as 80° and then get the correct answer for angle *BDE* but their reasons were either missing, incomplete or incorrect. Candidates would benefit from being reminded that showing a correct calculation does not constitute a correct reason stated. Common errors made were thinking that angle *y* was double or half angle *EFB*, assuming that angle *DBF* was a right angle and that *BDE* was also an isosceles triangle, and thinking that angles *EBF* and *EFB* were the 2 equal angles in triangle *BEF*. Some candidates clearly did know the reasons they were using to calculate angle *y* but were not able to state them correctly, e.g. confusing equilateral with isosceles, complementary with supplementary and stating quadrilateral in a circle rather than cyclic quadrilateral.

Question 18

(a) Most candidates were able to give the correct algebraic expression for at least one of the 2 dimensions of the cuboid. The most common errors were related to the expression for the width, with candidates giving answers such as $4 - x$ or $3x - 4$ and some candidates giving numerical rather than algebraic values as answers.

(b) Some good responses to this question were seen, however many candidates found this question challenging. Some tried to solve the given quadratic equation rather than derive and simplify it.

Many candidates formed an equation for the volume of the cuboid rather than the surface area. Some did not recognise that the areas of the faces were in pairs, multiplying the area of one of the faces by 4 rather than 2. Some omitted necessary brackets around the expression.

(c) There were many good responses to this question with very few candidates giving the solutions without showing some method. Most candidates recognised that they needed to use the quadratic formula to solve this equation, however there were still some candidates that tried to factorise it or solve it as a linear type of equation. The most common errors seen when substituting into the formula were the use of a short division line or short square root, the use of –16 in place of –(–16) or -16^2 in place of $(-16)^2$ and the use of 100 in place of –100. Despite the formula being included in the list of formulae at the start of the paper, there were candidates that wrote it down incorrectly. Completing the square was rarely used as a method to solve the equation.

(d) Many candidates were able to answer this question well, particularly if they had got the correct answer to the previous part. Some candidates used a value for *x* that was less than 4 to get the height of the cuboid, not realising that *x* needed to be greater than 4 in order to get a positive value for the width.

Question 19

(a) This question proved challenging for many candidates but there were some fully correct responses seen. Many candidates did not recognise that the cosine rule was required to find the angle. Common errors of those using the cosine rule were to substitute the values for the sides in the wrong position in the formula or if they did set up the correct implicit equation it was then processed incorrectly by combining $12^2 + 6^2 - 2 \times 12 \times 6 \times \cos A$ as $36 \cos A$. The most common error seen was candidates writing down that the angle *ADB* was twice the angle *ACB* and then setting up a calculation using 96° with the sine rule.

(b) Candidates also found this part challenging. Stronger responses used the sine rule to calculate the side *AC* and were able to correctly solve the problem following through from their answer to **part (a)**.

Question 20

There were many good responses to this question, and it was pleasing to see candidates leaving their final answer as a fraction rather than giving a rounded off decimal equivalent. Many candidates were able to use the correct process to solve the algebraic fraction and were able to write the left-hand side correctly over a common denominator and then go on to expand all brackets in clearing the fraction. Some of the most common errors seen here were the simplification of the quadratic denominator as $x^2 - x - 2$ or sign errors when collecting terms when all brackets had been removed. In some responses, the expression $3(x-1)(x+2)$ was not expanded correctly, with answers such as $3x - 3 + 3x + 6$ or $(3x - 3)(3x + 6)$ seen. There was also a number of candidates that attempted incorrect cancellation of individual terms in the numerator and denominator of the algebraic fraction before eliminating the denominator, or that tried to deal with the right-hand side of the equation by subtracting 3 from the numerator.

Question 21

(a) This part was answered well with most candidates able to give the upper bound of the width of the rectangle. Common wrong answers were 5.4 + 0.1 = 5.5 and 5.4 + 0.5 = 5.9

(b) Most candidates found calculating the lower bound of a difference challenging, with many of them calculating the length of the rectangle before considering bounds. Some candidates recognised that the lower bound of the perimeter was required but also used the lower bound of the width of the rectangle. Candidates should be encouraged to write down the upper and lower bounds of both dimensions before choosing the appropriate ones to use in their calculation.

Question 22

(a) Few candidates were able to identify the 'plane of symmetry'. Some candidates assumed that the base of the pyramid was an isosceles, rather than an equilateral, triangle. Common wrong answers for the number of planes were 1, 4, and 8, indicating that there may be some confusion with edges/faces and vertices.

(b) (i) This question was quite challenging for many candidates, although there were some good responses seen. There were a number of candidates that showed a circular calculation where they used 19.6 with the ratio to get the length of *OM* and then used that to get back to 19.6. Some candidates did not realise that the triangle *ABC* was an equilateral triangle and so tried to calculate *AC*. Other common errors seen were calculating $34^2 + 17^2$ rather than $34^2 - 17^2$ and not giving the length of *OB* as a more accurate value than 19.6 before rounding. Better responses included intermediate working to a greater degree of accuracy than was asked for in the final answer.

(ii) There were some good responses seen to this question. Many candidates, however, were unable to identify the required angle or the triangle *BVO*. Of those candidates that did use triangle *BVO*, quite a few calculated angle *OVB* rather than *VBO*, or assumed the vertical height, *VO* was 82. Other common errors seen were using the wrong dimensions, e.g. 17 or 29.4 rather than 19.9, calculating angle *VCB* or adding on 30° after calculating the correct angle.

(iii) Few candidates correctly found the volume of the pyramid. Many candidates stated the formula for the volume of a pyramid, but then did not indicate any further strategy to reach a solution. Some candidates recognised the fraction of $\frac{1}{3}$ was needed but then went on to multiply this by the given length of one side of the triangle and the slope height to reach an answer. Of the candidates that did attempt to calculate the area of the base, many of them calculated the area of the wrong face or used the wrong dimensions. Common incorrect answers were $\frac{1}{2} \times 34 \times 82$, $\frac{1}{2} \times 34 \times 34$, $\frac{1}{2} \times 34 \times 17$ and $\frac{1}{2} \times 17 \times 29.4$. Many candidates did not recognise that they needed to calculate the vertical height and so used 82 as the height of the pyramid. Common errors seen when calculating the vertical height were using the wrong value for *OB* (29.4, 17 or 9.8) or used an incorrect calculation involving Pythagoras' theorem. Candidates need to appreciate that the vertical height of the pyramid should be less than the slant height and so a value greater than 82 is incorrect for *VO*. Candidates are reminded that the formula for the volume of a pyramid is given at the start of the paper and that they are not expected to recall this. There were some good responses seen but again candidates should be encouraged to look at the accuracy of their intermediate work so that they reach an answer correct to 3 significant figures.

Question 23

There were some good attempts seen to this question with a number of different approaches. Some candidates attempted to draw tree diagrams to help them identify the relevant probabilities but made errors with the probabilities on the second set of branches, writing that the denominator of the second fraction was 11 or 13 rather than 12. One of the most efficient ways to calculate the probability of getting 2 pencils of different colours would be to calculate the probability of 2 pencils of the same colour and subtracting it from 1 but very few candidates used this approach and of those that did calculate the probability of 2 pencils of the same colour then omitted the final stage. Another efficient strategy seen was to calculate the probability of a red and not red, a green and not green and a yellow and not yellow, although some candidates attempted to do this but made errors with their calculations or used the wrong denominators. Most candidates that attempted this question tried to calculate the probabilities of the separate combinations e.g. red and yellow, red and green, green and yellow etc. but either did not consider all the possible outcomes, made errors with their calculations or used the wrong probabilities. Some candidates also added probabilities when they should have been multiplied, or multiplied when they should have been added e.g. $\frac{4}{13} \times \frac{7}{12} \times \frac{2}{12}$ rather than $\frac{4}{13} \times \left(\frac{7}{12} + \frac{2}{12}\right)$. Candidates should be encouraged to check their work carefully, especially when it involves multiple calculations, to ensure they have not made any slips which can affect their final answer.