MATHEMATICS

Paper 9709/11
Paper 11

Key messages

The 2020 revised syllabuses contain a statement in the rubric on the front cover that states, 'No marks will be given for unsupported answers from a calculator.' This means that working has to be shown to justify solutions of quadratic equations, trigonometric equations and simultaneous equations. In the case of quadratic equations, for example, it would be necessary to show factorisation or use of the formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down the solution is not sufficient and hence marks are not awarded.

General comments

The paper proved very challenging for many of the candidates and consequently few high scoring scripts were seen. In advanced level papers the knowledge and use of basic algebraic and trigonometric methods from IGCSE or O Level is expected, as stated in the syllabus.

Comments on specific questions

Question 1

All successful attempts at this question involved the elimination of y to obtain a quadratic equation in x. Occasionally calculation of the range of values of m which made the discriminant of this quadratic negative was seen. Those who chose to eliminate x often made little progress.

Question 2

Candidates who identified that integration of the given first derivative was required were usually able to proceed with the integration and calculation of the constant of integration to find the equation of the curve.

Question 3

Use of the chain rule to connect the given and required rate of change was seen very occasionally.

Question 4

This topic which is new to this paper within the revised syllabus. The question required the identification of three transformations and the interpretation of their effect on the equation of the cosine curve. The effect of the one way stretch parallel to the y-axis was the most correctly described transformation. Some completely correct solutions were seen.

Question 5

Most successful attempts at **part (a)** involved calculation of all the terms of the expansion rather than deduction and calculation of the required terms. Those who correctly found the powers of 2 and a in each term usually went on to find the correct value of a.

In **part (b)** few candidates realised that multiplication of the terms found in **part (a)** by 1 and $-x^3$ respectively must give a sum of zero.

Question 6

This proved to be a straightforward question for candidates who were able to differentiate the ‘function of a function’ correctly and knew the connection between the first derivative and the gradient. The resulting equation led to two values of x but very few candidates checked back to find the negative solution was the only one which was valid.

Question 7

The algebra involved in finding a common denominator in **part (a)** caused some problems with multiplication and sign errors appearing frequently. Some very good solutions with clearly quoted trigonometric relationships were seen.

The given result for **part (a)** was used effectively in **part (b)** to reach the positive value of $\tan\theta$ but the negative alternative was often ignored.

Question 8

In **part (a)** most candidates were able to quote the formula for the sum to infinity of a geometric progression and make some progress towards the given result through elimination of S and a.

Part (b) could be completed using only the given result from **part (a)** and the relationship between the terms stated in the question. It was noted that often only those who had completed **part (a)** went on to attempt **part (b)** and eliminated r and R to find S in terms of a.

Question 9

Circle geometry is new to this paper within the revised syllabus and **part (a)** proved to be a very accessible start to this question. The relationship between the gradient of the diameter and the gradient of the tangent was often used correctly to find the required equation.

Those who used the standard form for the equation of a circle, centre (a, b), radius, r, $(x-a)^2+(y-b)^2=r^2$ often did better in **part (b)** than those who chose the general form for a circle, centre $(-g,-f)$, $x^2+2gx+y^2+2fy+c=0$ where there was some confusion in the use of negative signs and $c=g^2+f^2-r^2$.

When candidates had found answers to **parts (a)** and **(b)** they were usually able to complete **part (c)** by eliminating y from their two equations to obtain a quadratic in x which led to the two solutions.

Question 10

In **part (a)** use of the relationship between a radius and its tangent and basic trigonometry led to an expression for CD but the basic trigonometry was often a stumbling block.

The arc length formula required in **part (b)** was usually quoted correctly and those who realised $CA=CD=CB$ and had an expression for CD often used it successfully to find the required perimeter.

Correctly answering **part (c)** depended on candidates knowing how to find the sector area EOF and the area of the kite $OECF$. Various successful methods were successfully used to find the kite area and the formula for the sector area was often quoted. Occasionally the two were used correctly to find the required area.

Question 11

Part (a) was nearly always completed successfully reflecting a very good understanding of the formation of composite functions.

Those who used the non-expanded form of their answer to **part (a)** in **part (b)** were able to find the inverse much more easily than those who used the expanded form. Occasionally the answer to **part (a)** was ignored and $(\mathrm{fg})^{-1}=\mathrm{g}^{-1}(\mathrm{f}^{-1})$ used instead. The domain of the inverse could be found either by considering the range of fg or the form of the inverse. Some correct answers were seen.

Both composite functions were usually quoted correctly in **part (c)** although some omitted the '–3' on the left-hand side of the equation. The resultant quadratic equation was usually solved correctly, and some considered the domains of the functions to select the positive solution. Some candidates did not show working for the solution of the quadratic, which resulted in the loss of a method mark.

Question 12

Part (a) was most easily solved by elimination of *y* between the two equations to give a quadratic in $x^{\frac{1}{2}}$.

Those who solved the quadratic by factorisation were more successful than those who isolated $x^{\frac{1}{2}}$ and squared both sides of the resulting equation.

Showing that the gradient of the curve was zero at $x = 1$ was sufficient to gain both marks in **part (b)**. Those who attempted this part were usually able to correctly complete the necessary differentiation.

It was rare to see a correct plan for finding the required area in **part (c)**. The use of the limits $x = 3$ to $x = 9$ for the area between the line and the *x*-axis and the limits $x = 4$ to $x = 9$ for the area between the curve and the *x*-axis meant most methods which relied on combining the two equations proved unsuccessful. Nevertheless, some completely correct solutions which found the two areas separately were seen.

MATHEMATICS

Paper 9709/12
Paper 12

Key messages

The 2020 revised syllabuses contain a statement in the rubric on the front cover that states, ‘no marks will be given for unsupported answers from a calculator.’ This means that working has to be shown to justify solutions of quadratic equations, trigonometric equations and simultaneous equations. In the case of quadratic equations, for example, it would be necessary to show factorisation or use of the formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down the solution alone is not sufficient; it is also not sufficient to quote the formula; values need to be shown substituted into it.

General comments

The paper was generally well received by candidates and many very good responses were seen. Candidates appeared to have sufficient time to complete the paper. Presentation of work was mostly good, although some of the answers appear to have been written in pencil and then superimposed in ink, which gives a very unclear image. Candidates should be strongly advised not to do this.

The two new topics on the 2020 syllabus: Transformations (**Question 11(d)** and (**e**)) and Circles (**Question 9**) were found to be challenging for many candidates. It is also important to note that clear reasoning needs to be given in all proof or ‘show that’ questions.

Comments on specific questions

Question 1

This question proved to be an accessible start for most candidates with many fully correct solutions seen. Many candidates wrote out a full expansion and were generally then able to identify the two terms which were required in order to form an equation with the 20. Weaker candidates sometimes either forgot to square and cube the (-2) from their expansions, only selected the term in x^3 or could not form an equation.

Question 2

Many candidates found an expression for the common ratio in terms of *p* and then used it in the sum to infinity formula. A significant number did not realise that they had to find the value of *p*, instead they quoted an expression containing it. Those who attempted to find the value of *p* often gave two expressions for the common ratio, equated them to form an equation in *p* only and were then able to solve it correctly. Others tried to use the expressions for *a*, *ar* and ar^2 but were generally less successful. Some candidates incorrectly obtained a value of *r* greater than 1 but often did not identify that an error had been made and substituted it into the formula anyway.

Question 3

Almost all candidates were able to make some progress on this question, but fully correct solutions were not common. Candidates generally eliminated *y* from the equations and formed a quadratic in *x* which also contained *m*. Some candidates however either tried to find a value for *m* or substituted different numerical values for it. The need to use the discriminant was also well known and often $m^2 - 8m + 17$ or a multiple of this was seen. Candidates were then very often unable to make any further progress or attempted to solve this expression equal to 0. Stronger candidates realised the need to use completing the square (or a valid alternative method) to show that the expression would always be positive and were then able to explain that the line would intersect the curve in two distinct points for all values of *m*.

Question 4

There was a mixed response to this question with many fully correct solutions. A considerable number of candidates misunderstood the question or were unable to make any progress. Candidates who substituted values for *n* were usually able to find values for *a* and *d*. A significant number incorrectly subtracted the terms to find *d* and hence thought that it equalled 7. Having found *a* and *d* some candidates used the sum formula rather than the *n*th term. Many candidates equated the given expression to the sum formula but only those who obtained an identity and equated coefficients, rather than finding an equation, made progress. A number of candidates left their final answer as 98.5, which was not sufficient for the final accuracy mark.

Question 5

Most candidates were able to find the correct answer to **(a)** and find the inverse functions in **(b)** and equate them correctly. Weaker candidates made errors in simplifying to the correct quadratic equation. Some candidates did not show their method for solving their quadratic equation, therefore not all marks could be awarded.

Question 6

Many fully correct solutions were seen in both parts of this question. Several candidates did not realise that they could use the given answer from **(a)** to help them solve **(b)** and hence some omitted this part. The use of the word ‘hence’ in **(b)** indicated to candidates that this part involved the use of **(a)**. Some candidates were unable to correctly combine the algebraic fractions and therefore made little progress, even though they identified the correct trigonometric identities they needed to use. Several candidates made errors in adding algebraic fractions. This is a fundamental skill which candidates would benefit from practicing more extensively. In part **(b)** candidates who used their result from **(a)** were usually able to obtain $\tan^3 x = \frac{1}{2}$. Some candidates found **(b)** difficult and often did not realise the need to cube root. A significant number of candidates incorrectly included 141.6° with their correct answer and did not therefore obtain the accuracy mark.

Question 7

Both parts of this question were generally well answered, and most candidates were confident with the techniques required. In part **(a)** most candidates substituted 4 into the given differential, although some candidates differentiated again before substituting. Some candidates only substituted once they had applied the chain rule and this sometimes led to errors in the calculation. Some weaker candidates misunderstood the information required and so applied the chain rule incorrectly.

In part **(b)** again most candidates integrated correctly although some weaker candidates used the equation of a straight line. Sometimes $\frac{6}{\frac{1}{2}}$ was simplified to equal 3, or $\frac{-4}{-\frac{1}{2}}$ became 2 or more commonly –8. Some candidates omitted the constant of integration. It is also important to note that when a question asks for the equation of a line or a curve then ‘*y* =’ or ‘f*(x)* =’ should be included.

Question 8

Both parts of this question brought a range of responses with many candidates showing confidence in the procedures needed, whereas others made little progress. In part **(a)** the majority were aware of the formula required for the area of the triangle ($\frac{1}{2}r^2\sin\theta$) but many were unable to correctly identify the required angle as $\pi - 2\theta$. Others split the isosceles triangle in half, found the lengths $r\sin\theta$ and $r\cos\theta$, and used these successfully to find half of the triangle area, although some then forgot to double their answer. The area of the sector formula was very well known and since the required angle was θ, many correct statements for **(a)** were seen. Some candidates however simplified their answer unnecessarily.

In part **(b)** candidates were generally confident in finding the arc length *BD*, however the length of *DC* was found to be more challenging. Those who had split the triangle in **(a)** were often able to double $r\cos\theta$ to obtain *AC* and then subtract *r.* Other methods such as using the cosine rule were sometimes successfully used. Premature approximation of the angle sizes sometimes led to a loss of accuracy in the final answer.

Question 9

This question was on circles, which was a new topic on the 2020 syllabus. This question was challenging for many candidates with **(b)** and **(c)** being omitted by a significant number. In part **(a)** candidates were generally able to find the radius of the given circle although some assumed that since an equation was asked for, it must be a straight line. Those who used the formula $(x-a)^2+(y-b)^2=r^2$ were usually able to correctly answer the question although some forgot to square the radius. Those using the formula involving *f, g* and *c* were often less successful.

In part **(b)** many different methods were used to show that *DC* was a tangent to the circle. The intended approach was to find *C*, the gradient of the diameter and the gradient of *DC* and then show that the product of the gradients was equal to –1 and hence state that *DC* was perpendicular to the diameter and therefore was a tangent. Candidates who followed this approach generally made progress but the reasoning at the end was often insufficient.

Part **(c)** was usually only answered well by those who realised the symmetry of the situation. The number of marks available for this part should have been an indication that a significant amount of algebraic work was not required.

Question 10

Most candidates were confident in answering **(a)** and using the calculus techniques required. Candidates should however take care to ensure they have correctly writing down the given function; several candidates missed out the $-x$ term. Part **(b)** should have indicated to these candidates that a mistake had been made as no stationary points existed. Weaker candidates who did include the $-x$ term were often unable to differentiate or integrate it correctly. Some candidates also appeared confused over which part of their answers to multiply or divide by –2 for those who remembered that it was required.

In part **(b)** those candidates who put their differential equal to zero were much more successful than those who put y = 0, because the resulting equation was much more straightforward. Those who obtained a cubic equation by putting *y = 0* and used only a calculator function to solve it did not show sufficient working for all marks to be awarded. Candidates are reminded to show their method clearly.

A significant number of candidates omitted **(c),** often after having been unable to find the correct coordinates of *M* in **(b)**. It is important to remember that both limits in a definite integral need to be shown substituted into the expression.

Question 11

Many fully correct answers were seen in each part of this question, however a significant number of candidates omitted one or more of the question parts. In part **(a)** some weaker candidates substituted 0 and π into the given equation, obtaining 5 for each answer and then gave these as the both greatest and least values of *y*. Some candidates did not identify the connection between **(a)** and **(b)** and their graphs did not correspond with their earlier answers. Other candidates used their calculators in degrees mode and obtained a straight line. Part **(c)** was found challenging by many, but those who attempted to draw the corresponding lines on their graphs were often successful. In parts **(d)** and **(e)** the correct terminology was not often used. The transformations should have been described as stretches and translations and their size and directions clearly indicated. For stretches this is best done by describing which axis it is parallel to as well as the corresponding scale factor, and for translations by use of a column vector.

MATHEMATICS

Paper 9709/13
Paper 13

Key messages

The 2020 revised syllabuses contain a statement in the rubric on the front cover that states, 'no marks will be given for unsupported answers from a calculator.' This means that working must be shown to justify solutions of quadratic equations, trigonometric equations and simultaneous equations. In the case of quadratic equations, for example, it would be necessary to show factorisation or use of the formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down the solution is not sufficient and hence marks are not awarded.

The two new topics on the revised syllabus: transformations (**Question 1(b)**) and circles (**Question 11**) were not attempted as successfully as many other topics.

General comments

The paper was generally well received by candidates and many very good solutions were seen. In general, candidates seemed to have sufficient time to finish the paper. Presentation of work was mostly good, although some of the answers seem to be written in pencil and then superimposed in ink giving a very unclear image once the script has been scanned. Candidates should be strongly advised not to do this.

Comments on specific questions

Question 1

Part **(a)** was successfully answered by almost all candidates. Part **(b)**, however, was found to be more challenging for candidates. Candidates were expected to identify the type of transformation i.e. translation, and then state the magnitude and direction of the translation, which was most succinctly done by using a column vector. Many candidates used a wordy description but often did not give the required information in a mathematically acceptable form. For example, the directions had to be in terms of the *x* or *y* directions or axes. Words such as 'left' or 'down' were often seen but not accepted.

Question 2

In part **(a)**, most candidates were able to integrate the given function accurately and apply at least one limit correctly to their integral. Many candidates, however, did not realise that the limit of infinity would result in that part of the integral tending to zero. Part **(b)** was well answered overall, but some candidates differentiated f(x) rather than using their integral from **(a)**.

Question 3

Overall, this question was answered well by candidates who set up and solved a 3-term quadratic in $\tan^2\theta$, finding both solutions in the given range. However, a significant number often did not show their method for solving the quadratic equation. Common errors seen included not showing the factorisation (or equivalent) or not giving the secondary solution. Several candidates also obtained $\tan^2\theta = \frac{2}{3}$ but then calculated the answer to $\tan\theta = \frac{2}{3}$ (33.7°) and gave answers following $\tan^2\theta = -1$ of 45° and 135°.

Some candidates used trigonometric identities to form an equation in either $\sin^2\theta$ or $\cos^2\theta$ which required substantial extra work and often errors were made.

Question 4

This question was generally well answered. Most candidates applied a correct method with the vast majority following the familiar route of finding the discriminant and then, often with a diagram, determining which inequalities to apply. The most common errors were that candidates omitted to show the factorisation (or equivalent), errors in collecting terms at the beginning, omitting a term or making sign errors and several left their answer as –22, 2 rather than applying correct inequalities.

Question 5

Most candidates found the appropriate three terms accurately. Weaker candidates sometimes struggled after this to find a suitable relationship that would lead them to a correct answer. Stronger candidates often found a relationship and then equated two expressions for the common ratio *r*. Algebraic errors, however, were not uncommon, which prevented some candidates from finding the value of $\frac{a}{b}$.

Question 6

Most candidates showed good algebraic skills in finding the correct inverse in **(a)**. The most successful strategy was to multiply both sides of the equation by the denominator. Candidates who worked with $\frac{1}{x}$ or $\frac{1}{y}$ were generally less successful. Several candidates identified from **(b)** that f(*x*) could be expressed in an alternative form and worked with this expression when finding the inverse in **(a)**. This made the algebra more demanding and some candidates following this path did not reach the correct answer. Several candidates transposed *x* and *y*, i.e. writing *x* = f(*y*) as their first step in the working rather than the last step and for this particular question this seemed to be the more successful strategy.

Part **(b)** was very well answered by most candidates, aided by the fact that the correct answer was given so that slips could be corrected. Part **(b)** intended to aid candidates with **(c)** but most candidates did not notice the hint given in **(b)** and were unable to successfully answer **(c)**.

Question 7

In part **(a)** most candidates started with $d = -\frac{\tan^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$ and demonstrated they knew the operations needed to obtain the given answer. However, many candidates made errors in subtracting the fractions and went on to swap the signs at a late stage in order to try to reach the given answer. In all cases where the answer is given in the question, it is important that candidates communicate each line of their method clearly, without omitting detail. Some perceptive candidates kept the minus sign with the '*d*' and so avoided making sign errors along the way.

In part **(b)**, most candidates applied the correct formula but working out $\cos^2\left(\frac{1}{6}\pi\right)$ and $\cos^4\left(\frac{1}{6}\pi\right)$ proved challenging for a number of candidates. Many candidates, however, reached the correct answer successfully.

Question 8

In part **(a)**, most candidates found the first derivative accurately, while more errors were seen in finding the second derivative. In part **(b)** most candidates equated their first derivative to zero and most were then able to deal with the negative power and arrive at a simple quadratic equation. Careful reading of the question reveals there is only one stationary point, but many candidates did not reject one of the solutions. Another common mistake was to forget to find the corresponding *y* value. Most candidates were able to determine that the stationary point was a minimum by showing that the second derivative was positive.

Question 9

In part **(a)** the most efficient and straightforward way to find angle at *A* was to draw the perpendicular from *O* to the mid-point of *AB* and then angle $A = \cos^{-1}\left(\frac{6}{8}\right)$. Whilst many candidates used this method, several candidates used other, less direct methods with varying degrees of success. For example, the cosine formula was used, sometimes intending to find angle at *A* and sometimes finding angle *AOB*. Sometimes the angle was first found in degrees and then converted to radians, which often lost accuracy during the conversion. Answers from some candidates were given to only 2 significant figures which was not sufficient for the accuracy mark in **(a)** and also lead to loss of accuracy to answers in **(b)** and **(c)**.

In part **(b)**, most candidates successfully found the area of the sector *ABC* and subtracted the area of triangle *AOB* using their angle from **(a)** in either radians or degrees. Again, many candidates used the more direct method which was to find the area of triangle *AOB* using $\frac{1}{2}bc\sin A$, in favour of first using Pythagoras' Theorem to find the length of the perpendicular from *O* to *AB* (= $\sqrt{28}$) from which the area of the triangle is $6\sqrt{28}$.

Part **(c)** was the most straightforward of the three parts and most candidates answered this well.

Question 10

Although **(a)** was answered reasonably well, the combination of *k* and *x* with positive and negative indices caused difficulties for weaker candidates. Even though *k* is described in the question as a positive constant some candidates treated *k* as a variable. Some candidates did not identify that the gradient is given as 3 when *x* is $\frac{1}{4}$ and they substituted into the function without differentiating.

In part **(b)** most candidates correctly integrated the first and second terms. For those who did not obtain a fully correct integration often made an error with the third term, most often with an attempt to integrate it with respect to *k* rather than *x*. Candidates knew to substitution after the integration, though many candidates made errors when substituting the lower limit of $\frac{1}{4}k^2$. Some candidates dealt with the upper and lower limits separately before combining them, which avoided error. The final mark was often not gained due to the lack of factorisation, or similar, or by not rejecting the negative solution.

Question 11

This question was found to be the most demanding on the paper. In part **(a)**, most candidates were able to substitute $x = -6$ and $y = 6$ into the LHS of the equation of the circle to obtain 200 and to compare this with 100 on the RHS, or to find the distance *CT* to be $\sqrt{200}$ and compare this with the radius of the circle (10. The comparison in either case was often done by using the symbol $\neq$, which merely shows that *T* does not lie **<u>on</u>** the circle, which is not sufficient. In order to obtain the final mark, it was necessary to use the symbol > and to state the conclusion that *T* lies outside the circle. Other methods were occasionally seen. These included finding that the minimum value of an *x*-coordinate lying on the circle is $8 - 10 = -2$ and hence that *T*, having an *x*-coordinate of –6, must lie outside the circle. Another method was to substitute $x = -6$ into the equation of the circle. This gives a quadratic equation in *y* which has no real solutions, which means that the line $x = -6$ does not intersect the circle and hence that *T* lies outside the circle.

A reasonable proportion of candidates answered this question well in part **(b)**, some aided by a diagram, which was also helpful in tackling **(c)** and **(d)**. Candidates needed to realise that the required angle was $\sin^{-1}\left(\frac{10}{\sqrt{200}}\right)$ which simplifies to $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ and hence to the given answer of 45° exactly. Many candidates attempted, unsuccessfully, to use the gradient of the line *CT*, whilst others assumed that the length of the tangent from *T* to the circle was 10 without establishing that this was the case.

In part **(c)**, many candidates were able to find the gradient of *CT* and hence of *AB* but were unable to make further progress. Successful candidates realised that *ACBT* was a square and hence that the line *AB* passed through the mid-point of *CT* and were thus able to find the equation of *AB*.

In part **(d)**, the *x*-coordinates of *A* and *B* are obtained by substituting the equation of *AB*, found in **(c)**, into the equation of the circle. Those candidates who had succeeded in finding the correct equation of *AB* in **(c)** went on to arrive at a simple quadratic equation in *x* which gave the two values of *x* as required.

MATHEMATICS

Paper 9709/21
Paper 21

Key messages

Candidates are reminded of the importance of reading questions carefully to ensure that they are giving their answer in the required form and to the appropriate level of accuracy. It is essential that clear and sufficient working is shown to support the answer a candidate gives for a question. Looking at the mark allocation for a question or question part should give an indication of the amount of work required for the solution.

General comments

There was a small entry for this paper and a wide range of abilities were seen. It was evident that some candidates were not sufficiently prepared for the examination.

Comments on specific questions

Question 1

Some candidates appeared to be unaware of the basic rule for the subtraction of two logarithmic terms. Of those that were able to obtain the result $\ln\left(\frac{2x+1}{x-3}\right)=2$, rewriting the expression in non-logarithmic form was problematic. For stronger candidates who obtained $\left(\frac{2x+1}{x-3}\right)=e^2$, rearranging the expression to the required form was usually done well although it was notable that many gave the answer as $x=\frac{-3e^2-1}{2-e^2}$. This was of course acceptable, but multiplying the numerator and denominator of the fraction by -1 would have given a neater result.

Question 2

Most candidates were able to provide a completely correct solution, showing a good understanding of both the factor theorem and remainder theorem. Any errors were usually arithmetic.

Question 3

It was essential that candidates realised that as an exact answer was required, they need to work with exact quantities rather than making use of their calculator. It was intended that the shaded area be found by calculating the area enclosed by the curve, the coordinate axes and the line $x=1$, and then subtracting this from the area of the trapezium shown in the diagram. Attempts to find the equation of the line AB and then use of a subtraction of the curve equation in order to find an expression to integrate, were less successful and more time consuming.

It was also important that solutions were set out well and with sufficient detail so that the processes involved could be followed and marks awarded where appropriate.

Question 4

(a) The most efficient method of dealing with this problem was to obtain two equations, $2x-5=x+6$ and $2x-5=-x-6$, or equivalent, taking into account the possible changes in sign. Some candidates did this, but others chose to square both sides of the equation and obtain a quadratic equation, thus eliminating having to decide about the correct change of sign. Correct solutions were common.

(b) It was essential that candidates realise that the use of the word 'hence' at the start of the question indicated that the use of the solution to **(a)** was required. By comparing the equation in **(a)** to that in **(b)**, the similarity between the two should have been clear, with x replaced by 2^{-y}. Equating the positive solution to **(a)** to 2^{-y}. then gave the required answer.

Many candidates did not consider the word 'hence' and often produced a lot of incorrect work. The mark allocation for this part of the question should have indicated that not too much work was required.

Question 5

(a) Most candidates used the memory function on their calculator to find the iterations to the required level of accuracy. With the starting value given, only a few iterations were required. It was important that the last two iterations given should round to the same value correct to 4 significant figures. Sometimes candidates forgot to write down the last iteration even though they had recognised it as being the last iteration needed. Candidates needed to ensure that the final answer was given to the required level of accuracy. A common error was to give the value from the last iteration, rather than to the accuracy required.

(b) This part of the question required knowledge of the fact that the initial equation be used such that $\alpha = \dfrac{6+8\alpha}{8+\alpha^2}$ or $x = \dfrac{6+8x}{8+x^2}$. Either equation was acceptable and some candidates were able to obtain one of these equations and hence solve to obtain the value of α.

Question 6

It was essential that candidates realised that the answers to each of the parts of the question needed to be exact in order to obtain full marks in each part.

(a) Use of the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$ was needed. The elimination of the term in $\cos\theta$ then lead to an expression in $\sin\theta$ only. Many candidates obtained a correct exact value.

(b) Very few candidates were able to obtain a correct solution for this part. It was essential that candidates recognised that as $\sec\theta = \dfrac{1}{\cos\theta}$ and as $\cos^2\theta + \sin^2\theta = 1$, using the exact value from **(a)** could lead to an exact value for $\sec\theta$.

(c) There was more success in this part of the question with candidates recognising that use of the appropriate double angle formula together with the exact answer to **(a)** was needed.

Question 7

(a) As candidates were required to show a given answer, it was essential that each step of their solution was shown clearly and in enough detail. Errors in the differentiation of the parametric equations and subsequent simplification were fairly common, with evidence of contrived answers seen. Candidates are advised not to alter what is an incorrect result to the required result as they may not obtain method marks that they would have otherwise obtained.

(b) Candidates should note that even if they do not get the required result in **(a)** they should carry on with subsequent parts of the question using the given result. Using their incorrect result is not an option. A standard piece of mathematical process was completed correctly by few candidates. An exact value for R was needed. Errors in obtaining the value of $\tan\alpha$ were common leading to $\alpha = 0.896$.

(c) Very few correct solutions were seen with candidates again not recognising the meaning of the word 'hence' in the context of this question. The solution of the equation $10\cos t - 8\sin t = 5$ was required and in order to do this the result obtained in **(b)** needed to be equated to 5.

Question 8

(a) Most candidates were able to make a reasonable attempt at the differentiation of the given function making use of the quotient rule. Errors in the simplification of the result often meant that candidates did not obtain the terms $16x^3 - 12x^2$ in the numerator of their derivative. This then meant that many candidates had a three-term cubic equation which they could not easily solve when they equated their numerator to zero. For those candidates that obtained the correct x-coordinates, the y-coordinates were not calculated, and candidates needed to check that they had given the answer required fully.

(b) Very few complete correct solutions were seen. Most candidates were able to use algebraic long division to obtain a correct quotient and remainder, but when integration of their result was attempted very few candidates realised that the remainder had to be considered as $\frac{\frac{1}{2}}{2x-1}$ and integrate to obtain $\frac{1}{4}\ln(2x-1)$.

MATHEMATICS

Paper 9709/22
Paper 22

Key messages

Candidates are reminded of the importance of reading questions carefully to ensure that they are giving their answer in the required form and to the appropriate level of accuracy. It is essential that clear and sufficient working is shown to support the answer a candidate gives for a question. Looking at the mark allocation for a question or question part should give an indication of the amount of work required for the solution.

General comments

There was a small entry for this paper and a wide range of ability was seen. It was evident that some candidates were not sufficiently prepared for the examination.

Comments on specific questions

Question 1

Some candidates appeared to be unaware of the basic rule for the subtraction of two logarithmic terms. Of those that were able to obtain the result $\ln\left(\frac{2x+1}{x-3}\right)=2$, rewriting the expression in non-logarithmic form was problematic. For stronger candidates who obtained $\left(\frac{2x+1}{x-3}\right)=e^2$, rearranging the expression to the required form was usually done well although it was notable that many gave the answer as $x=\frac{-3e^2-1}{2-e^2}$. This was of course acceptable, but multiplying the numerator and denominator of the fraction by –1 would have given a neater result.

Question 2

Most candidates were able to provide a completely correct solution, showing a good understanding of both the factor theorem and remainder theorem. Any errors were usually arithmetic.

Question 3

It was essential that candidates realised that as an exact answer was required, they need to work with exact quantities rather than making use of their calculator. It was intended that the shaded area be found by calculating the area enclosed by the curve, the coordinate axes and the line $x = 1$ and then subtracting this from the area of the trapezium shown in the diagram. Attempts to find the equation of the line *AB* and then use of a subtraction of the curve equation in order to find an expression to integrate, were less successful and more time consuming.

It was also important that solutions were set out well and with sufficient detail so that the processes involved could be followed and marks awarded where appropriate.

Question 4

(a) Most candidates made a reasonable attempt to differentiate the given equation using the quotient rule and then equate to zero. The solution to the resulting equation needed to be exact and because the quadratic did not factorise, some candidates mistakenly thought they had made an error.

(b) The trapezium rule is often challenging for candidates. It is important that the interval width is calculated correctly, bearing in mind that the interval value will usually be an integer. Making use of the diagram to split the shaded area into three intervals would have helped some candidates. Three intervals meant that four *x*–coordinates were needed. Candidates needed to note that in this question, an answer correct to 2 decimal places was required.

Question 5

(a) Candidates needed to recognise that implicit differentiation was involved as well as the differentiation of a product involving implicit differentiation. As the answer was given, it was essential that each step of the solution was shown in full. Many correct solutions were seen, but the fact that the answer was given may have given candidates the opportunity to go back and check their work and make appropriate corrections.

(b) As the answer to **(a)** was required to find the equation of the tangent and as it was also a given answer, many candidates were able to find the equation of the tangent and gained full marks. The final answer required was an equation equated to zero and some candidates did not do this. There were a few candidates who chose to find a perpendicular gradient and found the equation of the normal instead.

(c) Very few correct solutions were seen. Candidates were expected to equate the numerator of the derivative to zero and then to comment on the fact that neither *y* nor e^{2x} could be zero and hence there were no stationary points. Many candidates recognised that e^{2x} could not be zero but only after attempting to solve $e^{2x} = 0$ using a calculator and stating that they had an error. Candidates should be encouraged to present their arguments in a more mathematical manner. A statement that e^{2x} is never negative would have been sufficient for a method mark. Most candidates did not even consider y and so full marks were rarely gained.

Question 6

(a) Most candidates were able to identify that the integral of $\frac{8}{4x+1}$ involved $\ln(4x+1)$. Most candidates did not rewrite $\frac{8}{\cos^2(4x+1)}$ as $\sec^2(4x+1)$ before attempting integration. As a result, very few completely correct solutions were seen. It was also expected that the arbitrary constant be included in the final answer in order to gain full marks.

(b) Many candidates recognised that they needed to make use of a double angle formula to write $4\cos^2\left(\frac{x}{2}\right)$ in the form $p + q\cos x$, and many obtained the correct values of *p* and *q*. Most candidates were able to make a reasonable attempt at integration and substitution of the given limits but often resorted to the use of a calculator to evaluate their answer in spite of an exact answer being required.

Question 7

(a) Most candidates were able to gain as least one mark by using the factor theorem and the substitution of $x = 3$ into the given function. As expected, a result of 0 was obtained, but it was expected that candidates make some comment on their result. A comment such as ‘ When $x = 3$, $f(x) = 0$ so $(x - 3)$ is a factor’, or similar, would have been sufficient.

(b) Most candidates were able to use algebraic long division and obtain a correct quotient. Candidates were then expected to rearrange the quotient, which had been equated to zero, to show a given result. It was essential that candidates showed each step of their rearrangement carefully as correct manipulation of negative terms was needed.

(c) Most candidates used the memory function on their calculator and found the iterations to the required level of accuracy. As no starting value was given, it was expected that candidates refer back to the given diagram and use a negative initial value. It was important that the last two iterations given should round to the same value correct to 5 significant figures. Sometimes candidates forgot to write down the last iteration even though they had recognised it as being the last iteration needed. Candidates needed to ensure that the final answer was given to the required level of accuracy. A common error was to give the value from the last iteration, rather than to the accuracy required.

MATHEMATICS

Paper 9709/23
Paper 23

Key messages

Candidates are reminded of the importance of reading questions carefully to ensure that they are giving their answer in the required form and to the appropriate level of accuracy. It is essential that clear and sufficient working is shown to support the answer a candidate gives for a question. Looking at the mark allocation for a question or question part should give an indication of the amount of work required for the solution.

General comments

There was a small entry for this paper and a wide range of abilities were seen. It was evident that some candidates were not sufficiently prepared for the examination.

Comments on specific questions

Question 1

Some candidates appeared to be unaware of the basic rule for the subtraction of two logarithmic terms. Of those that were able to obtain the result $\ln\left(\frac{2x+1}{x-3}\right) = 2$, rewriting the expression in non-logarithmic form was problematic. For stronger candidates who obtained $\left(\frac{2x+1}{x-3}\right) = e^2$, rearranging the expression to the required form was usually done well although it was notable that many gave the answer as $x = \frac{-3e^2 - 1}{2 - e^2}$. This was of course acceptable, but multiplying the numerator and denominator of the fraction by -1 would have given a neater result.

Question 2

Most candidates were able to provide a completely correct solution, showing a good understanding of both the factor theorem and remainder theorem. Any errors were usually arithmetic.

Question 3

It was essential that candidates realised that as an exact answer was required, they need to work with exact quantities rather than making use of their calculator. It was intended that the shaded area be found by calculating the area enclosed by the curve, the coordinate axes and the line $x = 1$, and then subtracting this from the area of the trapezium shown in the diagram. Attempts to find the equation of the line AB and then use of a subtraction of the curve equation in order to find an expression to integrate, were less successful and more time consuming.

It was also important that solutions were set out well and with sufficient detail so that the processes involved could be followed and marks awarded where appropriate.

Question 4

(a) The most efficient method of dealing with this problem was to obtain two equations, $2x - 5 = x + 6$ and $2x - 5 = -x - 6$, or equivalent, taking into account the possible changes in sign. Some candidates did this, but others chose to square both sides of the equation and obtain a quadratic equation, thus eliminating having to decide about the correct change of sign. Correct solutions were common.

(b) It was essential that candidates realise that the use of the word 'hence' at the start of the question indicated that the use of the solution to **(a)** was required. By comparing the equation in **(a)** to that in **(b)**, the similarity between the two should have been clear, with *x* replaced by 2^{-y}. Equating the positive solution to **(a)** to 2^{-y}. then gave the required answer.

Many candidates did not consider the word 'hence' and often produced a lot of incorrect work. The mark allocation for this part of the question should have indicated that not too much work was required.

Question 5

(a) Most candidates used the memory function on their calculator to find the iterations to the required level of accuracy. With the starting value given, only a few iterations were required. It was important that the last two iterations given should round to the same value correct to 4 significant figures. Sometimes candidates forgot to write down the last iteration even though they had recognised it as being the last iteration needed. Candidates needed to ensure that the final answer was given to the required level of accuracy. A common error was to give the value from the last iteration, rather than to the accuracy required.

(b) This part of the question required knowledge of the fact that the initial equation be used such that $\alpha = \dfrac{6+8\alpha}{8+\alpha^2}$ or $x = \dfrac{6+8x}{8+x^2}$. Either equation was acceptable and some candidates were able to obtain one of these equations and hence solve to obtain the value of α.

Question 6

It was essential that candidates realised that the answers to each of the parts of the question needed to be exact in order to obtain full marks in each part.

(a) Use of the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$ was needed. The elimination of the term in $\cos\theta$ then lead to an expression in $\sin\theta$ only. Many candidates obtained a correct exact value.

(b) Very few candidates were able to obtain a correct solution for this part. It was essential that candidates recognised that as $\sec\theta = \dfrac{1}{\cos\theta}$ and as $\cos^2\theta + \sin^2\theta = 1$, using the exact value from **(a)** could lead to an exact value for $\sec\theta$.

(c) There was more success in this part of the question with candidates recognising that use of the appropriate double angle formula together with the exact answer to **(a)** was needed.

Question 7

(a) As candidates were required to show a given answer, it was essential that each step of their solution was shown clearly and in enough detail. Errors in the differentiation of the parametric equations and subsequent simplification were fairly common, with evidence of contrived answers seen. Candidates are advised not to alter what is an incorrect result to the required result as they may not obtain method marks that they would have otherwise obtained.

(b) Candidates should note that even if they do not get the required result in **(a)** they should carry on with subsequent parts of the question using the given result. Using their incorrect result is not an option. A standard piece of mathematical process was completed correctly by few candidates. An exact value for *R* was needed. Errors in obtaining the value of $\tan\alpha$ were common leading to $\alpha = 0.896$.

(c) Very few correct solutions were seen with candidates again not recognising the meaning of the word 'hence' in the context of this question. The solution of the equation $10\cos t - 8\sin t = 5$ was required and in order to do this the result obtained in **(b)** needed to be equated to 5.

Question 8

(a) Most candidates were able to make a reasonable attempt at the differentiation of the given function making use of the quotient rule. Errors in the simplification of the result often meant that candidates did not obtain the terms $16x^3 - 12x^2$ in the numerator of their derivative. This then meant that many candidates had a three-term cubic equation which they could not easily solve when they equated their numerator to zero. For those candidates that obtained the correct x-coordinates, the y-coordinates were not calculated, and candidates needed to check that they had given the answer required fully.

(b) Very few complete correct solutions were seen. Most candidates were able to use algebraic long division to obtain a correct quotient and remainder, but when integration of their result was attempted very few candidates realised that the remainder had to be considered as $\dfrac{\frac{1}{2}}{2x-1}$ and integrate to obtain $\frac{1}{4}\ln(2x-1)$.

MATHEMATICS

Paper 9709/31
Paper 31

Key messages

Candidates should be reminded of the importance of sketching a diagram to aid with solving inequalities. This was particularly helpful for **Question 1**. They should also know the important features when sketching graphs of standard functions, which was needed for **Question 5(a)**.

Candidates should ensure that they are confident with differentiation and in particular the use of the chain rule, as was required for **Question 10(a)**. They should also ensure they understand how to apply the law of logarithms and that log has base 10, while ln has base *e*, see **Question 4**.

Candidates are also reminded that calculators cannot be used in answering complex number questions. This was important for **Question 7(a)**.

General comments

The standard of work on this paper was extremely high. A considerable number of candidates performed well on many of the questions, with the exception of **Question 5(a)** and the final mark in **Question 11(b)** which proved challenging. On occasion some work was difficult to read and candidates should be reminded to set out their work clearly to ensure it is legible.

Candidates are generally aware of the need to show sufficient working in their solutions. They should, however, be reminded that where a question asks for the answer to be given in a specific form, they must obtain this form to be able to gain the relevant accuracy marks.

Comments on specific questions

Question 1

Most candidates squared both sides of the inequality and solved the resulting quadratic equation, gaining three marks. However, the majority of these candidates then did not interpret the two solutions so could not be awarded the final mark. A few candidates chose to sketch a graph and reached a correct inequality as the sketch enabled them to visualise it. This avoided introducing solutions to the quadratic equation that were not solutions of the original problem.

Question 2

Some candidates did not realise that both the loci were circles since the structure of the two inequalities was similar. Without overlapping circles their shaded region was also incorrect. A significant number of candidates had the smaller circle centred at an incorrect point, i.e. not (1, –i). Since the question asks for a sketch, rather than an accurate drawing, it was not essential to use compasses to draw the circles. However, when the final part of such a question involves a numerical calculation, an accurate diagram can be useful. The sketch does not need to be large, but it should clearly show the features that are important in solving the problem. The sketch should have an indication of scale on the real and imaginary axes, with equal scales on both. The shading needs to clearly identify the region that the candidate believes represents points satisfying both the given inequalities.

Question 3

This question was answered very well by the majority of candidates. Only a few did not know how to use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$. Some candidates did not use the chain rule for differentiating and mistakenly found $\frac{dx}{d\theta} = \sin 2\theta$ and $\frac{dy}{d\theta} = 2 + \cos 2\theta$. Having reached $\frac{dy}{dx} = \frac{(2+2\cos 2\theta)}{(2\sin 2\theta)}$, some candidates did not realise they needed to use double angle formulae in both the numerator and denominator. Those candidates who differentiated before applying double angle formulae usually did better than those who used the double angle formulae before trying to differentiate. If errors arose before the differentiation took place, then the incorrect differentiation received no credit. Occasionally candidates detected a sign error in their differentiation once they realised, they would not reach the stated answer.

Question 4

Many candidates produced accurate, concise solutions. Others scored very low marks through omitting to apply log laws. It is essential to deal with any coefficients of log terms other than unity before attempting to combine log terms. Candidates need to know which base is associated with ln and log terms since, despite the presence of 'log' in the question, it was common to see e^{-1} or e^{1} as opposed to 10^{-1} or 10^{1}. Other errors were seen in incorrectly rejecting the negative solution, leaving answers in surd form, or rounding to 3 significant figures rather than 3 decimal places as requested.

Question 5

(a) Candidates could gain full marks for this part by drawing clear sketches of both graphs and indicating or stating that the graphs intersected at two points, hence showing that the equation had two roots in the given interval. Many candidates were unable to sketch the graph of $y = \operatorname{cosec} x$. Candidates should be encouraged to use their knowledge of graph transformations when sketching functions such as the exponential one and to show asymptotic behaviour correctly. They should include some indication of scale on both axes and make sure that they clearly represent the features of the function they are sketching. In this case, the cosec graph is U shaped, it is symmetrical in the given domain, $y\left(\frac{\pi}{2}\right) = 1$ and there are asymptotes at $x = 0$ and $x = \pi$. Common mistakes in sketching the exponential graph were drawing a straight line, not including $y(0) = 2$ and not indicating the asymptote at $y = 1$ even for $x > \pi$.

(b) This question was answered well, with many candidates gaining full marks. However, a few candidates worked in degrees despite the fact that in **(a)** it states that $0 < x < \pi$ indicating that radians should be used. Candidates should be aware that, in general, radians will be used where trigonometric functions are compared with other functions. Some candidates did not give the results of each iteration to 4 decimal places. or state the root to 2 decimal places. As with all calculator work, candidates should be reminded to check by repeating the calculation.

Question 6

(a) The best approach for this question was to expand $R\cos(\theta)$ and equate terms, leading to $R\cos = \sqrt{6}$ and $R\sin = 3$. Then to find R and α using Pythagoras and simple trigonometry. Errors arose through quoting formulae and ending with $\tan = \frac{\sqrt{6}}{3}$ or with $\cos = \sqrt{6}$ and $\sin = 3$. This question requested an exact value of R and a value of α to 2 decimal places so $R = 3.87$ and $\alpha = 50.8°$ received no credit.

(b) Most candidates realised that there was a link between **(a)** and **(b)** so successfully reached $\cos^{-1}\left(\frac{2.5}{\sqrt{15}}\right)$. However, some did not realise that they should replace the θ in **(a)** by $\frac{x}{3}$ in **(b)**. Many candidates obtained the value of $x = 301.7$ but the second value of $x = 2.9$ was seen far less often. Some candidates who used a rounded value could not be awarded marks because their values for x were outside the allowed tolerance.

Question 7

(a) Candidates needed to substitute the number $-1 + \sqrt{5}i$ into the function $f(x) = 2x^3 + x^2 + 6x - 18$ and show, with some clear but straightforward calculation, that this evaluated to zero. In this way they verified that $-1 + \sqrt{5}i$ is a root of $f(x) = 0$. In such questions, candidates need to expand all brackets and reduce each term to either a real number or a term in i (not a power of i) in order to demonstrate convincingly that the expression is equal to zero. A solution of the form $2(-1 + \sqrt{5}i)^3 + (-1 + \sqrt{5}i)^2 + 6(-1 + \sqrt{5}i) - 18 = 0$ was insufficient. Candidates may find it easier to break the calculation into parts, firstly finding $(-1 + \sqrt{5}i)^2$ and then using this to find $(-1 + \sqrt{5}i)^3$ before substituting. It is better not to include ' = 0' from the start as it leads to the statement 0 = 0 which proves nothing. Candidates should be encouraged to replace i^2 with -1 directly and to collect like terms as this reduces the chance of errors.

(b) Many candidates produced good solutions to this part. Most realised that the conjugate, $-1 - \sqrt{5}i$, was also a root. They could then find a real quadratic factor and hence the linear factor, $2x - 3$, leading to the third root, $\frac{3}{2}$. Some candidates were confused about the difference between a root and a factor and so incorrectly wrote $2x - 3$, rather than $\frac{3}{2}$, as the root. Many also did not present $-1 - \sqrt{5}i$ as a root at all but simply used it in their calculations, but the question required them to find the other roots. It was not necessary to verify that $-1 - \sqrt{5}i$ was also a root. A fairly common mistake was to write $2x^3 + x^2 + 6x - 18 = (x - (-1 - \sqrt{5}i))\,(x - (-1 + \sqrt{5}i))\,(x - a)$ and to find a. However, this led to errors since it took no account of the leading coefficient 2.

Some candidates had already done much of the necessary work in **(a)**. For this to be credited in **(b)**, they needed to mention **(a)** in their answer to this part. Many candidates simply used their calculators to find the remaining two roots and this was insufficient to gain credit. A number of candidates tried to divide the cubic by $-1 - \sqrt{5}i$ or $-1 + \sqrt{5}i$ or their product of 6. Once again, such candidates were confused between the concept of a root and a factor. A small number of candidates tried to divide the cubic by $x - (-1 + \sqrt{5}i)$ but were almost always unsuccessful as it led to complex working.

Question 8

This was a reasonably straightforward question on first order differential equation with separable variables. The majority of candidates were able to separate the variables correctly and commence integration of $\frac{1}{y}$ to ln *y*. Few used modulus signs, but in this question this was not necessary. Most candidates were able to split the fraction in *x* successfully and integrate both parts correctly. Some candidates chose to integrate by parts twice and often successfully arrived at the correct terms. Others simply integrated $\frac{1}{x}$ and $12x^2$ and multiplied the results together.

Question 9

(a) Most candidates scored all or most of the marks on this question. Most successful candidates used the form $\frac{A}{(1\ x)} + \frac{B}{(2+3x)} + \frac{C}{(2+3x)^2}$. Some candidates made arithmetical errors when evaluating *A*, *B* and *C* through multiplication and substitution. The most common error was to multiply throughout by an expression involving $(2 + 3x)^3$. Those who chose to compare coefficients were more likely to make an error.

(b) This question proved more challenging for some candidates. Most were able to find the first terms of the expansion of $1 - x$ without difficulty, although sign errors occurred in a significant minority of cases. Many candidates were able to start the expansions of $\left(1+\left(\frac{3}{2}\right)x\right)^{-1}$ and of $\left(1+\left(\frac{3}{2}\right)x\right)^{-2}$, but made errors combining *B* and *C* with either 2^{-1} or, more commonly, 2^{-2}. Some candidates correctly found all the required terms only to make arithmetic slips when combining their three expansions.

Those who solved **(a)** in the form $\frac{A}{(1-x)}+\frac{(Bx+C)}{(2+\ 3x)^2}$ were less successful in this part, either forgetting to multiply their second expansion by $Bx + C$ or doing so incorrectly.

Question 10

(a) This should have been a straightforward application of the product rule for differentiation, but many candidates struggled because they were unable to differentiate $e^{\frac{-x}{2}}$. Some candidates who initially performed it correctly saw their 2 in the denominator drift into the numerator. Most knew they needed to set their derivative to zero, but some candidates had trouble solving the equation. If they cancelled the exponent term throughout, they could obtain $x = 4$ easily. Instead of this many candidates decided to take ln which was unnecessary and did not often lead to the answer.

(b) This part also required the application of a basic technique, integration by parts, although here the technique had to be applied twice. Candidates often found the integration of $e^{\frac{-x}{2}}$ difficult, with similar errors to **(a)** seen. The integration would be better done as a continuous piece of working rather than on different parts of the page as this often led to sign errors or missing terms. To minimise errors, candidates should make no attempt to substitute limits until all the integration has been performed and all similar terms collected together.

Question 11

(a) Many excellent solutions were seen, but there were some candidates who scored just a single mark because of arithmetical errors. Most candidates realised that two of the equations were independent of *a* and so could be solved immediately for μ and λ. From these two values it was easy to calculate *a* and find the position vector of the point of intersection. Candidates should be encouraged to check the point of intersection in the equation of both lines as this would help them to identify numerical errors.

(b) Some excellent solutions to this question were seen from candidates who realised that the scalar product required the direction vectors of the two lines and not the equation of the line or the position vectors of points on the line. However, poor algebra was seen in many good solutions, for instance the length of one of the vectors seen as $\sqrt{(a^2 + 2^2 + (-1)^2)} = \sqrt{(a^2 + 5)} = \sqrt{5}a$.

Few candidates took the modulus of the scalar product or used $\pm\frac{1}{6}$, the cosine of the given acute angle. Finding the angle between two given vectors using the scalar product produces a unique acute or obtuse angle. However, when finding the acute angle between the direction vectors of two lines, for example **c** and **d**, it is essential to choose the combination of ± **c** and ± **d** that produces a positive scalar product. Similarly, when finding the obtuse angle, a negative scalar product is needed. However, in questions such as this one, where a component of one direction vector is unknown **(a)**, a positive or negative scalar product cannot be achieved without placing some restriction on the value of *a*. To cover all possibilities, the values of *a* that give an acute angle between the direction vectors from the scalar product of **c** and ±**d** can be found. This is easy to do by taking the modulus of the scalar product or $\pm\frac{1}{6}$, the cosine of the given acute angle. It could be argued that in order to determine *a* in the scalar product and in the length of the vectors we can square both sides of the equation so the incorrect solutions now introduced are the same as those added by considering ± earlier. However, all this is doing is including a sign error (by considering only one sign at the start) then another sign error that corrects this. This explains why candidates obtained both values for *a* as shown in the mark scheme, even without using modulus or $\pm\frac{1}{6}$. If they had considered the scalar product $(2a - 3)$ using $a = 1$, they would have realised that it is negative for this value and would lead to an obtuse angle not an acute one. (A similar process works when looking for obtuse angles.)

MATHEMATICS

Paper 9709/32
Paper 32

Key messages for candidates

- Candidates should be reminded to read the questions carefully to ensure they answer the questions as they have been set.
- Candidates should make sure that they are familiar with the standard notation, as shown in the specification.
- Practising the basic methods in calculus will ensure that candidates are familiar with all the standard techniques and patterns.
- Candidates should be reminded to show all the stages in their working.
- Candidates should ensure that their work is clearly presented and that they do not write over their solutions. If candidates wish to replace a solution they are reminded to cross through the solution they do not want to be marked first.
- Candidates should check their algebra and arithmetic carefully.
- If a question asks for an exact answer then decimal working is not appropriate.

General comments:

Most candidates offered solutions to all ten questions, and they appeared to have sufficient time to review their solutions as necessary.

Much of candidate work was clearly presented and many candidates had taken notice of the instruction to show all necessary working. However, several candidates did not use brackets correctly in their work and it was common for brackets to be omitted when they were needed.

There were many candidates whose response to a request for an exact answer was simply to show more decimal places in their working. Decimal approximations are not appropriate when an exact answer is required.

Comments on specific questions

Question 1

Many candidates made the correct first step, forming an equation without logarithms. Some candidates achieved the next step, to rearrange and obtain an expression for e^{-3x} or e^{-x} and solved to obtain the correct value for *x*. A few candidates who used the correct method did not give the final answer to the required accuracy.

The false reasoning of removing the bracket in the given equation to give $\ln 1 + (-3x) = 2$ was very common.

Question 2

(a) Many candidates gave a fully correct answer. There were a number of slips in the arithmetic to simplify the coefficients, but the most common errors were using powers of *x* rather than 6*x* or using an incorrect index for the expansion.

(b) A significant number of candidates gave no response to this part of the question. Some responses showed some correct thinking, but only considered one side of the inequality. A few responses just repeated all or part of the answer to **(a)**.

Question 3

(a) Many candidates made a correct first step to obtain an equation such as $y\ln 2 = (1-2x)\ln 3$. Several candidates omitted the brackets and started with $y\ln 2 = 1-2x\ln 3$. In their subsequent working this confirmed that they had not intended to use brackets. The question asked candidates to show that the graph of *y* against *x* is a straight line. The simplest way to do this is either to compare their equation with the form $ay+bx=c$ or to rearrange their equation to make *y* the subject and then compare with $y=mx+c$. Several candidates made no comment at all about the equation. The incorrect comparison between the equation for $y\ln 2$ and $y=mx+c$ followed by a statement that the gradient was $\pm 2\ln 3$ or $\pm 2x\ln 3$ was common. The question asked for the exact value of the gradient, so a decimal approximation was not correct. Fully correct solutions were rarely seen.

(b) Several candidates were able to use their exact equation for the line to obtain the exact *x*-coordinate. The question required an exact answer, so those candidates who started with an equation with decimal coefficients scored no marks. Some candidates obtained a correct denominator of $2\ln 3 + 3\ln 2$ and did not simplify this or made errors in the process. Some candidates clearly understood that they needed to rearrange the equation, but their working contained errors in the algebra.

Question 4

(a) Many candidates gave concise, fully correct solutions to this question. Longer solutions involved steps such as expanding $\dfrac{\sin(\theta+60°)}{\cos(\theta+60°)}$. A common error was to rewrite $\tan(\theta+60°)$ as $\tan\theta+\tan 60°$. Some candidates rewrote $2\cot\theta$ as $\dfrac{1}{2\tan\theta}$.

(b) Starting with the given quadratic equation, many candidates scored at least 2 marks. The most common errors were due to premature approximation of decimal values or giving incorrect over-specified answers. Several candidates included additional incorrect solutions, and several claimed that $\tan\theta = -5.556$ gave no solution within the range. A minority of candidates rewrote the given equation as $\tan\theta\left(\tan\theta+3\sqrt{3}\right)=2$ and equated each factor to 2.

Question 5

(a) Most candidates knew how to find the gradient of a function given in parametric form. Many knew the derivative of $\tan\theta$. Some derived this from first principles by using the quotient formula to differentiate $\dfrac{\sin\theta}{\cos\theta}$, although that was not necessary in this question. Candidates had more difficulty with differentiating $\cos^2\theta$. A common incorrect answer was $\pm\sin^2\theta$. Several candidates did not try to differentiate the function directly and started by using the double angle formula to express *y* in terms of $\cos 2\theta$ before attempting to differentiate.

(b) Only the strongest candidates answered this question correctly. Among other candidates the most common approach was to look for the point(s) where the gradient was zero. These candidates did not appear to understand that the turning point of the original function was not necessarily going to be at the point where the gradient took its maximum value. Those candidates who started by differentiating $-2\sin\theta\cos^3\theta$ often scored at least three of the marks available. Although *P* is clearly shown on the diagram as a point with a negative *x*-coordinate, several candidates gave a positive final answer.

Question 6

(a) Most candidates knew the correct method for the division of complex numbers, and there were many correct solutions. There were some slips in the arithmetic, with a few candidates who did not know what to do with i^2 and a few who attempted to use $1-i$ in place of $1+i$. The minority of candidates who started by multiplying the equation by $1-i$ often did not get as far as considering the real and imaginary parts of their equation.

(b) Many of the Argand diagrams were produced with a good degree of accuracy. However, some very clearly had uneven scales, and a few had the real and imaginary axes the wrong way round. There were several blank responses.

(c) The given answer here appeared not to help many candidates. As the answer is given, it is important to say where the equation comes from, so an equation showing the relationship between the arguments was expected. In their working, many candidates implied that in the division of two complex numbers the arguments are added. It was very common for candidates to state that $\arg(1-i)=\frac{\pi}{4}$. Those candidates who simply calculated decimal approximations to both sides of the given equation scored no marks.

Question 7

(a) Most candidates recognised that they needed to start by separating the variables. Many candidates obtained $-\frac{1}{3}e^{-3t}$. Common errors at this point were sign errors and putting the 3 in the wrong place. The integration of $\cos^{-2}2x$ proved to be much more difficult. Those candidates who did not recognise this as $\sec^2 2x$ were not able to make any further progress. Common errors in attempts to complete the correct integration centred on the 2. The coefficient of $\tan 2x$ was often incorrect, and sometimes the answer was given as a multiple of $\tan x$. Candidates who completed the integration correctly usually obtained the correct final answer.

(b) There were many answers where the candidate simply stated 'it gets bigger' or 'it gets smaller', but there was also evidence of candidates engaging with the process of finding the limit and considering the value of e^{-3t} for large values of t.

Question 8

(a) The majority of candidates started by using a correct method to find $\overrightarrow{AB}$ and $\overrightarrow{CD}$. The question asked candidates to establish a relationship between the lengths of these two vectors, but only a minority of candidates went on to consider the magnitudes. There were several slips in the arithmetic.

(b) Most candidates understood the correct process for finding the angle between two vectors. The majority used their versions of the correct pair of vectors and used the scalar product to find an angle. Many candidates expected the answer to be an acute angle, but the correct answer here is an obtuse angle, as indicated by the negative value of the scalar product.

(c) Many candidates understood how to form the equations of the two lines and how to look for a point of intersection. There were several slips in the arithmetic, and some candidates used incorrect equations having confused directions and position vectors, while others used the same parameter in both line equations so they could not solve them correctly. Candidates were asked to show that the lines do not meet. It was not sufficient to get to a statement such as $\frac{17}{3}=\frac{7}{3}$ and to make no further comment.

Question 9

(a) The majority of candidates used the correct form for the partial fractions, and many obtained the correct values for the coefficients. Apart from slips in the arithmetic, common errors were to use the form $\frac{A}{3x+2}+\frac{B}{x^2+4}$ or to rewrite x^2+4 as $(x+2)^2$ or as $(x-2)(x+2)$. Some candidates used 8 in place of 18 in their working.

(b) Most candidates used the correct form for the integral of $\frac{A}{3x+2}$, although the coefficient was not always correct. The integration of $\frac{Bx+C}{x^2+4}$ proved to be much more difficult, with only a minority of candidates realising that this needed to be rewritten as two separate terms. Many chose an incorrect form involving logarithms. A few candidates expected something involving inverse tangents and tried to use that on the combined term.

Question 10

(a) Most candidates started with a correct use of the rule for differentiation of a product and went on to obtain the given form. There were many correct and concise solutions, but some that were quite lengthy. Some candidates made no attempt to rearrange their answer to the given form.

(b) The majority of candidates were confident in using the iterative process. The question asked candidates to use an initial value of 3, but some candidates did not do this. The most common errors were to work in degrees, or to use an incorrect formula. There were several solutions using $\tan$ in place of $\tan^{-1}$ and some where there was no obvious explanation for the numbers listed. In this question candidates needed to work to the accuracy specified.

(c) Only a minority of candidates used the correct method to integrate to find a volume, and even fewer used the correct method to integrate $\cos^2 x$. Many candidates invested a lot of effort into attempts to integrate $\sqrt{x}\cos x$. Those candidates who started with a correct form for the integral often claimed that the integral of $\cos^2 x$.was $\sin^2 x$. Candidates who integrated correctly usually obtained the correct final answer, but some used the incorrect upper limit $\frac{3}{2}\pi$.

MATHEMATICS

Paper 9709/33
Paper 33

Key messages

Candidates should be reminded of the importance of sketching a diagram to aid with solving inequalities. This was particularly helpful for **Question 1**. They should also know the important features when sketching graphs of standard functions, which was needed for **Question 5(a)**.

Candidates should ensure that they are confident with differentiation and in particular the use of the chain rule, as was required for **Question 10(a)**. They should also ensure they understand how to apply the law of logarithms and that log has base 10, while ln has base *e*, see **Question 4**.

Candidates are also reminded that calculators cannot be used in answering complex number questions. This was important for **Question 7(a)**.

General comments

The standard of work on this paper was extremely high. A considerable number of candidates performed well on many of the questions, with the exception of **Question 5(a)** and the final mark in **Question 11(b)** which proved challenging. On occasion some work was difficult to read and candidates should be reminded to set out their work clearly to ensure it is legible.

Candidates are generally aware of the need to show sufficient working in their solutions. They should, however, be reminded that where a question asks for the answer to be given in a specific form, they must obtain this form to be able to gain the relevant accuracy marks.

Comments on specific questions

Question 1

Most candidates squared both sides of the inequality and solved the resulting quadratic equation, gaining three marks. However, the majority of these candidates then did not interpret the two solutions so could not be awarded the final mark. A few candidates chose to sketch a graph and reached a correct inequality as the sketch enabled them to visualise it. This avoided introducing solutions to the quadratic equation that were not solutions of the original problem.

Question 2

Some candidates did not realise that both the loci were circles since the structure of the two inequalities was similar. Without overlapping circles their shaded region was also incorrect. A significant number of candidates had the smaller circle centred at an incorrect point, i.e. not (1, –i). Since the question asks for a sketch, rather than an accurate drawing, it was not essential to use compasses to draw the circles. However, when the final part of such a question involves a numerical calculation, an accurate diagram can be useful. The sketch does not need to be large, but it should clearly show the features that are important in solving the problem. The sketch should have an indication of scale on the real and imaginary axes, with equal scales on both. The shading needs to clearly identify the region that the candidate believes represents points satisfying both the given inequalities.

Question 3

This question was answered very well by the majority of candidates. Only a few did not know how to use $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$. Some candidates did not use the chain rule for differentiating and mistakenly found $\frac{dx}{d\theta}=\sin 2\theta$ and $\frac{dy}{d\theta}=2+\cos 2\theta$. Having reached $\frac{dy}{dx}=\frac{(2+2\cos 2\theta)}{(2\sin 2\theta)}$, some candidates did not realise they needed to use double angle formulae in both the numerator and denominator. Those candidates who differentiated before applying double angle formulae usually did better than those who used the double angle formulae before trying to differentiate. If errors arose before the differentiation took place, then the incorrect differentiation received no credit. Occasionally candidates detected a sign error in their differentiation once they realised, they would not reach the stated answer.

Question 4

Many candidates produced accurate, concise solutions. Others scored very low marks through omitting to apply log laws. It is essential to deal with any coefficients of log terms other than unity before attempting to combine log terms. Candidates need to know which base is associated with ln and log terms since, despite the presence of 'log' in the question, it was common to see e^{-1} or e^{1} as opposed to 10^{-1} or 10^{1}. Other errors were seen in incorrectly rejecting the negative solution, leaving answers in surd form, or rounding to 3 significant figures rather than 3 decimal places as requested.

Question 5

(a) Candidates could gain full marks for this part by drawing clear sketches of both graphs and indicating or stating that the graphs intersected at two points, hence showing that the equation had two roots in the given interval. Many candidates were unable to sketch the graph of $y=\operatorname{cosec} x$. Candidates should be encouraged to use their knowledge of graph transformations when sketching functions such as the exponential one and to show asymptotic behaviour correctly. They should include some indication of scale on both axes and make sure that they clearly represent the features of the function they are sketching. In this case, the cosec graph is U shaped, it is symmetrical in the given domain, $y\left(\frac{\pi}{2}\right)=1$ and there are asymptotes at $x = 0$ and $x = \pi$. Common mistakes in sketching the exponential graph were drawing a straight line, not including $y(0)=2$ and not indicating the asymptote at $y=1$ even for $x > \pi$.

(b) This question was answered well, with many candidates gaining full marks. However, a few candidates worked in degrees despite the fact that in **(a)** it states that $0 < x < \pi$ indicating that radians should be used. Candidates should be aware that, in general, radians will be used where trigonometric functions are compared with other functions. Some candidates did not give the results of each iteration to 4 decimal places. or state the root to 2 decimal places. As with all calculator work, candidates should be reminded to check by repeating the calculation.

Question 6

(a) The best approach for this question was to expand $R\cos(\theta)$ and equate terms, leading to $R\cos=\sqrt{6}$ and $R\sin=3$. Then to find R and α using Pythagoras and simple trigonometry. Errors arose through quoting formulae and ending with $\tan=\frac{\sqrt{6}}{3}$ or with $\cos=\sqrt{6}$ and $\sin=3$. This question requested an exact value of R and a value of α to 2 decimal places so $R = 3.87$ and $\alpha = 50.8°$ received no credit.

(b) Most candidates realised that there was a link between **(a)** and **(b)** so successfully reached $\cos^{-1}\left(\frac{2.5}{\sqrt{15}}\right)$. However, some did not realise that they should replace the θ in **(a)** by $\frac{x}{3}$ in **(b)**. Many candidates obtained the value of $x = 301.7$ but the second value of $x = 2.9$ was seen far less often. Some candidates who used a rounded value could not be awarded marks because their values for x were outside the allowed tolerance.

Question 7

(a) Candidates needed to substitute the number $-1 + \sqrt{5}\mathrm{i}$ into the function $\mathrm{f}(x) = 2x^3 + x^2 + 6x - 18$ and show, with some clear but straightforward calculation, that this evaluated to zero. In this way they verified that $-1 + \sqrt{5}\mathrm{i}$ is a root of $\mathrm{f}(x) = 0$. In such questions, candidates need to expand all brackets and reduce each term to either a real number or a term in i (not a power of i) in order to demonstrate convincingly that the expression is equal to zero. A solution of the form $2(-1 + \sqrt{5}\mathrm{i})^3 + (-1 + \sqrt{5}\mathrm{i})^2 + 6(-1 + \sqrt{5}\mathrm{i}) - 18 = 0$ was insufficient. Candidates may find it easier to break the calculation into parts, firstly finding $(-1 + \sqrt{5}\mathrm{i})^2$ and then using this to find $(-1 + \sqrt{5}\mathrm{i})^3$ before substituting. It is better not to include ' = 0' from the start as it leads to the statement 0 = 0 which proves nothing. Candidates should be encouraged to replace i^2 with -1 directly and to collect like terms as this reduces the chance of errors.

(b) Many candidates produced good solutions to this part. Most realised that the conjugate, $-1 - \sqrt{5}\mathrm{i}$, was also a root. They could then find a real quadratic factor and hence the linear factor, $2x - 3$, leading to the third root, $\frac{3}{2}$. Some candidates were confused about the difference between a root and a factor and so incorrectly wrote $2x - 3$, rather than $\frac{3}{2}$, as the root. Many also did not present $-1 - \sqrt{5}\mathrm{i}$ as a root at all but simply used it in their calculations, but the question required them to find the other roots. It was not necessary to verify that $-1 - \sqrt{5}\mathrm{i}$ was also a root. A fairly common mistake was to write $2x^3 + x^2 + 6x - 18 = (x - (-1 - \sqrt{5}\mathrm{i}))\,(x - (-1 + \sqrt{5}\mathrm{i}))\,(x - a)$ and to find a. However, this led to errors since it took no account of the leading coefficient 2.

Some candidates had already done much of the necessary work in **(a)**. For this to be credited in **(b)**, they needed to mention **(a)** in their answer to this part. Many candidates simply used their calculators to find the remaining two roots and this was insufficient to gain credit. A number of candidates tried to divide the cubic by $-1 - \sqrt{5}\mathrm{i}$ or $-1 + \sqrt{5}\mathrm{i}$ or their product of 6. Once again, such candidates were confused between the concept of a root and a factor. A small number of candidates tried to divide the cubic by $x - (-1 + \sqrt{5}\mathrm{i})$ but were almost always unsuccessful as it led to complex working.

Question 8

This was a reasonably straightforward question on first order differential equation with separable variables. The majority of candidates were able to separate the variables correctly and commence integration of $\frac{1}{y}$ to ln y. Few used modulus signs, but in this question this was not necessary. Most candidates were able to split the fraction in x successfully and integrate both parts correctly. Some candidates chose to integrate by parts twice and often successfully arrived at the correct terms. Others simply integrated $\frac{1}{x}$ and $12x^2$ and multiplied the results together.

Question 9

(a) Most candidates scored all or most of the marks on this question. Most successful candidates used the form $\frac{A}{(1\ x)} + \frac{B}{(2+3x)} + \frac{C}{(2+3x)^2}$. Some candidates made arithmetical errors when evaluating A, B and C through multiplication and substitution. The most common error was to multiply throughout by an expression involving $(2 + 3x)^3$. Those who chose to compare coefficients were more likely to make an error.

(b) This question proved more challenging for some candidates. Most were able to find the first terms of the expansion of $1 - x$ without difficulty, although sign errors occurred in a significant minority of cases. Many candidates were able to start the expansions of $\left(1+\left(\frac{3}{2}\right)x\right)^{-1}$ and of $\left(1+\left(\frac{3}{2}\right)x\right)^{-2}$, but made errors combining B and C with either 2^{-1} or, more commonly, 2^{-2}. Some candidates correctly found all the required terms only to make arithmetic slips when combining their three expansions.

Those who solved **(a)** in the form $\frac{A}{(1-x)}+\frac{(Bx+C)}{(2+\ 3x)^2}$ were less successful in this part, either forgetting to multiply their second expansion by $Bx + C$ or doing so incorrectly.

Question 10

(a) This should have been a straightforward application of the product rule for differentiation, but many candidates struggled because they were unable to differentiate $e^{\frac{-x}{2}}$. Some candidates who initially performed it correctly saw their 2 in the denominator drift into the numerator. Most knew they needed to set their derivative to zero, but some candidates had trouble solving the equation. If they cancelled the exponent term throughout, they could obtain $x = 4$ easily. Instead of this many candidates decided to take ln which was unnecessary and did not often lead to the answer.

(b) This part also required the application of a basic technique, integration by parts, although here the technique had to be applied twice. Candidates often found the integration of $e^{\frac{-x}{2}}$ difficult, with similar errors to **(a)** seen. The integration would be better done as a continuous piece of working rather than on different parts of the page as this often led to sign errors or missing terms. To minimise errors, candidates should make no attempt to substitute limits until all the integration has been performed and all similar terms collected together.

Question 11

(a) Many excellent solutions were seen, but there were some candidates who scored just a single mark because of arithmetical errors. Most candidates realised that two of the equations were independent of a and so could be solved immediately for μ and λ. From these two values it was easy to calculate a and find the position vector of the point of intersection. Candidates should be encouraged to check the point of intersection in the equation of both lines as this would help them to identify numerical errors.

(b) Some excellent solutions to this question were seen from candidates who realised that the scalar product required the direction vectors of the two lines and not the equation of the line or the position vectors of points on the line. However, poor algebra was seen in many good solutions, for instance the length of one of the vectors seen as $\sqrt{(a^2 + 2^2 + (-1)^2)} = \sqrt{(a^2 + 5)} = \sqrt{5a}$.

Few candidates took the modulus of the scalar product or used $\pm\frac{1}{6}$, the cosine of the given acute angle. Finding the angle between two given vectors using the scalar product produces a unique acute or obtuse angle. However, when finding the acute angle between the direction vectors of two lines, for example **c** and **d**, it is essential to choose the combination of ± **c** and ± **d** that produces a positive scalar product. Similarly, when finding the obtuse angle, a negative scalar product is needed. However, in questions such as this one, where a component of one direction vector is unknown **(a)**, a positive or negative scalar product cannot be achieved without placing some restriction on the value of a. To cover all possibilities, the values of a that give an acute angle between the direction vectors from the scalar product of **c** and ±**d** can be found. This is easy to do by taking the modulus of the scalar product or $\pm\frac{1}{6}$, the cosine of the given acute angle. It could be argued that in order to determine a in the scalar product and in the length of the vectors we can square both sides of the equation so the incorrect solutions now introduced are the same as those added by considering ± earlier. However, all this is doing is including a sign error (by considering only one sign at the start) then another sign error that corrects this. This explains why candidates obtained both values for a as shown in the mark scheme, even without using modulus or $\pm\frac{1}{6}$. If they had considered the scalar product $(2a - 3)$ using $a = 1$, they would have realised that it is negative for this value and would lead to an obtuse angle not an acute one. (A similar process works when looking for obtuse angles.)

MATHEMATICS

Paper 9709/41
Paper 41

Key messages

- Non-exact numerical answers are required correct to 3 significant figures as stated on the question paper. Candidates are advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation or a work/energy equation. Such a diagram would have been particularly useful in **Questions 5**, **6** and **7**.
- In questions such as **Question 4** in this paper, where acceleration is given as a function of time, it is important for candidates to realise that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The questions were generally answered well by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1** and **2** were found to be the easiest questions whilst **Questions 6** and **7** proved to be the most challenging. The presentation of the work was good in most cases with candidates writing clearly using black pen.

In **Question 6**, the angle θ was given exactly as $\sin^{-1}0.08$. There is no need to evaluate the angle in this case and problems such as this can often lead to exact answers and so any approximation of the angle can lead to a loss of accuracy.

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction. In fact, in some cases, such as in **Question 5** in this paper, it was impossible to achieve a correct given answer unless this value was used.

Comments on specific questions

Question 1

(a) Most candidates answered this question well. It involved dealing with the conservation of momentum before and after the collision. The majority of candidates found the initial momentum and dealt well with the case of the two particles coalescing and equating the two expressions for momentum.

(b) Again, most candidates knew the definition of kinetic energy and used the expression $\text{KE} = \frac{1}{2}mv^2$ and applied it to this problem. Almost all candidates found the KE before and after collision. Some did not complete the solution by subtracting the two values to find the required loss of KE.

Question 2

(a) Most candidates answered this question correctly. It was necessary to use the relationship $P = Fv$ to find P with the force used being the resistance force and v being the given constant speed. Some candidates incorrectly introduced the weight into their equation.

(b) The driving force, DF, acting on the car needed to be found using DF = P/v from the given value of P. Most candidates found this correctly and then applied Newton's second law to the motion. This motion is produced by two forces, the driving force and the resistance and these combined are equated to ma where $m = 1400$ and a is the required acceleration. A few candidates forgot to include the resistance force in their calculations, but most found the required value correctly.

Question 3

This was a problem requiring resolution of forces, generally in two perpendicular directions. Almost all candidates resolved in the direction of the 12 N force and also perpendicular to this. Most candidates made a good attempt at this part of the question. The main errors seen were to confuse the sine and cosine of the angles and there were also some incorrect signs of the forces. Overall the resolving aspect of the question was done well. Some candidates had difficulty in solving the equations for P and θ. The most straightforward method for finding P is to square and add the $P \sin \theta$ and $P \cos \theta$ values and then take the square root. Once P has been found, θ can be obtained from either the $P \sin \theta$ or $P \cos \theta$ values. This was the method that most candidates who attempted this part used.

Question 4

In this question it was important to realise that acceleration is not constant and that calculus methods needed to be used. Most candidates made a good attempt at this question. It was necessary to integrate the given expression for a in order to find the velocity, v. Since the question asked for distance travelled, then this required a further integration in order to find the displacement, s. Once the velocity was found, the value of t when $v = 0$ was required as the question referred to the particle coming to instantaneous rest. This value of t needed to be used as one of the limits of integration in the expression for s in order to find the displacement at this time. As the velocity was negative throughout the period of motion, this value of displacement was negative. As the distance travelled was required, the final answer needed to be positive. Almost all candidates solved this problem correctly. A few made errors on the integration and some forgot to reverse the sign for their final answer.

Question 5

(a) This question concerned the motion of two particles moving vertically over a fixed pulley. It was necessary to express Newton's second law for each particle during the motion. The forces acting on each particle were their weight and the tension in the string. Clearly the heavier 0.8 kg particle will move downwards after release. These two equations form simultaneous equations in a, the acceleration of the particles and T the tension in the string. Care needed to be taken when solving this as one of the answers is given. This meant that candidates needed to be extra careful to show all working when finding the solution. Some candidates incorrectly assumed that $a = 6$ and then used this to find T. This did not prove the given answer. Some candidates used Newton's law applied to the system which gave an equation for a directly and then the solution only needed one further equation involving T. This method was perfectly acceptable. Most candidates made good attempts at this problem. However, some candidates used mass rather than weight when using Newton's second law.

(b) In this part of the question the speed of the particles as the 0.8 kg particle reaches the ground needed to be found. This could be achieved by using the constant acceleration equation $v^2 = u^2 + 2as$ with $u = 0$, $a = 6$ and $s = 0.5$ which gave the required value of v. This was the speed with which the 0.2 kg particle was moving upwards at this time. The 0.2 kg particle came to rest at a height given by using the equation $v^2 = u^2 + 2as$ but this time using u as the velocity found as the 0.8 kg particle reaches the ground, $v = 0$, and $a = -g$. When these values were used s was the extra displacement travelled upwards by the 0.2 kg particle. Hence the total height travelled was 0.5 + 0.5 + this extra height. This is where some errors were seen as the initial 0.5 m was not noticed by some candidates. Overall, this question was answered well.

Question 6

(a) In this question it was a requirement that energy methods rather than the use of Newton's second law were used. Given this restriction, candidates needed to work out the gain in potential energy as the system moves from the bottom to the top of the hill. As the speed of the system is reduced as it travels up the hill, the loss of kinetic energy of the system needed to also be evaluated. As it travels the 800 m up the hill, work must be done against the resistances. The aim of the question was to determine the work done by the driving force and this could be achieved by using the correct combination of these energy terms. The energy equation should have taken the form of 'work done by DF + loss of KE = Gain in PE + WD against resistances'. The only unknown in this equation was the driving force. Most candidates attempted to evaluate some of these terms. However, often the terms were combined with the wrong sign. Some candidates also incorrectly included driving force rather than work done by the driving force in the energy equation. Some candidates did not include all of the relevant terms.

(b) In this part of the question Newton's second law needed to be applied to either the car, the trailer or to the system. Two of these equations were needed to find the two required answers. If the system equation was used then the acceleration could be found directly as this equation did not involve the tension in the tow-bar. One other equation was then required to find this tension. Most candidates made a good attempt at this part. An error seen was to include too many forces, particularly including the tension in the tow bar, when looking at the system. It is always a good strategy for candidates to draw a force diagram for each element of the system and also for the system itself.

Question 7

(a) This question could be approached in several different ways. One method was to use energy principles for the stage *AB* where there is no friction and to determine the speed of the particle at *B*. Then a further approach using energy methods could be used for the stage from *B* to *C* where there is a frictional element to find the required speed of the particle at *C*. An alternative approach was to apply Newton's second law over the two separate regions *AB* and *BC*, again finding the speed at *B* to link the two stages and finally to determine the speed at *C*. It was also possible to use an energy method on one of the stages and Newton's law on the other. A third, more direct method, uses energy principles but does not require any information to be found at *B*. The PE loss from *A* to *C* can be found. The KE gain will simply be $\frac{1}{2}\times 0.2\times v^2$ where v is the required speed at *C*. The frictional force and the work done against it could be found using $F = \mu R$. The energy equation could then be expressed as PE loss = KE gain + WD against friction and the required speed could be determined. Candidates used a variety of these methods. Most found the speed at *B*. Errors occurred with the use of an incorrect PE and sign errors in the energy equation or in Newton's second law. Most made a reasonable attempt at this even though it was the most challenging question on the paper.

(b) There were several approaches that could be taken to this problem, similar to those in **(a)**. The motion from *A* to *B* is unchanged and so use could be made of the answer found in the solution to **(a)** and then either energy or Newton's second law applied to the motion from *B* to *C*. Again, it was possible to solve the problem very simply without using any information at *B*. As the particle starts and finishes at rest, then there is no change in KE. Application of the energy equation would be PE loss = WD against friction. The frictional force is μR and since R can be found, the energy equation gives the required value of μ. Candidates found this question challenging but some good solutions were seen.

MATHEMATICS

Paper 9709/42
Paper 42

Key messages

- Non-exact numerical answers are required correct to 3 significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation or a work/energy equation. Such a diagram would have been particularly useful here in **Questions 3**, **6** and **8**.
- In questions such as **Question 7**, where acceleration is given as a function of time, it is important to realise that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The questions were generally very well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1** and **2** were found to be the easiest questions whilst **Questions 5, 6** and **8(b)** proved to be the most challenging. The presentation of the work was good in most cases with candidates writing clearly using black pen.

One of the rubric points on the front of the question paper was to take g = 10 and it was noted that almost all candidates followed this instruction. In fact, in some cases, such as in **Question 6** in this paper, it was impossible to achieve a correct given answer unless this value was used.

Comments on specific questions

Question 1

(a) Most candidates answered this question well. It involved applying the definition of momentum to the moving particle before the collision. The majority of candidates found this momentum correctly.

(b) This question was answered well by most candidates. It involved dealing with the conservation of momentum before and after the collision. The majority of candidates used the initial momentum found in **(a)** and applied it by equating this to the sum of the momentum of the two particles after the collision. One of the few errors seen was a numerical error when multiplying 0.2 by 0.3.

Question 2

(a) In this part of the question candidates needed to use the relationship $P = Fv$ with v = 20 in order to find the driving force. Most candidates used this correctly and it was then necessary for them to apply Newton's second law to the motion. This consists of two forces, the driving force and the resistance and these combined are equated to ma where m = 1800 and a is the given acceleration. Some candidates used the resistance force in their calculations but with incorrect sign although most found the value of the required power correctly.

(b) As this question asked for the steady speed, candidates needed to remember that this is achieved when the net force acting on the car is zero. This means that the driving force exactly balances the resistance. The driving force was given by DF = *P*/*v* using the power found in **(a)** and equating this to the given resistance gives a simple equation in *v*. Most candidates found this correctly. The correct resistance however needed to be used as the balancing force. An error made by some candidates was to incorrectly use the weight as the balancing force.

Question 3

There were a number of possible approaches to this question. Probably the most popular and direct approach was to resolve forces horizontally and vertically. In this case the horizontal component of the *T* N tension balances the horizontal component of the 20 N tension. This equation gives *T* directly. The vertical resolution is a 3-term equation involving the vertical components of *T* N and 20 N balancing the weight *mg*. This enabled the mass, *m*, to be found. Most candidates used this approach. Some candidates confused sine and cosine in the resolved equations and other treated the weight as *m* rather than *mg*. Other possible methods used by candidates were the triangle of forces approach and the use of Lami's theorem.

Question 4

(a) In this question it was necessary to use the definition of acceleration as the gradient of the curve in a velocity-time graph. This could be done in this case by considering the triangle whose base lies between *t* = 0 and *t* = *T*. Almost every candidate scored this mark.

(b) It was necessary here to consider the distance travelled as the area under the velocity-time graph. There were several different ways of dealing with this, but as the question referred to the distances before and after the car begins to move at constant speed it was best to look at the two stages before this happened and the two stages after as separate distances. The area (and hence the distance travelled) before constant speed begins could be thought of as a triangle with base from t = 0 to t = T and a trapezium for the stage from *t* = *T* to *t* = *T* + 5. The area after constant speed begins could be evaluated either as one trapezium or as the sum of a rectangle and a triangle. Once these expressions were found, an equation for *V* needed to be set up by relating these two areas. There were several ways of expressing the given condition such as using the fact that the area after constant speed begins is twice the area before it begins. A variety of approaches were seen to this problem. Some candidates missed out one or more parts of the area or included the same area twice. Also, some used the wrong factor when setting up the equation such as using $\frac{1}{3}$ when the factor should be 3. There were many good solutions to this problem.

Question 5

(a) Most candidates found this question challenging. A variety of possible methods of approach were possible. One approach was to find the time taken for the particle to reach its highest point (4 seconds). As it was given that the particle was above the building for 4 seconds, then 2 seconds of this is on the upward journey and 2 seconds was on the downward journey. Hence the time taken to reach the top of the building was 2 seconds less than the time to greatest height, namely 2 seconds. Use of the constant acceleration equation $s = ut + \frac{1}{2}at^2$ with $u = 40$, $a = -g$ and $t = 2$ gave the required height of the building. An alternative approach was to use the equation $v^2 = u^2 + 2as$ with $u = 40$, $v = 0$ and $a = -g$ to find s the greatest height achieved by the particle. In order to find the position of the top of the building the distance that the particle falls from rest in the next 2 seconds needed to be found. This could be found using the equation $s = ut + \frac{1}{2}at^2$ with $u = 0$, $a = g$ and $t = 2$. The distance found here was then subtracted from the greatest height to determine the height of the building. Candidates often incorrectly assumed that the given 4 seconds was in fact the time taken to reach the top of the building. Many other variations on these methods were seen.

(b) In this part of the question the time t seconds was given as the time after the first particle is projected. This meant that the time after the second particle was projected was $t - 1$ seconds and many candidates wrongly used either the same t values for both or used $t + 1$ for the second particle. One method of approach was to write the displacement for the first particle as

$s_1 = 40t - \frac{1}{2}gt^2$ and the displacement for the second particle could then be written in the form $s_2 = 20(t-1) - \frac{1}{2}g(t-1)^2$. Since the height of the building is 60 m, the time at which the particles were at the same height above ground could be found by solving the equation $s_1 = s_2 + 60$ to find the required time, t. Many candidates made an error when using this method by simply equating s_1 and s_2. Again, a variety of methods were seen to approach this problem. Some candidates found the position and velocity of the first particle after one second and then looked at the subsequent motion as if both particles started at this time. Another method was to track the motion of both particles at one second intervals. Both of these techniques, if followed correctly, led to a correct solution.

Question 6

(a) This question stated that a constant force of 40 N was applied to the block acting up the plane. As it was given that the block begins to move up the plane, the net force up the plane was positive. If F was the frictional force then resolving forces up the plane gave $40 - 5g\sin 30 - F > 0$. Since there was motion, friction was in the limiting case and so $F = \mu R$ where R could be found by resolving perpendicular to the plane. Substitution of this expression for F led to the required inequality for μ. Candidates using this approach often did not express their solution in terms of inequalities and only introduced the inequality on the final line and so did not offer a complete solution. An alternative approach, since it was given that motion takes place, was to introduce an acceleration term as $40 - 5g\sin 30 - F = 5a$. Use of $F = \mu R$ gave an equation in μ and a and using the condition $a > 0$ led to the correct inequality. There were some very well-worked solutions seen.

(b) In this part of the question the 40 N force acts horizontally. The effect of this was to change the expression for the normal reaction. It was no longer merely a component of the weight but also involved a component of the 40 N force. Since there was no motion in this case, resolving along the plane led to an equation which took the form $40\cos 30 - 5g\sin 30 - F = 0$ and resolving perpendicular to the plane led to the equation $R = 5g\cos 30 + 40\sin 30$. Since $F < \mu R$ substitution of the expressions for R and F led to the required value of μ. Although many excellent solutions were seen many candidates incorrectly thought that R was the same expression as in **(a)**. Other errors seen were to have incorrect signs and sine and cosine errors in their expressions for R and F. Since the answer in both parts of this question was given, it is particularly important to show all working thoroughly.

Question 7

(a) This question involved a given acceleration which is a function of time. This meant that the constant acceleration equations didn't need to be used but that calculus techniques needed to be employed. Integration of the expression for a was needed to determine the velocity. The constant of integration could be evaluated by using the condition given that the particle started with a velocity of 1.72 ms^{-1}. This two-term expression for the velocity could then be equated to 3 and solved for t in order to answer the question. Most candidates produced good solutions to this problem although several candidates assumed that the constant of integration was zero and were unable to find the correct solution. Some candidates incorrectly attempted to use the constant acceleration equations.

(b) To find the required displacement, the two-term expression for velocity which was found in **(a)** needed to be integrated. The constant of integration in this case was zero since the particle started from the origin O. Once the integration was performed, use of the correct limits was needed to find the required answer. Most candidates found this correctly although some who only used a one-term expression for the velocity were not able to find this correctly. Again, some candidates wrongly attempted to use the equations for constant acceleration.

Question 8

(a) This question involved a system of connected particles. There were three possible equations of motion and any two of them would have enabled the problem to be solved. Newton's second law could be applied either to particle *A* or to particle *B* or to the system of both particles. The system equation would not involve the tension in the string. Solving any two of these three equations would give the tension in the string and the acceleration of *B*. The three equations took the following forms. For *A* the equation was $T = 0.3a$. The equation for *B* took the form $3.5 + 0.5g\sin 30 - T = 0.5a$. The system equation was $3.5 + 0.5g\sin 30 = (0.3 + 0.5)a$. Often candidates incorrectly assumed that there was a weight force of 0.3 *g* also acting on *A,* even though the resolution of forces was in the horizontal. Most candidates offered good attempts at this question.

(b) This question required candidates to use energy techniques in their solution and almost all candidates followed this instruction. However, very few candidates were able to make a completely correct attempt at this problem. There were four elements which needed to be considered here. As the particles moved, the 0.5 kg particle lost potential energy. Both particles gained kinetic energy. There was a given loss of energy to friction. Finally, the force of 3.5 N would do work as it moves through the 0.6 m. The energy equation required the correct combination of all of these four elements. However, many candidates missed out one or more of the relevant terms. An error often seen was to only introduce KE for the 0.5 kg particle. Another error was not to include the work done by the 3.5 N force. Other candidates introduced a potential energy term for the 0.3 kg particle but there was in fact no change in height for this particle and hence no change in potential energy. The correct form of the energy equation is: WD by 3.5 N force + PE loss of 0.5 kg particle = WD against friction + KE gain by both particles. Very few candidates produced a correct solution to this problem due to one or more of these errors.

MATHEMATICS

Paper 9709/43
Paper 43

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper rather than correct to two significant figures as was sometimes seen, e.g. **Question 3(b)**.
- When resolving forces in equilibrium or when forming an equation of motion, a clear and complete force diagram can be helpful in ensuring that all relevant forces have been considered, e.g. **Question 6**.
- Candidates are reminded of the importance of key words in a question, e.g. '*rough horizontal* table' in **Question 3**; '*initial* acceleration' in **Question 5(b)**; '*limiting equilibrium'* in **Question 7(a)**.

General comments

Many candidates produced answers of a very high standard with clearly presented solutions. Most candidates attempted all questions. **Questions 1**, **2 (a)** and **(b)**, and **5(a)** were answered particularly well whilst **Questions 6(a)(ii)** and **7(b)** were the most challenging.

Comments on specific questions

Question 1

(a) This question required the application of the SUVAT formulae for motion under gravity. Although candidates knew all of the formulae and frequently found v accurately, some were confused by the use of $v\,\text{ms}^{-1}$ for the initial velocity and mistakenly used, for example, $u = 0$ and $a = -g$.

(b) This problem was straightforward for many candidates. Some of the SUVAT formulae were applied appropriately. However, common errors were to use $a = -g$ with $u = 0$ instead of $v = 0$ leading to $s = \frac{1}{2} \times -10 \times 3^2 = -45\,\text{m}$, or to use $a = g$ with $u = 30$ leading to $s = 30 \times 3 + \frac{1}{2} \times 10 \times 3^2 = 135\,\text{m}$.

Question 2

(a) The majority of candidates correctly multiplied the frictional force 40 N by the distance moved along the plane 15 m. Some candidates calculated the work done against gravity rather than the work done against friction. Others quoted $Fs\cos\theta$ and calculated $40 \times 15\cos 20$ instead of 40×15.

(b) A correct calculation for the change in potential energy as mgh was frequently seen for this part of the question. Occasionally the height was calculated incorrectly as 15cos20° instead of 15sin20°. The correct $5 \times 10 \times 15\sin 20°$ was occasionally rounded to 256 J instead of 257 J.

(c) Although candidates could total their answers for **(a)** and **(b)**, they often provided a new solution. The most common error was to find the difference between the work done against friction and the change in gravitational potential energy rather than finding the total.

Question 3

(a) Candidates were expected to draw a force diagram showing and labelling the four forces acting on the block. There were many diagrams with missing forces (usually the frictional force or the normal reaction or both). Some forces were unlabelled, and a few were incorrectly labelled, e.g. 4 kg instead of 4 N. Although the question referred to a horizontal table, several candidates believed that the block rested on an inclined plane and showed forces accordingly.

(b) Solutions were frequently fully correct despite incomplete diagrams seen in **(a)**. A common error was to resolve incorrectly, e.g. $R = 40 + 30\sin 24°$ leading to $\mu = 0.525$, or $R = 40$ leading to $\mu = 0.685$. A few candidates found $\mu = R/F$ (1.01) instead of $\mu = F/R$. This appeared to be either from an incorrect solution of $F = \mu R$ or from interpreting R to be the resisting force rather than the normal reaction. The final answer of 0.986 was sometimes seen as 0.99 correct to two significant figures rather than the required three significant figures.

Question 4

This question was often well attempted. Whilst the majority of candidates were familiar with this new topic, there were some who did not appear to have a knowledge of momentum. The strongest solutions recognised that sphere *A* could either continue in the same direction or change direction on collision, leading to two possible masses for sphere *B* and thus two values for the loss of kinetic energy. Some candidates found just one possible mass and the corresponding loss of kinetic energy. Others misinterpreted the loss of energy required and separately calculated the changes in energy for sphere *A* and for sphere *B* rather than the overall kinetic energy loss due to the collision. Those candidates who did not form a momentum equation were unable to calculate a value for the mass m kg and could thus only evaluate the kinetic energy change for sphere *A*.

Question 5

(a) Nearly all candidates solved the quadratic equation $4t^2 - 20t + 21 = 0$ (i.e. $v = 0$) and successfully found the two values of t required.

(b) This part proved to be more challenging with some misunderstanding of 'initial acceleration'. Most candidates knew that differentiation ($a = dv/dt$) was needed. Whilst the differentiation was almost always correct, candidates sometimes substituted $t = 1.5$ or $t = 3.5$ or both (the values found in **(a)**) instead of evaluating for the initial time $t = 0$.

(c) Many candidates found this part of the question involving the evaluation of v when $t = 2.5$ less demanding. However, some candidates completed the square to find the minimum velocity. A few candidates correctly found $v = -4$ ms^{-1} and then incorrectly stated the minimum velocity as $+4$ ms^{-1}.

(d) Most candidates knew that integration was needed and found $\int v dt$ as expected. Whilst the integration was almost always accurate, the choice of limits was sometimes unsuitable. Incorrect limits seen included 0 to 2.5, 1.5 to 2.5 or –4 to 0 (using y limits). Candidates who included a constant of integration C, instead of two limits, generally used $t = 0$ to obtain $C = 0$ and then mistakenly evaluated the integral for a single time t such as $t = 2.5$. Some candidates left the final answer as a negative displacement rather than stating the positive distance travelled. A few candidates attempted to use constant acceleration formulae despite the varying acceleration.

Question 6

(a) (i) Most candidates applied $P = Fv$ successfully, using $F = 400 + 250$, to obtain the power of the car's engine. Those who mistakenly used $F = 400$, the resistance for the car only, obtained 10 000 W for the power.

(ii) This question was challenging for many candidates. The main difficulty involved the driving force. Those who recognised that the driving force could be calculated as 39 000÷25 could then form two equations in the two unknowns (tension T N and acceleration a ms^{-2}). The application of Newton's Second Law to the system of car and caravan allowed the acceleration to be found directly. However, it was common to see missing terms, e.g. $1560 = 2400a$, $a = 0.65$ using DF = ma and some candidates ignored the resistance to motion. The caravan equation was more straightforward than the car or the system equations and was often correctly stated and used in an attempt to calculate the tension in the tow-bar. Those candidates who used a suitable method sometimes lost some accuracy by approximating for the acceleration, $a = 0.38$ correct to two instead of three significant figures.

(b) Candidates knew that they needed to apply $32\,500 = Fv$ or $v = 32\,500/F$. The expected answer was $F = 1600\,g \times 0.05 + 800\,g \times 0.05 + 400 + 650$ or equivalent. Common errors included one or more missing terms, sign errors or a missing g in the weight components. Those who recognised that $\sin(\sin^{-1}0.05) = 0.05$ did not need to find the angle of the plane 2.87°. A few candidates were unsure how to use $\sin^{-1}0.05$ and calculated, for example, $2400\,g\sin 0.05°$.

Question 7

(a) Key facts given in this question included a smooth plane P, a rough plane Q and limiting equilibrium. Many candidates produced accurate solutions by resolving parallel to plane P to obtain $T = 2\,g\sin 10°$; resolving parallel to plane Q and applying $F = \mu R$. Others treated the system as a whole without direct reference to the tension in the string. Thus $3\,g\sin 20° = \mu R + 2\,g\sin 10°$. Those candidates who did not consider the direction of motion sometimes obtained a negative value either for the frictional force or for the coefficient of friction. In this case an explanation was expected for any sign change made. A few candidates who overlooked the smooth plane, attempted to include a second μR for plane P. Others who initially ignored 'equilibrium', attempted to calculate the tension in the string by solving a pair of equations of motion including non-zero acceleration.

(b) Candidates were often able to calculate the acceleration of the system, usually by applying Newton's Second Law to each particle and solving the resulting equations. However, finding the required time proved too challenging for the majority of candidates. The difficulty arose from interpreting the one metre difference in vertical height of the particles. Successful solutions often used $s = \frac{1}{2}at^2$ and $s\sin 10° + s\sin 20° = 1$ with s as the distance moved along the plane. Common errors included the assumption that $s = \frac{1}{\sin 10°}$ or $s = \frac{1}{\sin 20°}$ or $s = 1$, or that 0.5 was the vertical change in height for each particle, or that $s\sin 20° - s\sin 10° = 1$ suggesting that both particles moved upwards. A small number of candidates attempted a solution using an energy method. Those who considered each particle separately usually attempted to form an equation in KE and PE, not accounting for the work done by the tension in the string.

MATHEMATICS

Paper 9709/51
Paper 51

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often did not provide sufficient evidence of the calculation undertaken, especially when there were errors earlier in the solution.

Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. It is important that candidates realised the need to work to at least 4 significant figures throughout to justify a 3 significant figure answer. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent. There is no requirement for probabilities to be stated as a decimal.

General comments

To do well in this component, candidates need to have an understanding of all the topics in the syllabus. Although there were some strong performances, there were a number of candidates who left many of the questions unattempted.

The use of simple sketches and diagrams can help clarify both context and conditions. These were frequently present in good solutions.

Candidates appeared to have sufficient time to complete all that they were able to within the paper.

It is good practice to read the question again after completing a solution to ensure that all the requirements have been fulfilled. As this is an applied mathematics component, comments about data should be related to the context of the question.

Comments on specific questions

Question 1

(a) Many candidates found this question challenging. The strongest solutions used an outcome space to identify where Events A and B were fulfilled, and clearly indicated those combinations which fulfilled both, and so supported their final answer. A successful alternative approach was to simply list score combinations for the dice which met the criteria and then indicate clearly which were in both lists. A few candidates stated a value for $P(A \cap B)$ with no justification. Solutions which did not use a consistent approach to listing outcomes often omitted potential terms which resulted in a loss of accuracy. If scenarios are being listed, it is good practice to identify the order of data being stated, so that differences can be noted. For example, a score of 9 can be formed by (B6, R3) or (B3, R6) but was often just stated as (3, 6).

(b) As the question is worded 'Hence…', candidates needed to use their answer from **(a)** within their solution. The most successful approach was to use $P(A) \times P(B) = P(A \cap B)$ to check for independence. Candidates who used an outcome space in **(a)** were able to undertake this comparison efficiently. Candidates should be aware that a numerical comparison must be made and a conclusion stated. Weaker solutions often omitted one of these. The alternative approach of using the conditional probability relationship, e.g. $P(B|A) = P(B)$, was also used successfully by some candidates.

Question 2

Stronger solutions often contained a tree diagram. This is good practice as the diagram often clarifies the context of the problem and clarify the relationships that need to be used.

(a) Stronger candidates used a tree diagram to identify the outcomes that would form an appropriate equation for the required probability. The strongest solutions communicated clear algebraic manipulations necessary to solve the equation. Weaker candidates often had simple arithmetical or algebraic errors at this stage. Some probabilities greater than 1 were noted, which was unexpected at this level.

(b) This was a standard problem to find the probability of the same event occurring twice. Stronger candidates often used the tree diagram in **(a)** to identify the appropriate outcome probability. Weaker candidates simply stated this value rather than squaring required. Again, poor arithmetical accuracy was seen.

Question 3

Most candidates recognised that their understanding of the geometric approximation was being assessed in this question. Candidates should be aware that more than one technique can be assessed in a question.

(a) The majority of candidates recognised that the geometric approximation was the most appropriate approach to the problem. The most efficient method was to calculate the probability of not achieving a success for 6 throws. Alternatively, candidates calculated and summed the individual probabilities for being successful for the first time on each turn for the first 6 turns and then subtracted from 1. A few candidates then used the formula for summing geometric series to evaluate their expression before subtracting from 1. A small number of candidates attempted to use the binomial approximation to calculate the probability of at least 1 success in 6 throws.
The most common error was interpreting ‘more than 6 throws’ to include ‘throw 6’. Candidates need to be able to effectively determine the boundary requirements of conditions consistently.
A small number of candidates simply stated the probability of obtaining a success on a particular throw, often the seventh.

(b) A significant number of candidates did not identify that the required conditions had changed to a standard binomial approximation problem. Many solutions used the geometric approximation again, which was not appropriate for this part. Stronger candidates clearly stated an unsimplified expression for the probability of obtaining a success fewer than three times which was then subtracted from 1. A small number of candidates used the more complex approach of summing the probabilities for the eight ways that the condition was fulfilled. Again, the interpretation of the context condition was challenging for many, with ‘at least 3 successes’ being treated as ‘more than 3 successes’.
Premature approximation was noted as a common inaccurate answer of 0.475 was seen following a correct unsimplified expression. Candidates should be aware that all intermediate calculations should be to a higher degree of accuracy than their final answer.

Question 4

Solutions which included an outcome space were often more successful in this question. These simple diagrams can help clarify the context initially and can often help to justify answers with little additional working.

(a) Some good probability distribution tables were seen. Weaker candidates often included extra terms because of errors in potential outcomes stated within the workings. Solutions where an outcome space had been generated initially were normally accurate. The weakest candidates identified the potential Y values accurately but assumed that the probabilities would be equal.

(b) The strongest solutions identified the values from the probability distribution table in **(a)** and used them to find the required value using the conditional probability formula. Weaker candidates simply re-stated the value of $P(Y = 2)$. Some stronger candidates used their outcome space in **(a)** to obtain the required probability, identifying the terms that they were considering clearly on their diagram.

Question 5

This question was a fairly straightforward normal approximation context and was accessible to the majority of candidates. Good solutions often included a sketch of the normal distribution curve which aided the interpretation of the specific requirements for each part.

(a) A number of good solutions were seen, which included a clear statement of the normal standardisation formula with the appropriate values substituted, with accurate evaluation to find the probability. Weaker candidates stated the probability for Davin spending less than 4.2 hours playing, where a simple sketch of the normal distribution curve may have been beneficial.

(b) Many candidates used the appropriate critical value that was provided on the normal distribution function table. This is anticipated as a syllabus expectation. A significant number of candidates formed an equation between a probability and the standardisation formula, rather than a z-value. Any equations formed were normally solved accurately. The most common error was to calculate the minimum time for Davin to play less than 90 per cent of his days.

(c) Most candidates found this part challenging. The strongest candidates recognised the symmetrical relationship between 2.8 and 4.2 with the mean and used this property in the normal distribution to calculate the required probability using the value calculated in **(a)**. However, many candidates calculated P ($t < 2.8$) using the normal standardisation formula, although not always with the same success as in **(a)**. Many candidates used an appropriate calculation to determine the number of days where Davin fulfilled the condition. Candidates should be aware that this is not an approximation or rounding of their calculated value, but an interpretation of their value within the stated context. It was noted that some candidates did not attempt this final part of the question, particularly where much of the working space had been utilised in finding the probability. It is good practice to encourage candidates to read the question again after they have finished their solution to ensure that they have met the requirements fully.

Question 6

(a) Almost all candidates attempted to draw a cumulative frequency graph, although histograms and frequency polygons were also seen. Stronger candidates stated the cumulative frequency before drawing the graph. The strongest candidates used a vertical scale that allowed the values to be plotted accurately, identified the correct boundaries of the classes for plotting the cumulative frequencies, fully labelled both axes and drew a smooth curve. Candidates should be reminded that axes involving variables should include units when labelled.

(b) Many candidates misinterpreted this question and used their graph to estimate the time which at least 24 per cent of the students take. Stronger candidates identified that 76 per cent of the students would have a time of less than t minutes, calculated the value and drew lines on their graph to show how the estimate was achieved. Weaker candidates simply provided a value with little evidence that it had been read off the graph. Where a question specifies a technique must be used, candidates should ensure that this is communicated clearly within their work.

(c) The majority of candidates found the unstructured nature of this question challenging, and a number of good solutions were left incomplete after calculating the approximate mean. An effective approach was to use and extend the data table provided at the start of the question to obtain all the information necessary, and this was often also used to evaluate the individual terms. However, because there was often insufficient space for values to be written clearly, errors were noted. The strongest candidates generated a new table in the working space and allowed sufficient space for clear working. Some successful candidates simply used the appropriate mean and variance formulae with unsimplified expressions generated directly from the original data. A common error was to use the class width rather than the mid-value in both formulae. A small number of candidates gave the variance as the final answer, rather than the standard deviation. It is good practice to reread questions once a solution is completed to ensure that all the requirements of the part have been fulfilled.

Question 7

Most candidates recognised that permutations and combinations were being assessed in this question. Many good solutions had simple diagrams to help interpret the context in each part.

(a) The most common error in answering this question was to ignore the repeated Ps and not divide the number of arrangements by 2. Some candidates made their solutions more complex by considering the number of different arrangements that are possible of SHOPKPR and then multiplying by the number of ways that EEE could be inserted, initially with the Es individually identified and then the repeats removed. This was an equivalent approach. Candidates should be reminded that exact answers should not be rounded to 3 significant figures to ensure full credit is available.

(b) The most common approach was to calculate the number of different arrangements that were possible of SHOKEEER and then to multiply by the number of ways that the Ps could be inserted while fulfilling the given condition. Stronger candidates often used a simple diagram to help interpret this context. A common error was not removing the repeats of either the Es or Ps. Some candidates successfully used the alternative approach of calculating the total number of ways the letters could be arranged and then subtracting the number of arrangements with the Ps together.

(c) Many candidates found this part challenging. Stronger candidates simply used standard probability theory for contexts without replacement for selecting the 2 Es at the ends. A common error linked with this method was to assume that the Es were not identical and various multipliers were used to account for this. However, most candidates used an approach similar to the previous parts and calculated the total number of different arrangements and the number of arrangements with Es at the end and used these values to form the probability. Again, a common error was not to eliminate the repeated arrangements.

(d) Most candidates found this part challenging. Stronger candidates identified the possible scenarios that fulfilled the requirements before calculating the number of selections that were possible for each scenario. A common misconception was that if PEE was chosen, then the remaining P and E would be included with the remaining letters for selection in the remaining places. Many candidates treated the Ps and Es as if they were individually identifiable and multiplied appropriately to include this within their calculation. The omission of PEEE was not uncommon with a final total of 25 being stated.

MATHEMATICS

Paper 9709/52
Paper 52

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often did not provide sufficient evidence of the calculation undertaken, especially when there were errors earlier in the solution. In questions where a given result is to be shown, e.g. **Question 2(a)**, candidates should ensure that their reasoning is justified.

Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. It is important that candidates realised the need to work to at least 4 significant figures throughout to justify a 3 significant figure value. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent. There is no requirement for probabilities to be stated as a decimal.

General comments

Many good solutions were seen for **Questions 2**, **3** and **5**. The context in **Questions 4** and **6** was found challenging by many. Sufficient time seems to have been available for candidates to complete all the work they were able to, although a few candidates did not appear to have prepared well for all topic areas of the syllabus.

The use of simple sketches and diagrams can help clarify both context and conditions. These were frequently present in good solutions.

It is good practice to read the question again after completing a solution to ensure that all the requirements have been fulfilled. As this is an applied mathematics component, comments about data should be related to the context of the question.

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. Where a solution is deleted, the use of the Additional Page was the most effective way to provide the replacement solution if there is insufficient space remaining. However, a significant number of candidates used the space either side of their original attempt resulting in poor clarity.

Comments on specific questions

Question 1

Most candidates recognised that their understanding of the geometric approximation was being assessed within this question. Candidates should be aware that more than one technique can be assessed in a question and that the question numbering system does not always indicate that there is a relationship between parts.

(a) The majority of candidates recognised that the geometric approximation was the most appropriate approach to the problem. The most efficient method was to calculate the probability of not scoring a 4 in five throws and subtracting from 1. Alternatively, candidates calculated and summed the individual probabilities for obtaining a 4 in each of the potential scenarios. A few candidates then used the formula for summing geometric series to evaluate their expression. The most common error was interpreting ‘fewer than six throws’ to include throw 6. Candidates need to be able to effectively determine the boundary requirements of conditions consistently. Weaker candidates often attempted to use the binomial approximation to calculate the probability of ‘*at least one 4 obtained in five throws*’, which was a less efficient approach, and clear communication of the anticipated full expression needed to be stated. A small number of solutions simply stated the probability of obtaining a 4 on a particular, often the fifth, throw.

(b) A significant number of candidates did not identify that the required conditions had changed to a standard binomial approximation problem. Many solutions used the geometric approximation again, which was not appropriate. Stronger candidates clearly stated an unsimplified expression for the probability of obtaining a 4 fewer than three times which was then subtracted from 1. A small number of candidates used the more complex approach of summing the probabilities for the eight ways that the condition was fulfilled. Again, the interpretation of the context condition was challenging for many, with 'at least three times' being treated as more than three times. Premature approximation was noted as a common inaccurate answer of 0.224 was seen following a correct unsimplified expression. Candidates should be aware that all intermediate calculations should be to a higher degree of accuracy than their final answer.

Question 2

(a) As a 'show that' question, candidates should be aware that they must communicate clearly all of their reasoning for their approach and justify all calculations fully. Stronger candidates used a tree diagram to identify the different possible selections, clearly stating the outcome for each branch and then calculating the required probabilities. Alternative approaches involved listing the potential scenarios and using combinations to determine the probability. Weaker candidates simply stated the appropriate calculation with no explanation as to how it related to the context. A common incorrect solution was to simply to note that $\frac{5}{8}\times\frac{3}{7}=\frac{15}{56},$ which although true is not an appropriate justification.

(b) Although many fully correct probability distribution tables were seen, there was a high number of candidates who did not attempt to calculate any of the remaining probabilities. The omission of 0 red balls as an outcome was a common error. A few solutions had probabilities which did not sum to 1. It is good practice to use exact values where possible. Candidates should be reminded that there is no expectation for probabilities that have been calculated exactly as fractions to be restated as decimals.

(c) Where a probability distribution table had been found in **(b)**, candidates were normally able to apply the variance formula accurately with their values. Candidates should be aware that evidence of the unsimplified expression is anticipated to support their answer. A number of solutions included calculating E(X) which was given in the question.

Question 3

This question was a fairly routine normal approximation context and was accessible to all but the weakest candidates. Good solutions often included a sketch of the normal distribution curve which aided the interpretation of the specific requirements for each part.

(a) Many good solutions were seen which included a clear statement of the normal standardisation formula with the appropriate values substituted, with accurate evaluation to find the probability. Weaker solutions stated the probability for Pia taking less than 11.3 minutes for the run, where a simple sketch of the normal distribution curve may have been beneficial.

(b) The majority of solutions used the appropriate critical value that was provided on the normal distribution function table. This is anticipated as a syllabus expectation. Almost all solutions formed an appropriate equation with a z-value, which was solved accurately to determine the time. The most common error was to calculate the minimum time for Pia complete less than 75 per cent of her runs in. A small number of solutions found the anticipated critical value, but then found the complement of a probability rather than the negative z-value to form the equation.

(c) Many candidates found this part challenging. Stronger candidates recognised the symmetrical relationship between 8.9 and 11.3 with the mean and used this property in the normal distribution to calculate the required probability using the value calculated in **(a)**. However, many candidates calculated P (t < 8.9) using the normal standardisation formula, although not always with the same success as in **(a)**. Most solutions used an appropriate calculation to determine the number of days where Pia fulfilled the condition. Candidates should be aware that this is not an approximation or rounding of their calculated value, but an interpretation of their value within the stated context. It was noted that a significant number of candidates did not attempt this final part of the question, particularly where much of the working space had been utilised in finding the probability. It is good practice to encourage candidates to read the question again after they have finished their solution to ensure that they have met the requirements fully.

Question 4

The use of tree diagrams to help clarify thinking in probability questions has been highlighted in previous reports. A number of good solutions were seen where candidates had appropriately extended the tree diagram in **(a)** for 3 April and used this successfully to support their work in **(c)** and **(d)**. A number of solutions incorrectly had probabilities greater than 1, which should have been an indication that a fundamental error had been made in interpreting the question.

(a) Many fully correct diagrams were seen. However, figures were not always clear, especially where corrections were made. At times this resulted in candidates misreading their own values in later parts of the question. A few candidates misinterpreted the basic probability information provided in the question as being a conditional probability, which would produce a probability greater than 1 for 2 April being fine, after 1 April being rainy. This should have been a clear indication that the data was being misinterpreted.

(b) Many good solutions were seen. The best solutions clearly indicated the two possible scenarios that fulfilled the condition and stated an appropriate unsimplified calculation to support the answer. Many candidates used the tree diagram in **(a)** to provide the calculation framework, but some explanation was expected to support the solution, especially where all outcomes are evaluated. Weaker candidates often made arithmetical errors when evaluating the correct unsimplified expression. The weakest attempts simply stated the probability that both 1 April and 2 April were fine.

(c) The strongest candidates often used the tree diagram in **(a)** to clarify the required condition. These solutions identified the two potential scenarios, calculated the probabilities for each separately and summed the answers. A few candidates ignored 1 April in their solutions, assuming that it must be fine and then simply calculated the probability that 3 April was rainy for the two potential outcomes for 2 April. Weaker candidates did not use the same probabilities for fine days as stated at the beginning of the question, but simply repeated the values they had calculated in **(b)**. A common misinterpretation was to assume that X and Y were independent events. These solutions determined $P(Y)$ and then calculated $P(X) \times P(Y)$.

(d) Although many good solutions were seen, some candidates did not support their answers with clear calculations to communicate their method. A significant number of candidates calculated $P(Y)$ in **(c)**, which, if clearly identified, could be credited here. The majority of solutions used the value calculated in **(c)** as the numerator, although a few candidates recalculated $P(X \cap Y)$ here. Again, the strongest solutions often used the tree diagram in **(a)** to support the work here. It was noted that a common arithmetical error was that correct unsimplified expressions were being evaluated inaccurately.

Question 5

(a) Many fully correct back-to-back stem-and-leaf diagrams were seen. These included a single key with appropriate units to explain how to interpret the data within the diagram, clear labelling on the leaves and a consistent spacing of data entries to enable the appropriate interpretation of the information visually. Candidates should be reminded that it is recommended that diagrams and graphs are completed in pencil so that errors can be erased, which here would avoid the difficulties of trying to maintain the vertical alignment of values following an error. The strongest solutions often had working that ordered the values before the construction of the final diagram. Weaker solutions had data entries which did not align vertically, included commas to separate values or dashes to indicate that no values were present. Candidates should be aware that none of these are acceptable at this level. The weakest solutions did not order the values, or used the stem as either the unit value, or place value of the data.

(b) Almost all candidates found the median value accurately. However, determining the interquartile range was less successful. The strongest candidates correctly identified the mid-value of the data above and below their median value. Weaker solutions used other approaches or did not provide supporting evidence for their value.

(c) Many candidates found this part challenging. Stronger candidates provided a general interpretation in context from the data about the differences in the spread and central tendency of the snowfall in the two resorts. At this level it was not sufficient to make simple numerical comparisons from the data, or to provide standard definitions of the terms. The strongest comments were similar to '*Linva has more variation in the amount of snow than Dados*' and '*Dados generally has more snow than Linva*'.

Question 6

Most candidates were able to interpret the context appropriately and determine an appropriate approach to each part of the problem.

(a) Almost all candidates recognised that the use of combinations was the appropriate approach for this context. The strongest candidates identified that once the first group had been created, there was no flexibility for the other group, and simply stated and evaluated ${}^{9}C_{6}$. Strong solutions also often identified that there were ${}^{3}C_{3}$ ways of selecting the second group and correctly multiplied by this. Weaker candidates anticipated that the order of picking the groups would create a difference and inappropriately doubled their answer. A common misinterpretation was to consider that having picked the group of six, the group of three would also be picked from the original nine people.

(b) Many candidates found this part challenging. Stronger candidates identified the total number of groups that could be formed of five people, and then interpreted the context condition as the number of selections of two people that could be formed once the three Baker children had already been chosen. Using the basic principles of probability, an appropriate fraction was formed. A common error was to multiply the numerator by 2 assuming that the order of selection was important. An alternative approach used by a small number of candidates was to consider the probabilities for each required member of the group being selected and then to multiply by the number of ways that the three Baker children could be selected in a group of five.

(c) A simple diagram to interpret the context was often seen in stronger solutions. The most successful approach was to calculate the number of arrangements of the seven people who are not identified and then to multiply by the number of places that Mr Ahmed and Mr Baker could be inserted to fulfil the given condition. A few candidates used a variation of this by considering that one of the men was in the original group of eight, and then multiplied by the number of places the other could be inserted so as to fulfil the criteria. The main alternative approach was to calculate the total number of arrangements for the nine people and then to subtract the number of arrangements when the two men were together. A common error was to not realise that here order did matter and many candidates who used this method did not multiply this value by 2 before subtracting. Several more complex methods were seen but these tended to be unsuccessful as not all possible arrangements that fulfilled the criteria were considered.

(d) Again, a simple diagram to interpret the context was seen in stronger solutions. Solutions tended to be based upon the same approach as **(c),** although those candidates who attempted to subtract from the total number of arrangements were usually unsuccessful as it is a very complex process to identify all possible arrangements that need to be removed. Successful solutions again calculated the number of arrangements the initial group of seven could make and then multiplied by the number of possible places that the Mr Ahmed and Mr Baker could be inserted appropriately. The most common error was not recognising that order of the men was important and so not multiplying by 2.

MATHEMATICS

Paper 9709/53
Paper 53

Key messages

It is important that candidates realise that they must show their working and that unsupported answers will not be rewarded. Where calculators are used, all stages in their response must be evident. Candidates should also be aware that, if they make more than one attempt at a question, only one of their responses will be marked. They should clearly indicate their final answer.

General comments

There were many examples where candidates did not appear to have read the question carefully enough and so did not answer the question as it had been set. In other answers some candidates showed a lack of confidence about the distinction between 3 decimal places and 3 significant figures.

Comments on specific questions

Question 1

(a) This question was well answered by most candidates. Almost all candidates produced correct standardisation expressions using the mean and standard deviation and went on to find the correct area. Those who struggled to deal with the combination of a positive and negative *z*-value would usually have benefited from sketching the curve for the normal distribution.

(a) The majority of candidates used their tables the correct way round and knew to give the *z*-value to 4 significant figures. Only a few tried to equate their standardisation expression to a probability rather than a *z*-value. However, only the most careful candidates noticed that the required value of *t* was to be in minutes rather than seconds. A minority knew not to mix minutes and seconds in the standardisation expression while some others corrected themselves at the final stage and converted their answer of 67.6 seconds to 1.13 minutes. Most candidates ignored the mention of minutes and gave their final answer as 67.6.

Question 2

(a) This question was answered well by many candidates. They knew to subtract the probability of throwing a 6 from one and to raise the resulting probability $\left(\frac{5}{6}\right)$ to the power eight. The most frequent misunderstanding was to assume it was a geometric distribution and to find the probability that it took exactly eight throws rather than more than eight throws. Those who tried to subtract the probabilities of throwing a 6 on each of the first eight throws from one, complicated the question and sometimes made errors.

(b) Stronger candidates knew that the expected value is $\frac{1}{p}$ for a geometric distribution. As long as they knew that the probability of obtaining a pair of 6s from two fair dice is $\frac{1}{36}$, they usually wrote down the correct value. However, a common incorrect answer was the probability $\frac{1}{36}$ while $\frac{1}{6}$ and $\frac{1}{12}$ were other frequently seen incorrect responses.

(c) Most candidates knew how to find the probability of an event after 10 or 11 tries by summing p^9q and $p^{10}q$. Some worked with $p = \frac{5}{6}$ or $\frac{11}{12}$ instead of $\frac{35}{36}$ and a number of candidates did not recognise that they needed to sum the two probabilities of 10 and 11 throws and instead gave two separate answers. There was some confusion about the difference between 3 decimal places and 3 significant figures and the rounded answer of 0.043 was seen frequently. If candidates had written the fuller answer of 0.0425(015) before rounding, they would have gained credit. The same confusion caused others to approximate prematurely and add 0.0216 and 0.0210.

Stronger candidates answered successfully by subtracting the probabilities of taking '9 or fewer' and 'more than 11 throws' from 1.

Question 3

(a) This question was very well answered with the majority of candidates gaining full marks. Most solutions were well presented with a clear identification of the different scenarios. Most candidates recognised that there were three possible ways to satisfy the condition of more women than men and knew to multiply the combinations concerning the choice of women and the choice of men. A small number omitted the possibility of an all women committee.

A few candidates chose to subtract the number of ways where the number of women was the same or lower than the number of men from the total number of ways, making the question a little more complicated but equally accessible.

(b) There were many ways to answer this question, some more efficient than others. The two most commonly seen efficient methods were:

1. To subtract the number of ways of choosing a committee with both the sister and the brother (12C4) from the total number of ways of choosing a committee (14C6)
2. To add the number of ways of choosing a committee with neither the brother nor the sister (12C6) to the number of ways of choosing a committee with either the brother or the sister but not both (2 × 12C5).

There was a third efficient method seen a few times, which was to find the number of ways of choosing a committee with either the brother or the sister excluded (2 × 13C6) and then to subtract the number of ways with neither (12C6) as it had been double counted.

A significant number of candidates chose a much less efficient method, considering separately the different combinations of men and women and were rarely successful.

Question 4

(a) This question was answered well with most candidates attempting to use a binomial expansion with n = 7 and p = 0.35. A number of responses included an extra term, misunderstanding 'fewer than 3' and including the case where the train arrives late on 3 days. Others omitted the term 0.65^7, not realising that the train may not be late at all. Those who obtained the correct unsimplified answer sometimes approximated prematurely and could not be awarded the final accuracy mark.

(b) This question was also very well answered with most candidates confidently finding the correct mean and variance for a normal approximation and standardising correctly. A number of candidates remembered the continuity correction and most of them correctly used 40.5. The most challenging part of the question appeared to be finding the correct area with a significant number subtracting the correct final answer from 1.

Question 5

(a) Some candidates presented the number of ways of arranging the letters with V first and E last $\left(\frac{6!}{2!2!} = 180\right)$ as their final answer, and made no attempt to find the probability. It was unclear as to whether they did not know how to proceed or had misread the question. Those who realised that they needed to find the probability of this arrangement usually found the correct final answer, dividing by the total number of ways $\left(\frac{8!}{2!3!}\right)$.

(b) Stronger candidates recognised that they needed to make their working very clear when attempting this conditional probability question. They knew to divide the number of ways with both Rs and Es together (120) by the number of ways with the Es together (360) or to divide the probability of both the Rs and Es being together $\left(\frac{1}{28}\right)$ by the probability of the Es being together $\left(\frac{3}{28}\right)$. Either way, they knew to explain what the totals or probabilities represented. Many of those who worked out the probability of the Rs being together mistakenly assumed independence and multiplied that probability by the probability of the Es being together to find the probability of Rs and Es being together.

Question 6

(a) Stronger candidates recognised that in a ‘show that’ question where the answer is given, every stage of working needs to be apparent and explained. They knew to show that there were three possible ways of obtaining two heads and a tail and to show the corresponding three factor product of probabilities that corresponded to each of the ways. They then summed the three products to arrive at a total of $\frac{20}{45}$ which simplifies to $\frac{4}{9}$.

(b) This question was answered well by the majority of candidates. Most recognised that the possible values of X were 0, 1, 2 and 3 with only a small number omitting the possibility that there were no heads. Tables were generally well presented and many candidates checked that their probabilities summed to one.

(c) Most candidates were well prepared for this type of question and applied the correct variance formula to their probability distribution, remembering to show their working clearly and to subtract the square of the mean.

Question 7

(a) Stronger candidates knew to calculate the frequency densities before drawing a histogram and recognised that they needed to treat the discrete data as continuous so that the bar ends were 0.5, 5.5, 10.5, 20.5, 40.5 and 70.5 resulting in class widths of 5, 5, 10, 20 and 30 and frequency densities of 2, 1, 2.6, 1.6 and 0.6. The graph paper provided opportunities for sensible scales with most successful candidates choosing 4 cm to a unit on the vertical scale starting at zero and 1 cm to 5 incorrect notes on the horizontal scale. There were several ways of using the horizontal axis with many candidates preferring to start from 0.5 so that their bars were easy to draw on a solid graph line. Having calculated the frequency densities correctly, most candidates plotted the heights accurately and correctly used the class boundaries for their bars rather than the upper limits (5,10, 20, 40 and 70). Careful candidates also remembered to label their axes ‘frequency density’ and ‘no of incorrect notes’ and to show at least three values on each axis.

(b) Although most candidates were able to identify that the lower quartile lay in the 11 – 20 class and the upper quartile lay in the 21 – 40 class, the discrete nature of the data caused some confusion. Many candidates realised that they needed to subtract the lower end of the lower quartile class from the upper end of the upper quartile class to find the greatest possible value of the interquartile range. However, only the strongest candidates knew to use the limits (40 and 11) rather than the bounds that they used when drawing the histogram.

(c) Candidates who knew to show their working were generally successful. So long as they correctly identified the mid-points and knew the formula for an estimated mean from grouped data, they usually obtained the correct answer.

MATHEMATICS

Paper 9709/61
Paper 61

There were too few candidates for a meaningful report to be produced.

MATHEMATICS

Paper 9709/62
Paper 62

Key messages

Candidates should ensure that, for numerical answers, they have all relevant working clearly shown.

Final answers need to be to at least 3 significant figure accuracy, therefore during working at least 4 significant figure accuracy should be maintained.

Candidates must also ensure that they have read the question carefully.

When writing conclusions to hypotheses tests the answer must be given in context and they should not be definite or make any contradictory statements.

Candidates should be able to correctly choose and justify a valid approximating distribution when required.

General comments

There was a wide range of scripts in terms of quality of responses from candidates.

There were certain questions on this paper where candidates found them challenging, this being particularly true of **Questions 2, 3** and **7**. Candidates coped very well with most of **Question 5** and **Question 4** was reasonably well completed, although not all candidates expressed their conclusion in the correct way.

On the whole, candidates worked to the required level of accuracy and showed all the necessary steps in their working; it is important that candidates do not just give a numerical answer without full justification.

Timing did not appear to be a problem for candidates, with the majority of candidates being able to complete the question paper in the allotted time.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

Most candidates realised that a Poisson approximation was required but not all used the correct value of λ. Candidates should know, and check, the conditions required for a valid approximating distribution. Numerous candidates incorrectly chose a normal distribution, which was not valid here, and a few candidates did not use an approximating distribution at all but used the given binomial distribution to calculate the probability.

Question 2

(a) This question was not particularly well attempted by candidates. This question required a confidence interval for a proportion and this seemed to cause a problem for many candidates. The values for *p* and *q* should have been $\frac{56}{300}$ and $\frac{244}{300}$ respectively; many candidates did not give an expression of the correct form using these values. It was pleasing to note that of the candidates who did use an expression of the correct form, most used a correct *z* value and gave their answer as an interval as required.

(b) Very few candidates were able to give a fully correct answer to this question part. Of those who realised that they were checking to see if $\frac{1}{6}$ lay in the interval, very few were able to express their reasoning with the required level of uncertainty in the language they used. For example a statement such as '$\frac{1}{6}$ is within the confidence interval so the die is not biased' would not be acceptable as it is a definite statement.

Question 3

(a) This part required the knowledge that the area of the triangle under the line between 0 and 3 was equal to 1. Many candidates realised this and successfully showed that *c* was $\frac{2}{3}$.

(b) Whilst this part of the question could have been solved by a method involving scale factors, most candidates preferred to find the equation of the line and integrate from 2 to 3 (or equivalent). As the equation was not given, it had to be found from the information supplied in the question. Many candidates did not find the correct equation of the line, the most common errors were to use a gradient of $+\frac{2}{9}$ rather than $-\frac{2}{9}$, or to use $y=\frac{2}{3}$.

(c) If not found earlier, the equation of the line had to be found for this part of the question. Those who found the correct equation generally went on to successfully find E(X); those with an incorrect equation of the line could score marks for attempting to integrate xf(x), but it was not possible to proceed here without attempting to find f(x).

Question 4

The first 4 marks of this question were generally well attempted. Finding the estimates for the population mean and the unbiased estimate for the population variance and stating the hypotheses were a good source of marks for many candidates. A minority of candidates calculated and used the biased estimate; follow through marks were available for these candidates. Calculation of the test statistic was generally done well and the comparison with the critical value of 2.24 (or alternative comparison) was clearly shown by the majority of candidates who reached this stage of the question.

The final conclusion to the test, however, was not well answered. Candidates often realised that H_0 would be rejected but did not write the conclusion in context and with the required level of uncertainty in the language used. For example 'μ is not 8.9' is not acceptable, but 'there is evidence to show that μ is not 8.9' gives a level of uncertainty, as required, and would be acceptable. A statement such as 'there is evidence that μ has changed' would also be unacceptable, as although the language used has the correct level of uncertainty, the statement has no context linking it to the question.

Question 5

(a) It was important in this part that candidates put their answer in the context of the question. Generic answers such as 'the events occur independently' merely quotes general conditions, hence is not sufficient, whereas 'the customers arrive independently' relates the condition to the question, which was an acceptable answer.

(b) This part was particularly well attempted. An occasional error was seen where P($X \leqslant 3$) was calculated rather than P(X = 3).

(c) Again, this was well attempted with most candidates using Po(4.6) and finding the correct expression. Errors included an incorrect λ, omission of '1–' in the expression, and omission of the final term. There were also candidates who lost accuracy in the final answer by prematurely approximating individual terms in a correct expression, and there were some candidates who did not show relevant working.

(d) This part was not so well attempted. The correct probability of none arriving ($e^{-2.3}$) was found by many candidates, but relatively few went on to form a correct binomial expression for the probability required. A premature approximation of $e^{-2.3}$ resulted in some candidates not obtaining the final answer to the accuracy required, and occasionally ${}^5C_2$ was missing.

Question 6

(a) Many candidates gave correct hypotheses (though μ was sometimes used rather than p). The main error noted was to find the probability of exactly 2 offers in 20 rather than the tail probability of 2 or less offers in 20. The conclusion to the test needed to be in context and not definite, that is using language with a level of uncertainty. Answers such as 'the claim is wrong' would not be acceptable but 'there is no evidence to support the manufacturer's claim' shows context and has a level of uncertainty in the language.

(b) Some candidates were able to make a reasonable attempt at finding the probability of a Type II error. Common errors in finding the correct expression of $1 - P(X \leqslant 3)$ included omission of '1–', use of $\frac{1}{3}$ rather than $\frac{1}{7}$ and omission of a term in the expression.

(c) Context was important here when describing what was meant by a Type II error, generic definitions and those not in context were not acceptable. Whilst some candidates tried to use the context given, a common error was in referring to the proportion not being $\frac{1}{3}$ rather than being less than $\frac{1}{3}$. Answers such as 'Concluding that the proportion is $\frac{1}{3}$ when actually it is not', did not quite describe the context fully; 'Concluding that the proportion is $\frac{1}{3}$ when actually it is less', would be preferable.

Question 7

(a) Generally, this question part was not well attempted by candidates. Finding the mean and the variance was challenging for many candidates; the question must be read carefully to prevent misinterpretation errors. Standardising and attempting to find the probability was typically done well and candidates were able to recover marks here following earlier errors.

(b) Similarly, candidates struggled here to find values for the mean and variance; a common error was to calculate $140 \times 20 + 80 \times 50$ for the variance rather than $140 \times 20^2\ 50^2$ and some candidates confused units ($ and cents) or made errors when trying to convert between the two.

MATHEMATICS

Paper 9709/63
Paper 63

Key messages

Candidates must ensure that their conclusions to significance tests are given in a suitable form. They must be written in the context of the question and in a non-definite form. There must be no contradictions.

Candidates are required to show their methods and working for each question, as unsupported answers are not acceptable.

General comments

Overall, many candidates did show their methods and working clearly. Most candidates also gave their numerical answers to the required accuracy, usually given to 3 significant figures.

Comments on specific questions

Question 1

(a) The appropriate approximating distribution was the Poisson distribution, $\text{Po}\left(\frac{2}{3}\right)$. To find the probability that more than 1 flower was white required candidates to use 1 – P (0 or 1 white flowers). Some candidates omitted the P(1) term. Other candidates did not use the Poisson approximation and instead calculated with the binomial distribution. A special case mark was available for these candidates. A normal distribution was not an appropriate approximation for this question.

Some candidates prematurely approximated the $\frac{2}{3}$ parameter to 0.67 instead of using at least 0.667 (with 3 or more significant figures). To gain the correct answer it was preferable to use the exact $\frac{2}{3}$ value.

(b) The approximation to the original binomial distribution $\text{B}\left(200, \frac{1}{300}\right)$ was the Poisson, $\text{Po}\left(\frac{2}{3}\right)$, as $n > 50$ and the mean $\left(np = \frac{2}{3}\right) < 5$, or $n > 50$ and the probability $\left(p = \frac{1}{300}\right) < 0.1$. It was necessary for candidates to state these values, mention of large *n* or small *p* was not sufficient.

(c) The appropriate approximating distribution for **part (a)** was the Poisson distribution $\text{Po}\left(\frac{2}{3}\right)$.

For the second set of flowers the appropriate approximating distribution was the Poisson distribution. The distribution of the total number of white flowers was the Poisson distribution with parameter $\frac{11}{3}$. Using this Poisson distribution the probability P(0, 1, 2, 3) was required. Some candidates used other parameter values such as 3 or 6 or 3.5. Though these values were incorrect it was possible to gain a method mark. Some candidates prematurely approximated the $\frac{11}{3}$ parameter to 3.7 or inaccurately to 3.66 instead of stating at least 3.67 (with 3 or more significant figures). To gain the correct answer it was preferable to use the precise $\frac{11}{3}$ value.

Question 2

(a) The 90 per cent confidence interval for the proportion of adults was required. Thus, the proportion in the sample $\left(p = \frac{102}{250}\right)$ and the variance $\left(\frac{pq}{n} = \frac{(0.408 \times 0.592)}{250}\right)$ were required, as well as the value of *z* (1.645) for the 90 per cent interval was needed. Some candidates incorrectly used $z = 1.282$. Other candidates obtained 0.0009661 but then incorrectly inserted an extra 250. Some candidates had the correct variance but used 102 instead of the proportion.

The answer had to be presented as an interval, however different forms of the interval were acceptable.

(b) For this part, unbiased estimates were required. Many candidates applied the formulae correctly, however a few candidates found only the biased variance.

(c) The question required candidates to explain why the suggested plan would not provide a random sample. To explain this, it was necessary to refer to the chances of the houses being selected. As every house did not have an equal chance of being selected (or in most cases no chance of being selected) this plan would not give a random sample. Many candidates just stated that the plan was biased, which was not sufficient. Some candidates referred to the values of the properties or the distances to be travelled without referring to the unequal chances of selection.

Question 3

Many candidates correctly set up the new variable F – 0.5M and found the probability of this being F – 0.5M < 0. The mean (17) and the variance (1485.25) were found and used to standardise the value 0. Then the correct area was selected to give the probability. Some candidates did not find the correct variance as they used 0.5 instead of 0.5^2 or subtracted instead of adding the separate variances. Some candidates gave the larger area as their probability answer instead of finding the smaller 'tail' area. A sketch of a normal distribution centred on the mean of 17 could have helped with this choice. An alternative valid approach was to set up the new variable 2F – M. Other candidates did not set up the new variable and attempted to work within just one of the given distributions. These attempts were not valid and were unsuccessful.

Question 4

(a) Many candidates wrote down the answer for *k* (from using the area being equal to 1), this was thoroughly acceptable. Some candidates integrated f(*x*) = *k* with the limits 0 and *a* and used the property that the area was equal to 1 to obtain this answer. It was necessary to express the answer as $k = \frac{1}{a}$. It was not sufficient to end with $ka = 1$ or $a = \frac{1}{k}$.

(b) For the variance the integration of kx^2 with limits 0 and a was required. Also the mean $\left(\frac{a}{2}\right)$ was needed. This could be stated from the symmetry of the given diagram or found by integrating *kx* with limits 0 and *a*. These results were then used in $E(X^2) - (E(x))^2 = 3$. Some candidates carried out these steps to find *a* = 6 very efficiently. Other candidates did not square E(*X*), leading to a quadratic in *a*, or omitted this term altogether. The answer for *a* needed to be given as 6, not ±6.

Question 5

(a) The Poisson parameter (2.1) was equal to the variance of the distribution. Hence the standard deviation was $\sqrt{2.1}$ or 1.45. Many candidates obtained this correctly. A few candidates gave the answer as 2.1.

(b) For this 2-week period the Poisson parameter was 4.2. To find the probability that the number of absences was at least 2 it was necessary to find 1 – P(0 or 1). Many candidates obtained this correctly. Some candidates included the extra term P(2), whilst other candidates used an incorrect parameter value.

(c) For this 3-week period the Poisson parameter was 6.3. For the required number of absences the Poisson probabilities P(5 or 6 or 7) were needed. Many candidates obtained this correctly. Some candidates incorrectly found P(8) – P(4), whilst other candidates used an incorrect parameter value.

(d) Some candidates produced a very efficient hypothesis test, showing the hypotheses, calculating the 'tail' probability, comparing the significance level and stating a suitable conclusion. It was necessary to state the hypotheses for the Poisson distribution with parameter 6.3. It was necessary to calculate the probability P($\leqslant$ 2) and compare this value with the significance level 0.10 and to state a suitable conclusion. Some candidates omitted the hypotheses. Other candidates calculated only the single term P(2). This was not a valid method. Several candidates wrote down the correct comparison which was 0.0498 < 0.10. In some cases, the conclusion was correctly stated in context, in a non-definite form and with no contradictions, however many candidates omitted the context or stated the conclusion too definitely or included contradictions. Some candidates attempted to apply a normal distribution. This also was not valid.

(e) It was essential for the candidate to state a suitable reason for their choice of error type. The conclusion to the test in **part (d)** was to reject H_0. For this to have been an error H_0 must have been true. Hence a Type I error might have been made. Candidates could gain these marks on a follow through basis from their different conclusion in **part (d)**.

Question 6

(a) Many candidates answered this question efficiently and accurately, using the distribution of means of samples, standardising with $\frac{6.9}{\sqrt{30}}$ and finding the relevant area. Some candidates applied a continuity correction, which was not appropriate here. It was preferable to work to at least 4 significant figures with the accumulating combinations of *z* values and probability values. To find the final probability candidates needed to select the relevant area, a sketch could have helped here. Some candidates incorrectly subtracted the two large probability values.

(b) (i) As Anjan was considering a possible difference in mean journey time it was appropriate to use a two-tail test. It was acceptable to state difference or change or 'it could be higher or lower'.

(ii) Some candidates produced a very efficient hypothesis test, showing the hypotheses, calculating the probability, comparing the significance level and stating a suitable conclusion. As this was a two-tailed test the alternative hypothesis was H_1: $\mu \neq 38.4$. Some candidates used a one-tailed test. For the standardisation using the normal distribution it was necessary to use $\frac{6.9}{\sqrt{30}}$. Many candidates did write down the comparison of the *z* value with the critical value (1.429 < 1.645). Other candidates omitted this essential step. The comparison could have been between the corresponding probabilities (0.0765 > 0.05) or the critical value of *z* could have been used (40.2 < 40.47). Some candidates stated the conclusion correctly in context, in a non-definite form and with no contradictions. Other candidates omitted the context or stated the conclusion too definitely or included contradictions.

(iii) The given information did not indicate the nature of the distribution of the mean journey times. In particular, there was no reference to a normal distribution. Hence it was necessary to use the Central Limit Theorem. The statement 'Yes because the population distribution was not normal' was accepted. It was necessary to refer to the population and not just to 'it …' or not just 'the sample …'.