



# Cambridge International AS & A Level

CANDIDATE  
NAME

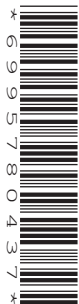
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## FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

May/June 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ( $g$ ) is needed, use  $10 \text{ ms}^{-2}$ .

## INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.



- 1 A particle  $P$  is projected with speed  $u$  at an angle of  $30^\circ$  above the horizontal from a point  $O$  on a horizontal plane and moves freely under gravity. The particle reaches its greatest height at time  $T$  after projection.

Find, in terms of  $u$ , the speed of  $P$  at time  $\frac{2}{3}T$  after projection. [5]

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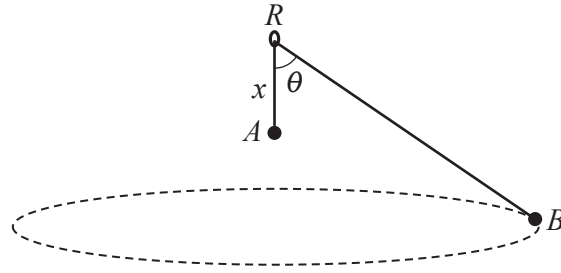
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A light inextensible string of length  $a$  is threaded through a fixed smooth ring  $R$ . One end of the string is attached to a particle  $A$  of mass  $3m$ . The other end of the string is attached to a particle  $B$  of mass  $m$ . The particle  $A$  hangs in equilibrium at a distance  $x$  vertically below the ring. The angle between  $AR$  and  $BR$  is  $\theta$  (see diagram). The particle  $B$  moves in a horizontal circle with constant angular speed  $2\sqrt{\frac{g}{a}}$ .

Show that  $\cos \theta = \frac{1}{3}$  and find  $x$  in terms of  $a$ . [5]

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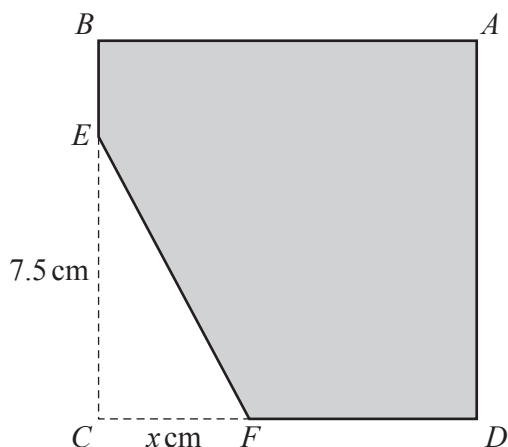




(b) Find the speed of  $P$  when the spring first returns to its natural length.

[4]

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A uniform square lamina  $ABCD$  has sides of length 10 cm. The point  $E$  is on  $BC$  with  $EC = 7.5$  cm, and the point  $F$  is on  $DC$  with  $CF = x$  cm. The triangle  $EFC$  is removed from  $ABCD$  (see diagram). The centre of mass of the resulting shape  $ABEFD$  is a distance  $\bar{x}$  cm from  $CB$  and a distance  $\bar{y}$  cm from  $CD$ .

- (a) Show that  $\bar{x} = \frac{400 - x^2}{80 - 3x}$  and find a corresponding expression for  $\bar{y}$ . [4]

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The shape  $ABEFD$  is in equilibrium in a vertical plane with the edge  $DF$  resting on a smooth horizontal surface.

- (b)** Find the greatest possible value of  $x$ , giving your answer in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are constants to be determined. [3]

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5 A particle  $P$  is moving along a straight line with acceleration  $3kv - kv^2$  where  $v$  is its velocity at time  $t$ ,  $u$  is its initial velocity and  $k$  is a constant. The velocity and acceleration of  $P$  are both in the direction of increasing displacement from the initial position.

(a) Find the time taken for  $P$  to achieve a velocity of  $2u$ . [3]

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6 A particle  $P$  of mass  $m$  is moving with speed  $u$  on a fixed smooth horizontal surface. The particle strikes a fixed vertical barrier. At the instant of impact the direction of motion of  $P$  makes an angle  $\alpha$  with the barrier. The coefficient of restitution between  $P$  and the barrier is  $e$ . As a result of the impact, the direction of motion of  $P$  is turned through  $90^\circ$ .

(a) Show that  $\tan^2 \alpha = \frac{1}{e}$ . [3]

Dotted lines for answer writing

The particle  $P$  loses two-thirds of its kinetic energy in the impact.

(b) Find the value of  $\alpha$  and the value of  $e$ .

[5]

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