Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1749707556

FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2019**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

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1 Prove by mathematical induction that $3^{3n} - 1$ is divisible by 13 for every positive integer n. [5]

2 The curve C has polar equation $r^2 = \ln(1 + \theta)$, for $0 \leqslant \theta \leqslant 2\pi$.

(i) Sketch C. [2]

(ii) Using the substitution $u = 1 + \theta$, or otherwise, find the area of the region bounded by C and the initial line, leaving your answer in an exact form. [5]

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3 **(i)** Write down the fifth roots of unity. [2]

(ii) Find all the roots of the equation

$$z^{10} + z^5 + 1 = 0,$$

giving each root in the form $e^{i\theta}$. [5]

4 **(i)** Use the method of differences to show that $\sum_{r=1}^{N} \frac{1}{(3r+1)(3r-2)} = \frac{1}{3} - \frac{1}{3(3N+1)}$. [4]

(ii) Find the limit, as $N \to \infty$, of $\displaystyle\sum_{r=N+1}^{N^2} \frac{N}{(3r+1)(3r-2)}$. [4]

5 The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & 0 & 4 \\ 5 & 2 & 1 & -3 \\ 4 & 0 & 1 & -7 \\ -2 & 4 & -1 & \alpha \end{pmatrix}.$$

It is given that the rank of $\mathbf{M}$ is 2.

(i) Find the value of α and state a basis for the range space of T. [4]

(ii) Obtain a basis for the null space of T. [4]

6 The curve C has equation

$$y = \frac{x^2}{kx - 1},$$

where k is a positive constant.

(i) Obtain the equations of the asymptotes of C. [3]

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(ii) Find the coordinates of the stationary points of C. [3]

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(iii) Sketch C. [3]

7 The line l_1 passes through the points $A(-3, 1, 4)$ and $B(-1, 5, 9)$. The line l_2 passes through the points $C(-2, 6, 5)$ and $D(-1, 7, 5)$.

(i) Find the shortest distance between the lines l_1 and l_2. [5]

(ii) Find the acute angle between the line l_2 and the plane containing A, B and D. [5]

8 Find the particular solution of the differential equation

$$9\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + x = 50\sin t,$$

given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0$. [10]

9 A cubic equation $x^3 + bx^2 + cx + d = 0$ has real roots α, β and γ such that

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -\tfrac{5}{12},$$
$$\alpha\beta\gamma = -12,$$
$$\alpha^3 + \beta^3 + \gamma^3 = 90.$$

(i) Find the values of c and d. [3]

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(ii) Express $\alpha^2 + \beta^2 + \gamma^2$ in terms of b. [2]

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(iii) Show that $b^3 - 15b + 126 = 0$. [4]

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(iv) Given that $3 + i\sqrt{(12)}$ is a root of $y^3 - 15y + 126 = 0$, deduce the value of b. [2]

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10 Let $I_n = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \cot^n x \, dx$, where $n \geqslant 0$.

(i) By considering $\frac{d}{dx}(\cot^{n+1} x)$, or otherwise, show that

$$I_{n+2} = \frac{1}{n+1} - I_n.$$ [5]

The curve C has equation $y = \cot x$, for $\frac{1}{4}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(ii) Find, in an exact form, the y-coordinate of the centroid of the region enclosed by C, the line $x = \frac{1}{4}\pi$ and the x-axis. [6]

11 Answer only **one** of the following two alternatives.

EITHER

A 3×3 matrix $\mathbf{A}$ has distinct eigenvalues 2, 1, 3, with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 0 \\ b \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

respectively, where b is a positive constant.

(i) Find $\mathbf{A}$ in terms of b. [9]

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(ii) Find $\mathbf{A}^{-1}\begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$. [2]

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(iii) It is given that

$$\mathbf{A}^n\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{A}^n\begin{pmatrix} -1 \\ 0 \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ b^{-1} \end{pmatrix}.$$

Find the values of n and b. [3]

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OR

The positive variables y and t are related by

$$y = a^t,$$

where a is a positive constant.

(i) (a) By differentiating $\ln y$ with respect to t, show that $\frac{dy}{dt} = a^t \ln a$. [3]

(b) Write down $\frac{d^2y}{dt^2}$. [1]

(ii) Determine the set of values of a for which the infinite series

$$y + \frac{dy}{dt} + \frac{d^2y}{dt^2} + \frac{d^3y}{dt^3} + \ldots$$

is convergent. [3]

A curve has parametric equations

$$x = t^a, \quad y = a^t.$$

(iii) Find $\frac{d^2y}{dx^2}$ in terms of a and t, and show that, when $t = 2$,

$$\frac{d^2y}{dx^2} = 2^{1-2a}(1 - a + 2\ln a)\ln a. \qquad [7]$$

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.