Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3951355360

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 The coefficients of x and x^2 in the expansion of $(2 + ax)^7$ are equal. Find the value of the non-zero constant a. [3]

2 The common ratio of a geometric progression is r. The first term of the progression is $(r^2 - 3r + 2)$ and the sum to infinity is S.

(i) Show that $S = 2 - r$. [2]

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(ii) Find the set of possible values that S can take. [2]

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3 Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

4 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} \qquad \text{and} \qquad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}.$$

The point P lies on AB and is such that $\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$.

(i) Find the position vector of P. [3]

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(ii) Find the distance OP. [1]

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(iii) Determine whether OP is perpendicular to AB. Justify your answer. [2]

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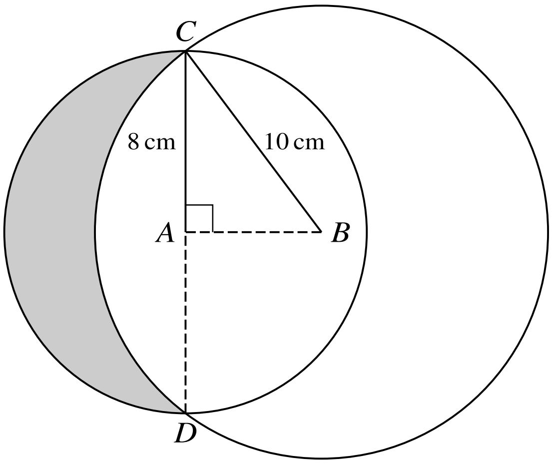
5 **(i)** Show that the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ may be expressed as $\cos^2\theta = 2\sin^2\theta$. [3]

(ii) Hence solve the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ for $0° < \theta < 180°$. [3]

6 The line $3y + x = 25$ is a normal to the curve $y = x^2 - 5x + k$. Find the value of the constant k. [6]

7



The diagram shows two circles with centres A and B having radii 8 cm and 10 cm respectively. The two circles intersect at C and D where CAD is a straight line and AB is perpendicular to CD.

(i) Find angle ABC in radians. [1]

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(ii) Find the area of the shaded region. [6]

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8 $A(-1, 1)$ and $P(a, b)$ are two points, where a and b are constants. The gradient of AP is 2.

(i) Find an expression for b in terms of a. [2]

(ii) $B(10, -1)$ is a third point such that $AP = AB$. Calculate the coordinates of the possible positions of P. [6]

9 **(i)** Express $9x^2 - 6x + 6$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

The function f is defined by $\mathrm{f}(x) = 9x^2 - 6x + 6$ for $x \geqslant p$, where p is a constant.

(ii) State the smallest value of p for which f is a one-one function. [1]

(iii) For this value of p, obtain an expression for $\mathrm{f}^{-1}(x)$, and state the domain of f^{-1}. [4]

(iv) State the set of values of q for which the equation $\mathrm{f}(x) = q$ has no solution. [1]

10 **(a)**

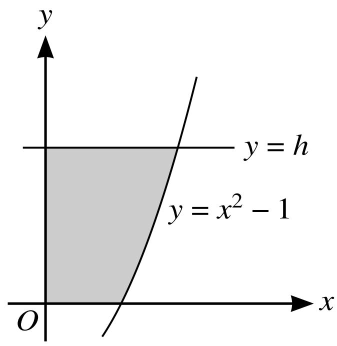

Fig. 1

Fig. 1 shows part of the curve $y = x^2 - 1$ and the line $y = h$, where h is a constant.

(i) The shaded region is rotated through 360° about the ***y*-axis**. Show that the volume of revolution, V, is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. [3]

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(ii) Find, showing all necessary working, the area of the shaded region when $h = 3$. [4]

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(b)

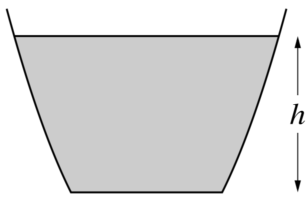

Fig. 2

Fig. 2 shows a cross-section of a bowl containing water. When the height of the water level is h cm, the volume, V cm^3, of water is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. Water is poured into the bowl at a constant rate of 2 cm^3 s^{-1}. Find the rate, in cm s^{-1}, at which the height of the water level is increasing when the height of the water level is 3 cm. [4]

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11 The function f is defined for $x \geqslant 0$. It is given that f has a minimum value when $x = 2$ and that $\mathrm{f}''(x) = (4x+1)^{-\frac{1}{2}}$.

(i) Find $\mathrm{f}'(x)$. [3]

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It is now given that $\mathrm{f}''(0)$, $\mathrm{f}'(0)$ and $\mathrm{f}(0)$ are the first three terms respectively of an arithmetic progression.

(ii) Find the value of $\mathrm{f}(0)$. [3]

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(iii) Find $f(x)$, and hence find the minimum value of f. [5]

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