Cambridge International AS Level

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MATHEMATICS **9709/21**

Paper 2 **May/June 2017**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	Take logarithms of both sides and apply power law to both sides	**M1**	Allow $y = \frac{\log 5}{4\log 3}$ for **M1 A1**
	Rearrange to the form $y = \frac{\ln 5}{4\ln 3}x$ or equivalent	**A1**	
	Obtain $m = 0.366$	**A1**	
	Total:	**3**	
2	State or imply non-modulus inequality $(4-x)^2 \leqslant (3-2x)^2$ or corresponding equation, pair of linear equations or linear inequalities	**M1**	
	Attempt solution of 3-term quadratic equation, of two linear equations or of two linear inequalities	**M1**	
	Obtain critical values -1 and $\frac{7}{3}$	**A1**	SR Allow **B1** for $x \leqslant -1$ only or $x \geqslant \frac{7}{3}$ only if first **M1** is not given
	State answer $x \leqslant -1, x \geqslant \frac{7}{3}$	**A1**	Do not accept $\frac{7}{3} \leqslant x \leqslant -1$ or $-1 \geqslant x \geqslant \frac{7}{3}$ for **A1**
	Total:	**4**	

Question	Answer	Marks	Guidance
3	Integrate to obtain form $ke^{\frac{1}{2}x+3}$ where k is constant not equal to 4	**M1**	
	Obtain correct $8e^{\frac{1}{2}x+3}$	**A1**	Allow unsimplified for **A1**
	Obtain $8e^{\frac{1}{2}a+3} - 8e^3 = 835$ or equivalent	**A1**	
	Carry out correct process to find a from equation of form $ke^{\frac{1}{2}a+3} = c$	**M1**	
	Obtain 3.65	**A1**	If 3.65 seen with no actual attempt at integration, award **B1** if it is thought that trial and improvement with calculator has been used.
	Total:	**5**	
4(i)	Use iteration correctly at least once	**M1**	
	Obtain final answer 2.08	**A1**	
	Show sufficient iterations to 4 dp to justify answer or show sign change in interval $(2.075,\ 2.085)$	**A1**	
	Total:	**3**	
4(ii)	State or clearly imply equation $x = \dfrac{2x^2 + x + 9}{(x+1)^2}$ or same equation using α	**B1**	
	Carry out relevant simplification	**M1**	
	Obtain $\sqrt[3]{9}$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(i)	State $R = 3$	**B1**	Allow marks for (i) if seen in (ii)
	Use appropriate trigonometric formula to find α	**M1**	
	Obtain 48.19 with no errors seen	**A1**	
	Total:	**3**	
5(ii)	Carry out evaluation of $\cos^{-1}\frac{1}{3}(= 70.528...)$	**M1**	**M1** for $\cos^{-1}\left(\frac{1}{R}\right)$
	Obtain correct answer 118.7	**A1**	
	Carry out correct method to find second answer	**M1**	
	Obtain 337.7 and no others between 0 and 360	**A1**	
	Total:	**4**	
6(i)	State or imply correct y-values 0, $\tan\frac{1}{6}\pi$, $\tan\frac{2}{6}\pi$	**B1**	Some candidates have their calculator in degree mode when working out $\tan\frac{\pi}{6}$ etc. this gives 0.00915 and 0.0183. Allow **B1**.
	Use correct formula, or equivalent, with $h = \frac{1}{12}\pi$ and y–values	**M1**	Must be convinced they have considered 3 values for y for **M1**
	Obtain 0.378	**A1**	
	Total:	**3**	

Question	Answer	Marks	Guidance
6(ii)	State or imply $\pi\int\left(\sec^2 2x-1\right)\,\mathrm{d}x$	**B1**	
	Integrate to obtain $k_1\tan 2x+k_2x$, any non-zero constants including π or not	**M1**	
	Obtain $\frac{1}{2}\tan 2x-x$ or $\pi\left(\frac{1}{2}\tan 2x-x\right)$	**A1**	
	Obtain $\pi\left(\frac{1}{2}\sqrt{3}-\frac{1}{6}\pi\right)$ or equivalent	**A1**	
	Total:	**4**	
7(i)	Differentiate x and y and form $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	
	Obtain $\dfrac{4t^3-6t^2+8t-12}{3t^2+6}$	**A1**	First 2 marks may be implied by an attempt at division
	Carry out division at least as far as kt or equivalent	**M1**	For **M1**, it must be division by a quadratic factor. Allow attempt at factorisation with same conditions as for division
	Obtain $\frac{4}{3}t$	**A1**	
	Obtain $\frac{4}{3}t-2$ with complete division shown and no errors seen	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
7(ii)	State or imply gradient of straight line is $\frac{1}{2}$	**B1**	Allow **B1** if $y=\frac{1}{2}x+\frac{9}{2}$ is seen
	Attempt value of t from their $\frac{\mathrm{d}y}{\mathrm{d}x}$ = their negative reciprocal of gradient of line	**M1**	
	Obtain $t=0$ and hence $(1,5)$	**A1**	
	Total:	**3**	
8(i)	Apply product rule to find first derivative	***M1**	
	Obtain $6x\ln\left(\frac{1}{6}x\right)+3x$ or equivalent	**A1**	Allow unsimplified for **A1**
	Identify $x=6$ at P	**B1**	
	Substitute their value of x at P into attempt at first derivative	**DM1**	dep *M
	Obtain 18	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
8(ii)	Equate their first derivative to zero and attempt solution of equation of form $k\ln\left(\frac{1}{6}x\right)+m=0$	***M1**	
	Obtain x–coordinate of form $a_1e^{a_2}$	**DM1**	dep *M
	Obtain $x=6\mathrm{e}^{-\frac{1}{2}}$ or exact equivalent	**A1**	
	Substitute exact x–value in the form $a_1e^{a_2}$ and attempt simplification to remove ln	**M1**	
	Obtain $-54\mathrm{e}^{-1}$ or exact equivalent	**A1**	
	Total:	**5**	