MATHEMATICS

Paper 9709/11
Paper 1 Pure Mathematics 1

Key messages

Since the change in syllabus, previous reports have highlighted the question paper rubric that states 'no marks will be given for unsupported answers from a calculator.' Although this message has been taken on board by most candidates, there is still a significant minority whose working does not contain enough detail. Clear working must always be shown to justify solutions.

For quadratic equations, for example, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. It is not sufficient to use calculators to solve equations and write down only the solution. It is also not sufficient to quote only the formula: candidates need to show values substituted into it. Candidates should ensure that factors must always produce the coefficients of the quadratic equation when expanded.

General comments

Some very good responses were seen but the paper proved challenging for a number of candidates. In AS and A Level Mathematics papers, the knowledge and use of basic algebraic and trigonometric methods from IGCSE or O Level is expected, as stated in the syllabus.

Before starting to write their solution, candidates are recommended to read the question and think carefully to ensure they are using the required approach. In the geometric progression question (**Question 7**) some candidates used the formulae and techniques for arithmetic progressions and therefore received no credit. In **Question 4**, some candidates produced all terms in the binomial expansion instead of focusing on the coefficient of one particular term.

Where a question asks for an answer to a certain degree of accuracy (e.g. **Question 7(b)** and the nearest millimetre), it is expected that candidates will provide the answer in this form to be awarded all the marks.

Comments on specific questions

Question 1

Candidates needed to start by multiplying by the denominator, $x - 1$, to create a quadratic equation which could then be solved. A small number did not fully multiply $(3x + 2)(x - 1)$, omitting terms. Others did not deal correctly with the 2 terms that remained on the right hand side, for example adding it instead of subtracting. It was essential to show working for solving the quadratic equation leading to two solutions, as stated in the Key messages. The majority of candidates used factorisation or the quadratic formula, with only a few completing the square. In some cases, the quadratic formula was quoted or used incorrectly, or factors did not match the quadratic equation, suggesting that candidates had derived their 'factors' from roots obtained on their calculator. A complete, correct method is required for candidates to be awarded full marks.

Question 2

(a) This question required candidates to substitute $x = 6$ into the given $\frac{dy}{dx}$ to find the gradient of the tangent then use their gradient and the point (6, 4) to find the equation of the tangent. Not all candidates understood that $\frac{dy}{dx}$ gives the gradient of the tangent at a particular point. Some candidates incorrectly integrated the function first, while others evaluated $\dfrac{-1}{\left(\dfrac{dy}{dx}\right)}$ and hence found the equation of the normal instead. In this question, the equation of the tangent was not required in a particular form so there was no need to simplify it or rearrange it.

(b) It was necessary to integrate the gradient function to obtain an expression for y, including a constant of integration. This could be evaluated by substituting (6, 4) and the final equation of the curve obtained with y as the subject. The most common errors related to integration, with incorrect powers often seen and many candidates forgetting to use the reverse chain rule to divide by ½, the coefficient of x, as well as by the new power, –3. A few candidates substituted the x and y coordinates the wrong way round.

Question 3

Most candidates differentiated the equation initially, though a few tried to square individual terms to deal with the $x^{\frac{1}{2}}$. Some errors were seen in the coefficient of $x^{\frac{1}{2}}$. To find the y-coordinate of P, a stationary point, it was necessary to set $\dfrac{dy}{dx} = 0$, substitute in $x = 9$, and then solve the resulting equation to find a. Using this value of a and $x = 9$ in the original equation of the curve led to the required y-value of 18. A number of candidates made no progress with solving the equation while some attempted to substitute their a-value into $\dfrac{dy}{dx}$ instead of into y. In some instances, power errors were seen in either the differentiation step or the equation-solving step.

Question 4

This question required candidates to focus on the term in x^2, so there was no need to produce the entire expansion in both cases. For the first two marks, it was necessary to evaluate the coefficient of x^2 in each binomial expansion. The most common error at this stage was to omit to square $\dfrac{2}{p}$. Some candidates had errors in their binomial coefficients, often through miscopying the powers they had written down. To find the possible values of p, candidates needed to create an equation by adding the terms in x^2 and equating them to 70. At this stage, they should have obtained a quartic which was recognisable as a quadratic in p^2 which could therefore be solved using quadratic techniques. It was acceptable to substitute another variable for p^2. Some candidates who successfully reached this point forgot to take square roots to find the four possible values of p. A significant number of candidates took square roots but did not find both positive and negative values.

Question 5

In this question, many candidates chose to annotate the diagram to work out their strategy.

(a) To find the perimeter of the shaded segment, it was necessary to work out the length of AB and add it to the given arc length. To find AB required deducing the radius (6) from the given perimeter. Using the formula arc length = $r\theta$, it was straightforward to evaluate angle AOB which was $\dfrac{4}{3}$. However, many candidates assumed that the length of AB was also 6, giving an equilateral triangle with all angles $\dfrac{\pi}{3}$. This meant that candidates could only achieve a maximum of $\dfrac{2}{4}$ marks in **part (a)** and $\dfrac{1}{2}$ marks in **part (b)**. It is important for candidates to evaluate angles rather than guess them by looking at the diagram.

(b) The area of the shaded segment was equal to sector area – triangle area. The area of the sector was found using $\dfrac{1}{2}r^2\theta$ and there were two possible ways of finding the area of the triangle. Some candidates chose to find the area using the formula $\frac{1}{2}$ $ab \sin C$ (where $a = b = 6$ and $C = \frac{4}{3}$) while others used $\frac{1}{2}$ x base x height where the base was 6 sin $\frac{2}{3}$ and height was 6 cos $\frac{2}{3}$. In both methods, some candidates used $\frac{4}{3}\pi$ for the angle instead of $\frac{4}{3}$. In the second method, a number of candidates used $\frac{4}{3}$ instead of $\frac{2}{3}$ for the angle in half the triangle.

Question 6

(a) This question required candidates to manipulate a trigonometric equation and express it in the given form. It was essential to include the right hand side initially ('= 1') and to show clearly all steps in the argument to justify the result. The first step was to choose a common denominator in order to combine the fractions on the left hand side. A number of candidates were unable to carry out this first step, or introduced errors and therefore did not reach a quadratic equation. Examples of poor notation were seen, for instance $\sin \theta^2$ instead of $\sin^2 \theta$ in the denominator. Some candidates did not multiply through by their common denominator and others did not use the identity $\cos^2 \theta = 1 - \sin^2 \theta$ correctly. It is always helpful in a 'show that' question to look at the required result for clues as to how to form an argument.

(b) This part required candidates to solve their quadratic equation in $\sin \theta$. Some chose to substitute another variable. It was important to check the interval to ensure that all values of θ were found. A number of candidates found only one correct value because they did not consider both the 3rd and 4th quadrants. As stated in the rubric on the front of the exam paper, answers should be given to 1 decimal place in the case of angles in degrees, but integer values or rounding errors were common in this question.

Question 7

This question stated that the distance the post sinks into the ground follows a geometric progression, but some candidates applied the formulae for an arithmetic progression.

(a) From the given information, candidates could find values of r and a. A number of candidates did not start this question because they did not identify any values. A few candidates thought that $r = \frac{5}{4}$ not $\frac{4}{5}$. To verify that the 9th impact is the first in which the post sinks less than 10 mm, it was necessary to calculate **both** the 8th and 9th terms using the formula, then to write a brief conclusion that the 9th impact is the first one below 10 mm.

(b) and **(c)** These parts of the question required candidates to apply correct formulae using their r and a values. In **part (b)**, many candidates used the formula for the sum of 20 terms correctly but a significant number did not give their answer to the nearest millimetre as required. In **part (c)**, most candidates successfully applied the formula for the sum to infinity, and could be awarded the method mark even if their a and r values were incorrect. Candidates who summed all the individual terms very often made calculation or rounding errors and so did not reach the correct answers. Using a formula appropriately is likely to be more efficient and more accurate.

Question 8

(a) A number of candidates misunderstood the notation f'(x) and so found the inverse function instead of the derivative. Some realised their error later so used their working in **part (b)**. Many candidates who successfully found the derivative were unable to interpret it: with a positive value in the numerator and a squared bracket in the denominator, $f'(x) > 0$ and hence f is an increasing function. Errors in the derivative included incorrect powers, or omitting to use the chain rule and therefore multiplying the result by 4.

(b) This part required a standard method of finding an inverse function. It was generally done well, though there were errors seen in numerical and algebraic manipulation.

(c) Only those candidates who had a correct inverse function made progress here, comparing the two functions and observing that $p = 8$.

Question 9

(a) and **(b)** Both required standard techniques for completing the square, with **part (b)** presenting more of a challenge to candidates who encountered problems in dealing with the $2x^2$. Not all candidates gave their answer in the required form using two sets of brackets.

(c) Many candidates had difficulty in interpreting the notation, or in understanding the mapping, or did not have a relevant result from **part (b)** to use. As a result, correct answers were relatively rare.

(d) Some candidates were able to describe at least one of the two transformations even without a result in **part (c)**. Fully correct responses were rarely seen. Accurate use of descriptive language is important, with examples given in the mark scheme.

Question 10

(a) To find the area of the region between the two curves required subtraction of the equations of the curves then integration (or vice versa). Common errors involved powers or fractions (e.g. when attempting to divide by $\frac{3}{2}$), and some candidates used limits in the wrong order.

(b) This problem-solving question proved to be very challenging for all but the most able candidates. Successful responses began by differentiating both curves to find the gradients of the two tangents, 3 and $\frac{1}{2}$. From this point onwards there were several possible strategies. Some candidates found the equations of the tangents then joined them to form a triangle, found the vertices and used the cosine rule to obtain $\alpha = 45°$. Others found the normal at a chosen point to connect the two tangents, creating a right-angled triangle whose sides could be determined, then from trigonometry found $\alpha = 45°$. A simpler method was to calculate $\tan^{-1} 3$ and $\tan^{-1} \frac{1}{2}$ then find the difference between the angles to be 45°. Other geometrical methods were possible and a diagram proved very helpful to candidates who attempted this part, as well as to the reader.

Question 11

(a) Candidates needed to substitute the equation of the line into the equation of the circle to eliminate x or y, then use the condition $b^2 - 4ac = 0$, solving the resulting quadratic (or quartic) equation to find $m = \pm 2$. Of those candidates who attempted this part, a number made arithmetic or algebraic errors so their quadratic equation was incorrect.

(b) The coordinates of the two points could be found either by substituting a value of m into the quadratic (or quartic) equation or by equating the tangent and normal then solving. Having found one point, some candidates used the symmetry of the diagram to deduce the second point. A few candidates chose to start again by equating the tangent and the circle.

(c) Almost all candidates found this question to be very challenging. Three principal methods were seen: the cosine rule; using sin, cos or tan of *BOA* to find the angle, doubling this to find *BOC* then halving again to find *ODC* (using the property that the angle on the circumference is half the angle at the centre); or calculating angle *ODB* and subtracting it from 90°, then subtracting the angle between *CD* and a vertical line drawn upwards from *D*.

MATHEMATICS

Paper 9709/12
Paper 1 Pure Mathematics 1

Key messages

Previous reports, since the change in syllabus, have highlighted that the question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' It is pleasing to see that this has been taken on board by most candidates. However there are still a significant minority for whom it has not and it is important to emphasise that clear working must always be shown to justify solutions. This includes integration and gradient questions. For quadratic equations it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Candidates should also remember that factors must always produce the coefficients of the quadratic when expanded.

Some candidates used the formulae and techniques for geometric progressions on the arithmetic progression question and vice versa and therefore received no credit. It is recommended that candidates pause and ensure they are using the required approach before embarking on their attempted solution.

General comments

The paper was well received by candidates and many very good scripts were seen. Candidates generally seemed to have sufficient time to finish the paper. Presentation of work was mostly good, although some of the answers were written in pencil which, when superimposed with ink, gives a very unclear image when the script is scanned. Centres are strongly advised that candidates should be reminded to write in black or dark blue ink, unless drawing a graph or diagram.

Comments on specific questions

Question 1

This question was a good start to the paper for many candidates. In **part (a)** candidates were able to find the midpoint of *AB*, its gradient and the negative reciprocal of the gradient. Common mistakes were using *A* or *B* rather than the midpoint or failing to calculate the negative reciprocal. Some weaker candidates made slips in calculating the gradient or failed to calculate difference in *y* divided by difference in *x*. In **part (b)** those who used $(x-a)^2+(y-b)^2=r^2$ as the equation of the circle were generally much more successful than those who attempted to use the expanded form. For some candidates there was some confusion whether the length *AB* was the radius or the radius squared.

Question 2

As mentioned in the key messages, some candidates used the formulae and techniques for geometric progressions on this arithmetic progression question, but the majority were able to make rapid and effective progress by equating the differences between consecutive terms. The resulting equation was very often solved correctly, and full marks obtained. A significant minority of candidates did not find *a* and left their final answer as 1275*a*. If this type of answer was expected, then the question would have asked for an answer in terms of *a*.

Question 3

In **part (a)** the vast majority of candidates realised that the discriminant was required for this question and were able to obtain an inequality or an equation in terms of k. Significantly fewer were able to obtain the correct final answer. Common mistakes were only considering positive values of k or deciding that the outside regions were the correct answer or that $k = 8$. In **part (b)** most candidates attempted to factorise, sometimes after dividing by 2 throughout although use of the formula was also fairly common. These were generally successful although the solution $\cos\theta = 1$ was sometimes dismissed as impossible. As mentioned in the key messages, the use of a calculator function to solve the quadratic was not accepted and should be strongly discouraged.

Question 4

Full marks were often awarded to candidates for this question but it was not always tackled in the most efficient way and some weaker candidates used the formulae and techniques for arithmetic progressions on this geometric progression question. Good candidates were able to quickly find ar and then divide 1764 by this to obtain r. Those who attempted to use the sum formula for the sum of the first three terms were generally less successful, usually because they forgot to subtract a. Some candidates misunderstood the question and attempted to add the sum of the first two terms and the sum of the first three terms. A minority of candidates found the sum of the first fifty terms rather than the fiftieth term.

Question 5

This question was found to be challenging for many candidates. Significant numbers missed it out, especially **part (b)**. In **part (a)** many candidates knew that the graph had to be shifted upwards as a result of the translation but fewer understood the effect of the stretch, with a factor of 2 often being used rather than 0.5 . In **part (b)** many seemed to not understand what was required, so practice on similar ones would be recommended for future candidates. For those who did understand what was required, +1 was often seen, but $\mathrm{f}(2x)$ much less frequently.

Question 6

Part (a) was generally very well done by candidates with many scoring all 3 marks. **Part (b)** was less well done, mainly because candidates did not follow the 'hence' instruction and reverted to the original form of the question rather than using the completed square form obtained in **part (a)**. Candidates are advised to be familiar with the meaning of terms such as 'hence'. In **part (c)** many candidates did not use the information provided by the completed square form from **part (a)** and sometimes differentiated to find the stationary point. Weaker candidates often missed out the sketch altogether, plotted points, plotted a graph with a maximum point or thought that the coordinates of the stationary point were where the graph crossed the axes. Practice on this type of question will significantly help future candidates.

Question 7

In **part (a)** nearly all candidates started working from the left hand side of this identity and realised the need to use a common denominator. After this step though, many were unsure how to proceed, and only a small number were able to provide a convincing proof. Many candidates then unnecessarily missed out **part (b)** rather than using the given answer from **part (a)**. Those who did use it were usually successful although some failed to consider that $\tan\theta$ could be $-\sqrt{3}$ as well as $\sqrt{3}$.

Question 8

The first two parts of this question were generally well completed but in the third part many seemed to not understand what was required. In **part (a)** nearly all candidates realised the need to integrate and most of these knew that the given point had to be used to find the value of c. Some unusual methods or algorithms related to integration were seen though, with a significant minority dividing by the coefficients of the two terms as well as the new powers. These candidates were invariably unsuccessful, and the use of an algorithm should be discouraged unless it is fully understood. In **part (b)** the vast majority of candidates knew to equate the given differential to 0, although solving the subsequent equation proved more challenging for many. Some weaker candidates equated the equation that they had found in **part (a)** to 0. About a third of candidates missed out **part (c)** and many who did attempt it did not realise that the required condition would be met when x was greater than their value found in **part (b)**.

Question 9

Parts (a) and **(b)** were very well done on this question but candidates struggled with the explanations required in **part (c)** and **part (e)** especially. The first two parts showed a very good understanding of composite functions with only the weakest candidates attempting to multiply them, but in **part (c)** many candidates did not realise that the presence of a minimum point meant that the function could not be one to one and therefore could not have an inverse. Please note that a comment that the function would fail the horizontal line test was insufficient to be awarded the mark. In **part (d)** many candidates could find the required function although simplifying it proved challenging for many with the 5 sometimes appearing in the numerator rather than the denominator. Some candidates misunderstood the question and attempted to find the inverse of the function found in **part (a)**. **Part (e)** proved to be the most challenging on the whole paper with only the very best candidates able to give a satisfactory explanation.

Question 10

About 10 per cent of candidates missed out this question altogether, although many fully correct solutions were seen. In **part (a)** many candidates realised the need to add the 2 different arcs although the use of $\frac{2\pi}{3}$ rather than $\frac{4\pi}{3}$ was common. Those working in radians were more successful than those attempting to convert the angles to degrees, and this approach should be discouraged. In **part (b)** those using $\frac{1}{2}ab\sin\theta$ were more accurate than those using trig to find the base and heights of the triangles. Some candidates seemed to not understand the meaning of the word difference. **Part (c)** proved challenging with many candidates not realising that the answer from **part (b)** should be added to the two sector areas. Some candidates did not give their answers in each part to the specified level of accuracy.

Question 11

This question was well answered by many candidates although around 20 per cent did miss out the final part. In **part (a)** those differentiating were generally more successful than those attempting to complete the square. A few candidates wrote down the correct answer with no working shown and were therefore penalised – see the key messages about calculator usage. In **part (b)** many fully correct solutions were seen although some candidates equated the two curves, presumably in an attempt to find the points of intersection which were given in the question. It is much better to show that it is the difference between the areas which is required in this type of question rather than equating and then integrating. In **part (c)** some candidates used the differential found in **part (a)** rather than finding the required one. There was also some confusion over the chain rule with –7.5 and 2 sometimes being divided rather than multiplied, although many correct answers were seen.

MATHEMATICS

Paper 9709/13
Paper 1 Pure Mathematics 1

Key messages

Since the change in syllabus the previous reports have each highlighted that the question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' Although this message has been taken on board by most candidates, there are still a significant minority for whom it has not. Clear working must always be shown to justify solutions. For example, when solving a quadratic equation it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for certain marks to be awarded. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Candidates should ensure that factors must always produce the coefficients of the quadratic equation when expanded. This message is particularly relevant to **Questions 1**, **10(a)** and **11(b)(i)**.

General comments

Nearly all candidates were able to attempt a significant number of questions and many very good scripts were seen. The requirement that candidates know and can use the algebra, geometry and trigonometry techniques studied at GCSE level seemed to be evident in the responses from most centres.
Almost all candidates finished the paper.

Comments on specific questions

Question 1

Most candidates recognised the requirement to use the trigonometric identity to obtain a quadratic in $\cos\theta$. The solution of this quadratic was nearly always seen but the method of solution was often omitted. Some correct answers were spoilt by the inclusion of an incorrect acute angle and some lost a mark needlessly by ignoring the accuracy requirements for this paper.

Question 2

(a) Completion of the square was well understood and applied correctly here by a large majority of candidates. Where errors were seen these usually involved incorrect calculation of 'b'.

(b) Those candidates who looked beyond the value of 'b' and used the domain of f were able to deduce the range of the function correctly.

(c) The use of the completed square form from **part (a)** was seen in most answers. Rearrangement of the function was then generally performed correctly up to the selection of the sign of the square root. The negative sign within the square root frequently became omitted in the final stages or even moved outside the square root. The final step of interpreting the square root, with reference to the domain of f, to give only negative values was the most significant error.

Question 3

(a) The binomial expansion was well understood and the required terms found accurately by most candidates.

(b) When the answer to **part (a)** was found correctly this part was nearly always also completed correctly.

(c) The use of the answers to **parts (a)** and **(b)** to obtain the three required terms was almost always seen. Where the answers to **parts (a)** and **(b)** were correct then the correct answer to this part usually followed.

Question 4

There was a mixed reaction to this question but more able candidates scored well. Most candidates realised the need to differentiate the given formula with respect to x although an intrepid few managed to rearrange the formula and find $\frac{\mathrm{d}x}{\mathrm{d}V}$. The most common error at this stage was the omission of the multiplication by the derivative of $9 - x$. The chain rule was often seen and used correctly with $x = 4$ substituted correctly. The most successful answers involved finding the rate of change of the height in mh^{-1} and then converting to $cm\,min^{-1}$. Attempts to convert the units before the chain rule was applied were usually unsuccessful. The conversion was often omitted altogether with answers being left in mh^{-1}.

Question 5

This question was surprisingly omitted by a significant number of candidates who were able to attempt all the other questions.

(a) This question proved challenging for many candidates. An incorrect factor of 2 was often given and some candidates divided 6 by 2 in the reverse order and hence obtaining the incorrect answer of $\frac{1}{3}$.

(b) Candidates found this part less challenging. Although there is little connection between the answers to this part and **part (a)** the answers were often interchanged.

(c) Most attempts gained at least one of the two available marks in this part usually because the x-coordinate was given correctly. It is always worth candidates considering if just writing down the centre of the ellipse, (8, 6), would be enough for 2 marks.

(d) Applying the translation by vector $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$ to their **part (a)** answer gained the candidates both marks.

Candidates were more successful on this part because full marks could be obtained by either writing the correct answer or by following through correctly from the answer given to **part (c)**.

Question 6

This question stated that the answer should be found without using calculator trigonometric functions. Nevertheless many candidates stated $\sin\alpha$ and/or $\tan\alpha$ with no clear evidence leaving Examiners with no option but to assume calculator functions had been used. Where calculator functions were used a maximum of one mark could be gained.

Some successful candidates used the trigonometric identity to find $\sin\alpha$ and went on to use $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$. Others used the 8, 15, 17 right angle triangle to justify their values of $\sin\alpha$ and/or $\tan\alpha$. Those who simplified the two fractions to $\frac{(1+\cos\alpha)}{\sin\alpha}$ were able to reach the final result in the fewest steps.

Question 7

(a) Use of the given first derivative and gradient to form a quartic equation was usually successfully completed. Those candidates with good algebra skills were able to rearrange and solve their equation by taking fourth roots and considering both the positive and negative results. Those who expanded the binomial term to obtain a five term quartic equation were rarely successful.

(b) Candidates were more successful in this part that in **part (a)**. The required integration was well understood as was the use of the constant of integration. It was expected that an expression for f(x) would be stated as a final answer and in most cases it was.

Question 8

In both parts it was expected that answers would be expressed to at least three significant figures. As calculators are allowed expressing answers in terms of inverse trigonometric functions was not accepted.

(a) Most candidates were able to find either angle *APQ* or *AQP* using an appropriate trigonometric ratio. From their results candidates usually recognised the required perimeter as the arc length multiplied by four (or two if they used angle *APB* or *AQB*.) Some candidates used the cosine rule to find the angle *PAQ* or *PBQ* and then found the appropriate angle required to find the arc length. There were many completely correct solutions but some answers were incorrect in spite of correct working because of premature approximation of intermediate answers. There were some candidates who appeared unprepared for a question which initially involved simple trigonometry.

(b) There were a variety of methods used to answer this question, the most straightforward of which was to work with the formula for the area of a segment $A = \frac{1}{2} r^2 (\theta - \sin\theta)$ where θ was angle *APB* or *AQB* and then doubling their result. However the majority of candidates chose to work with the difference between the area of a sector and the area of an appropriate triangle and an appropriate multiplication. Some candidates worked with sectors and triangles that did not match each other and Examiners commented on the difficulty of following the working of some candidates.

Question 9

(a) This area of the syllabus proved to be well understood, candidates knew how to use the general term of a geometric progression and many completely correct solutions were seen.

(b) This question was mostly completed correctly by those candidates who found the second and third terms of the geometric progression and equated these to the required arithmetic progression terms. Solving for the first term and the common difference and calculation of the required sum usually followed on successfully. Candidates who chose to equate algebraic expressions for the two geometric progression terms invariably assumed both progressions had the same first term and scored zero.

Question 10

(a) Most candidates achieved at least 3 out of 4 marks, as a correct quadratic equation was formed using substitution which allowed the coordinates of point *A* to be found. A mark was lost by a significant number of candidates who did not show any method for solving their quadratic equation. Faced with a positive and negative solution for x it was uncommon to see the wrong coordinates selected for point *A*.

(b) Candidates found this more challenging than **part (a)**. Many candidates seemed confused about which integration was required, possibly because of the presence of the straight line on the diagram. Those who integrated the correct expression for y^2 were usually able to find the correct limits using the equation of the circle and their coordinates from **part (a)**. To obtain the final accuracy mark the area had to be quoted in the form shown in the question.

(c) This proved to be very challenging for many candidates. Quite a few used the angle as $\tan^{-1}2$, presumably as the line gradient was 2, some used degrees incorrectly in the arc length formula and many converted their answers to decimals often before a correct exact form was seen. When exact answers are required, exact values should be used in all stages of the solution.

Question 11

(a) The use of the formula for the distance between two points was well understood and mainly used to solve for p in this part. A lesser used alternative was to find the equation of the perpendicular bisector of AB and substitute $(2p, p)$ for (x, y) to find an equation in p.

(b) (i) The quality of candidate's responses varied greatly for this question. Most candidates formed correct expressions for the gradients of AC and BC, but some then struggled to set them up as a product equal to –1. Common errors were to merely let the expressions for the gradients of AC and BC equal each other or to multiply one of the gradient expressions by –1 and then equate the gradient expressions. Some candidates formed a correct equation using their gradient expressions and obtained a correct quadratic equation, but then did not show any method for solving this quadratic equation. Those who chose to use Pythagoras and equated the sum of the squares of the lengths of AC and BC to the square of the length of AB usually scored well.

(ii) This proved to be quite a demanding question for candidates. Those who realised from the outset that AB must be a diameter were able to quickly form a circle equation and rearrange this to the required form. Attempts at finding the centre using the intersection of the perpendicular bisectors of AC and BC often resulted in errors. A few confused diameter with radius. The use of the given equation and substitution of the three coordinates was a less popular method but with correct algebraic elimination (quite frequently not seen) often led to the correct values of a, b and c required in the given form of the circle equation.

MATHEMATICS

Paper 9709/21
Paper 2 Pure Mathematics 2

Key messages

Candidates should be reminded that the mark allocation of a question relates to the amount of work that needs to be shown. A question that relies purely on the use of a calculator would be unlikely to attract 4 marks. Candidates should also be familiar with the rubric on the front of the paper which states 'You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator'. This was especially relevant for **Question 3**.

Candidates should also ensure that they read each question carefully and ensure that they have fully met the demands of the question and that their answers are in exact form if required or are to the correct level of accuracy as detailed in the question itself or otherwise in the rubric on the front of the question paper.

It is also essential that solutions are shown clearly and with sufficient detail.

General comments

It was evident that many candidates were well prepared for the examination and were able to produce a range of clear and well thought out solutions. It was also evident that there were some candidates who were not so well prepared as evidenced by incomplete or questions that were not attempted. There appeared to be no timing issues and most candidates had sufficient room on the script, making use of the additional sheet when needed.

Comments on specific questions

Question 1

Most candidates were expected to use either one of two methods to solve the given inequality. Either method used was equally successful, although those candidates who attempted to use two linear equations to find the two critical values tended to make fewer sign errors than those candidates forming a quadratic equation from squaring each side of the inequality.

Question 2

Few correct solutions were seen, with candidates being unable to manipulate logarithms correctly. It was expected that one of the initial steps in the solution should be $\ln 14 + \ln e^{-2x} = \ln 5^{x+1}$. However, many candidates were unable to manipulate the left-hand side of the equation correctly. Use of the power rule for the right-hand side of the equation was usually correct. General manipulation of correct terms was poor, leading to an incorrect solution.

Question 3

A significant number of candidates viewed this question as purely a calculator exercise and, as a result, were unable to gain any marks. It was expected that candidates make use of the identity $\sec^2\theta = 1 + \tan^2\theta$, and obtain the result that $\tan\theta = 4$. Candidates were awarded no marks for simply finding the actual value of θ from their calculator. The next expected step was that the compound angle formula

$\tan\left(\theta+\frac{\pi}{4}\right)=\dfrac{\tan\theta+\tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}$ be used with a substitution for $\tan\theta=4$ and $\tan\frac{\pi}{4}=1$ to obtain an exact value for $\tan\left(\theta+\frac{\pi}{4}\right)$. Unfortunately, many candidates used their calculator with the value of θ previously obtained.

The question was intended to test the candidates' knowledge of the use and application of trigonometric identities and compound angle, not the ability to use a calculator, which is not a syllabus objective.

Question 4

(a) Most candidates were able to make a reasonable sketch of the graph of $y=e^{-\frac{1}{2}x}$, however the graph of $y=x^5$ proved to be more difficult to sketch for some candidates. The question required that candidates were to show that there was only one real root for the given equation. This meant that there was an expectation that reference be made to there being only one point of intersection in order to obtain both marks, provided that both sketches were correct. Many candidates did not make this final step.

(b) Most candidates were able to gain marks from the correct use of the iteration process, obtaining iterations correct to 6 significant figures, however some candidates either truncated their answer or did not give the final result to the correct level of accuracy.

Question 5

Most candidates obtained some marks by recognising the need for implicit differentiation and that the term $4e^{2x}y$ needed to be differentiated as a product. Some candidates made errors in the simplification of their derivative and so did not obtain a correct result. Another common error was to omit the differentiation of the constant term 21.

Question 6

(a) It was pleasing to see that there were many completely correct solutions, usually using algebraic long division, which clearly showed both the quotient and the remainder. Some candidates chose to use synthetic division, which was acceptable provided an adjustment to division by $\frac{3}{4}$ was made when dealing with the quotient. Unfortunately, some candidates did not do this and gave an incorrect quotient of $12x^2+8x$.

(b) Most candidates realised that they needed to make use of the response to **part (a)**, obtaining an integrand of $2+\dfrac{2}{4x-3}$. The integration of this expression was completed well by candidates who were able to get this far, but it should be noted that an exact form was required. Candidates who had an incorrect quotient (usually from use of synthetic division) were less successful as their integrand contained an extra term of $6x^2$.

Question 7

(a) Most candidates realised that differentiation of a quotient was needed and were able to obtain a correct derivative. Problems arose when candidates did not realise that at the point A, the value of x is 1, and it was this value that was needed to be used in the derivative.

(b) Few correct responses were seen, with many candidates using the equation of the curve rather than the equation of gradient function. It was expected that a substitution of $x=3$ and $x=3.1$ be made into the gradient function, showing a change of sign which implied that the point B was a maximum point.

(c) Some candidates chose to make use of two separate trapezia to calculate an approximate area for the shaded region. Of the candidates who attempted use of the trapezium rule, most were successful, identifying the correct width for each trapezium, which has in the past proved problematic. It is essential that candidates ensure that their final answer is in the required form, with some not rounding to the required 2 decimal places.

Question 8

(a) Few correct solutions were seen. It was intended that the appropriate double angle formula be used for both terms for $f(\theta)$. This would result in $f(\theta) = 8\cos 2\theta + 6\sin 2\theta + 8$, the first two terms of which could be written in the form $R\cos(2\theta - \alpha)$. Many candidates did not make the link that a double angle was used in the required form and that there were no double angles in the original function. Candidates should read the questions carefully and try to take note of facts such as those mentioned.

(b) Of the few correct responses obtained in **part (a)**, few correct responses to **part (b)** were obtained, with errors usually being due to incorrect application of the order of steps in the solution of the trigonometric equation $\cos(2\theta - 0.6435) = 0.9$. Candidates who had incorrect values for R could have noticed possible errors when an equation that could be solved was obtained.

MATHEMATICS

Paper 9709/22
Paper 2 Pure Mathematics 2

Key messages

Candidates should read each question carefully to help ensure that they have fully met the demands of the question and that their answers are in exact form if required or are to the correct level of accuracy as detailed in the question itself or otherwise the rubric on the front of the question paper. It is also essential that solutions are shown clearly and with sufficient detail.

General comments

It was evident that many candidates were well prepared for the examination and were able to show their understanding of the syllabus objectives by producing a range of clear and well thought out solutions. There appeared to be sufficient timing to fully demonstrate knowledge and most candidates had sufficient room on the script, making use of the additional sheet when needed.

Comments on specific questions

Question 1

Most candidates were able to write correctly the given equation in terms of $\sin\theta$ and $\cos\theta$ and hence obtain an equation in terms of $\tan\theta$. Many correct solutions given to the correct level of accuracy were seen, however a few candidates truncated their answers to 78.6° and 258.6°. There were attempts by some candidates to square each side of the given equation and use the relevant trigonometric identities, but a common error was to omit to square 5 also.

Question 2

Candidates were able to use either of the two valid method for obtaining the solutions of the given equation. Many chose to attempt the solution of two linear equations usually with success although some candidates made errors when forming a linear equation where the signs of $4x$ and x were different. Candidates who chose to square each side of the given equation to obtain a quadratic equation appeared to be less successful in obtaining the correct solutions, due to sign errors in factorisation.

Candidates should read each question carefully. In this question, a common error was to have $p > q$, which then had a marked impact on the final part of the question.

It was evident that many candidates did not know how to deal with $|p-2|-|q-1|$. A number of candidates substituted in their values for p and for q and then squared each bracket to obtain an incorrect value. Other candidates did not retain the exact value required, again reinforcing that questions should be read carefully.

Question 3

It was essential that candidates realised that the relationship $y = Ax^k$ should be written as $\ln y = k\ln x + \ln A$.

It was expected that the coordinates of the given points be used to either find the gradient of the line and equate it to k, or use the coordinates of the given points to form two simultaneous equations which could be solved to obtain both k and $\ln A$.

There were a few common errors, the first arising from not writing down the correct logarithmic equation. The most common error occurred when candidates used the logarithms of the given coordinates in equations such as $\ln 2.87 = k\ln 0.56 + \ln A$ and/or $\ln 3.47 = k\ln 0.81 + \ln A$.

Some candidates found the gradient and successfully identified it as being the value of k, but then used equations such as $\ln 2.87 = k\ln 0.56 + \ln A$ and/or $\ln 3.47 = k\ln 0.81 + \ln A$ in an attempt to find the value of $\ln A$ and hence A.

An alternative method using the equations $e^{3.47} = Ae^{0.81k}$ and $e^{2.87} = Ae^{0.56k}$ to find the values of A and k was also acceptable but attempted by very few candidates.

Question 4

(a) This was by far the question for which most candidates had the greatest success with many getting full marks in both parts. It was expected that the value of a be found by using the factor theorem. It should be noted that if an awkward fraction is obtained in a question such as this, that the calculations be checked in case an arithmetic slip has been made. This is especially important if a remainder of zero is not obtained from subsequent work. With most candidates obtaining a value of $a = 6$, algebraic long division usually provided a correct quotient, with some candidates finding this quadratic factor by observation which was acceptable.

The demand of the question was to factorise the polynomial completely. Some candidates did not attempt to factorise the quadratic factor.

It should be noted that if synthetic division is used, then the final results should be checked and adapted so that the factors obtained match the form of the polynomial.

(b) Many correct solutions were seen with candidates recognising, through the use of the word 'Hence', that the factors from **part (a)** needed to be used. Most candidates that did this, realised that they could equate x from **part (a)** to e^{4y} and use the fact that an exponential term in this form is always positive to obtain the equation $3e^{4y} = 2$ which could then be solved.

Question 5

(a) It was essential that candidates realised that they need to differentiate the given equation and equate to zero in order to make a start. Many candidates did differentiate correctly to obtain $\ln(4x+1) + \frac{4x}{4x+1} - 3 = 0$ or equivalent but were unable to rearrange their equation to obtain the given result. Multiplication throughout by $4x+1$ was an initial step and simplification leading to $(4x+1)\ln(4x+1) = 8x+3$ would then lead to the given answer. There were other variations of this which were acceptable, but few completely correct responses were seen.

(b) The method expected was for candidates to form a new function such as $p(x) = \frac{2x+0.75}{\ln(4x+1)} - 0.25 - x$ and find $p(1.8)$ and $p(1.9)$, showing a change of sign between the two values. Some candidates formed the function $x - \left(\frac{2x+0.75}{\ln(4x+1)} - 0.25\right)$ which gave a similar response provided no sign errors were made. Other methods were acceptable but needed to be accompanied by a full and detailed explanation.

(c) Most candidates were able to gain full marks showing sufficient iterations to the required level of accuracy. Errors included not giving the final answer to the required level of accuracy.

Question 6

The most successful method used involved finding two separate areas and then combining them. Most candidates were able to make a reasonable attempt at integration of each of the separate functions, together with substitution of limits. There were some errors with signs and coefficients, with some candidates not simplifying the logarithms sufficiently. The main error arose when combining the two areas. It was essential that the magnitude of $\int_0^4 \left(3e^{-x} - 3\right) dx$ be considered correctly. Candidates who considered the area as a difference of the integrals of the two functions initially were usually successful.

Question 7

(a) An unstructured part which required candidates to use their problem-solving skills. Some very good responses were seen, gaining most of the available marks. Most candidates made an attempt to differentiate each of the parametric equations and use a correct method to find the gradient function of the curve. There were sign errors in some cases and occasionally the double angle was miscopied as $\sin\theta$ rather than $\sin 2\theta$. Equating the gradient function to 2 and the use of the double angle formula meant that many candidates were able to find a value for $\sin\theta$. Many candidates obtained the correct value of $\sin\theta = -\frac{1}{6}$, but did not then go on to find the correct value for θ, not taking into consideration that $\pi \le \theta \le \frac{3\pi}{2}$.

(b) Fewer correct methods and subsequent correct solutions were seen in this part. It was essential to use the equation of the straight line passing through the point *Q* together with the parametric equations of the curve in order to obtain a quadratic equation in terms of $\sin\theta$, a solution of which would give the value of at the point Q. The value of $\sin\theta$ at the point *Q* is negative as given by the information given in the question $\pi \le \theta \le \frac{3\pi}{2}$. Some candidates did not use this fact and used a positive value for $\sin\theta$. Use of the expression for the gradient function of the curve from **part (a)** was then needed to calculate the gradient of the curve at the point *Q*.

MATHEMATICS

Paper 9709/23
Paper 2 Pure Mathematics 2

Key messages

Candidates should be reminded that the mark allocation of a question relates to the amount of work that needs to be shown. A question that relies purely on the use of a calculator would be unlikely to attract 4 marks. Candidates should also be familiar with the rubric on the front of the paper which states 'You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator'. This was especially relevant for **Question 3**.

Candidates should also ensure that they read each question carefully and ensure that they have fully met the demands of the question and that their answers are in exact form if required or are to the correct level of accuracy as detailed in the question itself or otherwise in the rubric on the front of the question paper.

It is also essential that solutions are shown clearly and with sufficient detail.

General comments

It was evident that many candidates were well prepared for the examination and were able to produce a range of clear and well thought out solutions. It was also evident that there were some candidates who were not so well prepared as evidenced by incomplete or questions that were not attempted. There appeared to be no timing issues and most candidates had sufficient room on the script, making use of the additional sheet when needed.

Comments on specific questions

Question 1

Most candidates were expected to use either one of two methods to solve the given inequality. Either method used was equally successful, although those candidates who attempted to use two linear equations to find the two critical values tended to make fewer sign errors than those candidates forming a quadratic equation from squaring each side of the inequality.

Question 2

Few correct solutions were seen, with candidates being unable to manipulate logarithms correctly. It was expected that one of the initial steps in the solution should be $\ln 14 + \ln e^{-2x} = \ln 5^{x+1}$. However, many candidates were unable to manipulate the left-hand side of the equation correctly. Use of the power rule for the right-hand side of the equation was usually correct. General manipulation of correct terms was poor, leading to an incorrect solution.

Question 3

A significant number of candidates viewed this question as purely a calculator exercise and, as a result, were unable to gain any marks. It was expected that candidates make use of the identity $\sec^2\theta = 1 + \tan^2\theta$, and obtain the result that $\tan\theta = 4$. Candidates were awarded no marks for simply finding the actual value of θ from their calculator. The next expected step was that the compound angle formula

$\tan\left(\theta+\frac{\pi}{4}\right)=\dfrac{\tan\theta+\tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}$ be used with a substitution for $\tan\theta=4$ and $\tan\frac{\pi}{4}=1$ to obtain an exact value for $\tan\left(\theta+\frac{\pi}{4}\right)$. Unfortunately, many candidates used their calculator with the value of θ previously obtained.

The question was intended to test the candidates' knowledge of the use and application of trigonometric identities and compound angle, not the ability to use a calculator, which is not a syllabus objective.

Question 4

(a) Most candidates were able to make a reasonable sketch of the graph of $y=e^{-\frac{1}{2}x}$, however the graph of $y=x^5$ proved to be more difficult to sketch for some candidates. The question required that candidates were to show that there was only one real root for the given equation. This meant that there was an expectation that reference be made to there being only one point of intersection in order to obtain both marks, provided that both sketches were correct. Many candidates did not make this final step.

(b) Most candidates were able to gain marks from the correct use of the iteration process, obtaining iterations correct to 6 significant figures, however some candidates either truncated their answer or did not give the final result to the correct level of accuracy.

Question 5

Most candidates obtained some marks by recognising the need for implicit differentiation and that the term $4e^{2x}y$ needed to be differentiated as a product. Some candidates made errors in the simplification of their derivative and so did not obtain a correct result. Another common error was to omit the differentiation of the constant term 21.

Question 6

(a) It was pleasing to see that there were many completely correct solutions, usually using algebraic long division, which clearly showed both the quotient and the remainder. Some candidates chose to use synthetic division, which was acceptable provided an adjustment to division by $\frac{3}{4}$ was made when dealing with the quotient. Unfortunately, some candidates did not do this and gave an incorrect quotient of $12x^2+8x$.

(b) Most candidates realised that they needed to make use of the response to **part (a)**, obtaining an integrand of $2+\dfrac{2}{4x-3}$. The integration of this expression was completed well by candidates who were able to get this far, but it should be noted that an exact form was required. Candidates who had an incorrect quotient (usually from use of synthetic division) were less successful as their integrand contained an extra term of $6x^2$.

Question 7

(a) Most candidates realised that differentiation of a quotient was needed and were able to obtain a correct derivative. Problems arose when candidates did not realise that at the point *A*, the value of *x* is 1, and it was this value that was needed to be used in the derivative.

(b) Few correct responses were seen, with many candidates using the equation of the curve rather than the equation of gradient function. It was expected that a substitution of $x=3$ and $x=3.1$ be made into the gradient function, showing a change of sign which implied that the point *B* was a maximum point.

(c) Some candidates chose to make use of two separate trapezia to calculate an approximate area for the shaded region. Of the candidates who attempted use of the trapezium rule, most were successful, identifying the correct width for each trapezium, which has in the past proved problematic. It is essential that candidates ensure that their final answer is in the required form, with some not rounding to the required 2 decimal places.

Question 8

(a) Few correct solutions were seen. It was intended that the appropriate double angle formula be used for both terms for $f(\theta)$. This would result in $f(\theta) = 8\cos 2\theta + 6\sin 2\theta + 8$, the first two terms of which could be written in the form $R\cos(2\theta - \alpha)$. Many candidates did not make the link that a double angle was used in the required form and that there were no double angles in the original function. Candidates should read the questions carefully and try to take note of facts such as those mentioned.

(b) Of the few correct responses obtained in **part (a)**, few correct responses to **part (b)** were obtained, with errors usually being due to incorrect application of the order of steps in the solution of the trigonometric equation $\cos(2\theta - 0.6435) = 0.9$. Candidates who had incorrect values for R could have noticed possible errors when an equation that could be solved was obtained.

MATHEMATICS

Paper 9709/31
Paper 3 Pure Mathematics 3

Key messages

- Show clear working.
- Be prepared to answer questions on all topics on the syllabus.
- Write clearly and do not overwrite one solution with another – the result can be very difficult to read once scanned.
- Check your algebra and arithmetic carefully, and use brackets correctly.
- If a question asks you to obtain a given answer then take particular care to show full working.
- If a question asks for an exact answer then decimal working is not appropriate.
- Make sure that you answer the question.

General comments

A wide range of scores were seen from candidates however the median mark for the paper was low, with several candidates offering no response for some topics. Stronger candidates had prepared all the topics and showed a good understanding of the work covered.

Candidates gave good responses to those questions involving inequalities (**Question 1(b)**), trigonometric equations (**Question 4**), iterations (**Question 7(c)**), and partial fractions (**Question 10(a)**).

Comments on specific questions

Question 1

(a) There were several correct sketches. A few sketches did not consist of straight lines, but the most common error was to have the vertex on the vertical axis.

(b) A minority of candidates were able to use their sketch to conclude what the solution should look like. They then solved an appropriate linear equation or inequality and reached the correct solution. The most common approach was to square the given inequality to obtain a quadratic equation. This gave two critical values, and most candidates used both of them in their solution.

Question 2

Most of the candidates who attempted this question obtained a correct sketch. On some diagrams the circle was not in the correct position, and some candidates showed $\operatorname{Re} z = -2$ as a horizontal line.

Question 3

Candidates who were seemingly familiar with the rules of logarithms and indices usually scored well in this question. The majority of candidates started by attempting to take logs of both sides. The term $(3x-1)\ln 2$ was often seen without brackets, although many candidates recovered from this in their later work. The most common error was to rewrite 5×3^{-x} as 15^{-x}. Several candidates made errors in their attempts to find the value of *x*; some were sign slips when moving terms, some were misapplications of the rules of logarithms. The alternative approach of starting by applying the rules of indices was less common. Candidates often made an initial step such as $2^{3x-1} = \dfrac{2^{3x}}{2}$ but went no further.

Question 4

Most candidates recognised that they needed to start with the expansion of $\tan(x+45°)$, and many went on to obtain a correct quadratic in $\tan x$. There were several fully correct solutions. Most errors were due to slips in rearranging formulae, or writing $2\cot x$ as $\frac{1}{2\tan x}$. Candidates who attempted to use the expansions of $\cos(x+45°)$ and $\sin(x+45°)$ often made a correct start, but few got as far as simplifying their equation.

Question 5

(a) This topic was a challenge for most candidates. Those who knew how to combine the complex numbers usually scored well.

(b) Very few candidates showed understanding of what was required here. There were several blank responses, some answers that gave a negative real value, and some that appeared to be guesses.

Question 6

(a) Most candidates recognised that they needed to use the formula for $\cos 2A$ and there were some fully correct proofs of the identity. Common errors came from slips in the algebra, in particular errors when squaring $(2\cos^2\theta - 1)$.

(b) Only a minority of candidates used **part (a)** to show that this equation is equivalent to $8\cos^4\theta = 7$. Those who did usually obtained the correct answers.

Question 7

(a) There were several correct attempts to use the quotient or product rule to differentiate *y*. Errors in the signs or coefficients often prevented candidates from obtaining the given answer.

(b) There were several blank responses to this part of the question. Those candidates who understood what was required often gave a correct response. Candidates who were considering an equation of the form $\mathrm{f}(\alpha) = \alpha$ did not always present their solution clearly, and were not always able to establish a change in sign with their values.

(c) There were several blank responses to this part. If candidates attempted to use the iterative formula using radians they usually obtained the correct answer. The question specifies the level of accuracy required, so it is important that candidates work to this degree of accuracy.

Question 8

(a) The candidates who made a correct start to this question usually separated the variables correctly. The integral in *x* was often completed correctly. Some candidates recognised the correct form for the integral in *y*, but there were several errors in dealing with the coefficient. Those candidates who had a constant of integration usually used the initial conditions correctly to evaluate the constant.

(b) This part proved a challenge and there were many blank responses, often due to an incomplete response in **part (a)**. Exact working was not required, but several candidates succeeded in continuing with exact values. For the final answer a decimal value was required.

Question 9

(a) The majority of candidates attempted to differentiate using the product rule. There were some sign errors and some errors in the coefficients. Candidates who got as far as $1 + \frac{1}{3}(3 - x) = 0$ did not always reach the correct conclusion. Some candidates did not go on to find the value of *y*, or did not give an exact value.

(b) Most of the candidates who attempted this part of the question recognised the need to use integration by parts. Some used the function as printed in the question, and some multiplied out the bracket before integrating. The form of the answer was often correct, but with errors in the coefficients. Candidates who completed the integration almost always went on to use the correct limits.

Question 10

(a) The majority of those who attempted a solution used a correct form for the partial fractions and usually reached a correct conclusion. However, there were some candidates who offered no attempt. Most errors were due to slips in the algebra and arithmetic, often due to omission of brackets.

(b) This part proved challenging with many candidates offering no attempt to use the binomial expansion. A few candidates reached the correct conclusion but there were several errors in using the expansion and in simplifying the results. The expansion of $(1+x)^{-1}$ was often correct, but there were errors in taking out the factor of 2 in the other two expansions.

Question 11

This question was challenging for many candidates and there was a high proportion of blank responses to **parts (b)** and **(c)**.

(a) Several candidates were able to interpret the diagram and make some progress with this part of the question.

(b) In order to access this part of the question, candidates did need to have answers to **part (a)**. They were able to score 2 marks for the correct use of incorrect vectors. There was evidence of correct use of the scalar product by some candidates.

(c) The majority of candidates offered no solution to this part of the question. A small number of candidates demonstrated an understanding of what they needed to do, and they made some progress with the task.

MATHEMATICS

Paper 9709/32
Paper 3 Pure Mathematics 3

Key messages

- Show clear working.
- Be prepared to answer questions on all topics on the syllabus.
- Write clearly and do not overwrite one solution with another – the result can be very difficult to read once scanned.
- Check your algebra and arithmetic carefully, and use brackets correctly.
- If a question asks you to obtain a given answer then take particular care to show full working. You should obtain the conclusion as printed on the question paper.
- If a question asks for an exact answer then decimal working is not appropriate.
- Make sure that you answer the question.

General comments

The majority of the candidates for this paper found opportunities to demonstrate their skills. Most candidates offered responses to most questions, and they showed a good understanding of the topics covered. Notation was an issue, with many candidates losing marks in **Question 1** and in **Question 10(a)** through omission of essential brackets. There were also many instances in all questions of candidates making errors in simple algebra and arithmetic and losing marks after demonstrating a good understanding of the topic in their initial work.

Some questions led to errors if candidates could not see how to solve the problem. In **Question 2**, many candidates expected the quadratic factor to have real roots. In **Question 6(b)** only a few candidates recalled the basic result $\sin\theta = \sqrt{1-\cos^2\theta}$.

Candidates gave good responses to those questions involving partial fractions (**Question 10(a)**) and an iterative formula (**Question 9(c)**). Candidates who followed the expected trigonometric method scored well in **Question 4**.

Comments on specific questions

Question 1

Candidates that were seemingly familiar with the rules of logarithms and indices usually scored well in this question. The majority of candidates started by attempting to take logs of both sides. The term $(3x-1)\ln 2$ was often seen without brackets, although many candidates recovered from this in their later work. The most common error was to rewrite $5\times 3^{1-x}$ as 15^{1-x}. Many candidates made errors in their attempts to find the value of *x*; some errors were sign slips when moving terms and some were misapplications of the rules of logarithms.
The alternative approach of starting by applying the rules of indices was popular. Candidates often got as far as $3^x 8^x = 30$ or $3^x 2^{3x} = 30$ but did not simplify this further.

Question 2

(a) The majority of candidates used the factor theorem correctly, and they usually obtained the correct solution. There were several sign errors and arithmetic errors in the working. A minority of candidates substituted $\frac{3}{2}$ or $-\frac{2}{3}$ in place of $-\frac{3}{2}$.

Some candidates preferred to use algebraic division, which was more complicated than necessary here but assisted in **part (b)**. The division often introduced errors.

(b) Only a minority of candidates scored full marks in this part. Many attempted to apply the quadratic formula to find the roots of a cubic equation. Another common false approach was to rewrite the polynomial as $x^2(2x-1)=9$ and equate each factor in turn to 9.

Those candidates who attempted to divide by $(2x+3)$ and obtain the correct quadratic factor often assumed that it should have real factors. A minority of candidates did use the quadratic formula to demonstrate that there were no real roots, and a small number completed the square to demonstrate that the quadratic factor was always positive. Very few candidates with the correct factors demonstrated an understanding that for a negative result they required one factor to be positive and the other negative.

Question 3

There were several possible approaches to this question. The approach that produced the simplest solutions was to start by rewriting the equation as $y = 2\sin^2 x\cos x$ before differentiating. Occasionally candidates did further manipulation before differentiating, obtaining $y = 2\cos x - 2\cos^3 x$ and then applying the chain rule. Candidates who started by applying the product rule to the original equation often assumed that $\frac{\mathrm{d}}{\mathrm{d}x}\sin 2x = \cos 2x.$ If they applied the double angle formulae correctly, they could still obtain the subsequent method marks.

Candidates with a correct approach often made slips in the algebra and arithmetic when solving their equation. A few candidates gave an answer in degrees, which is inappropriate having used calculus and the question does specify an interval in radians.

Question 4

(a) Many candidates gave fully correct answers with minimal working. Where working was shown, it was often apparent that the candidates had used an incorrect expansion of $R\cos(x+\alpha).$ There were several scripts with $\cos\alpha = 4,$ and many with $\tan\alpha = -\frac{1}{4}$ or $\tan\alpha = \pm 4.$ The question specifies that the angle should be given correct to 2 decimal places, but writing an answer to 1 decimal place or more than 2 decimal places were common errors.

(b) Most candidates recognised the link between the two parts of the question and there were several fully correct solutions. Some candidates gave final answers to more accuracy than their working justified.

The most common errors were in adjusting from an expression in x in **part (a)** to an equation in $2x$ in **part (b)**. Some candidates overlooked the 2, and some thought that they should be working with $\cos(2x+2\alpha)$.

A minority of candidates ignored the result from **part (a)** and used the double angle formulae to form an equation in $\sin x$ and $\cos x.$ Very few of these solutions made progress, but a small number of candidates did obtain a correct equation in $\tan x.$

Question 5

(a) The simplest method here was to complete the square to obtain $(z-3\mathrm{i})^2-3=0$ and hence the roots. Candidates who applied the quadratic formula correctly were usually successful. The popular alternative of substituting $z=x+\mathrm{i}y$ was more complicated, and several candidates were unable to continue once they had formed equations by comparing the real and imaginary parts. The most common error was to use 6 in place of 6i when substituting values into the quadratic formula. Some candidates incorrectly assumed that the roots would be $z=x+\mathrm{i}y$ and $z=x-\mathrm{i}y$.

(b) There were many fully correct diagrams. Some diagrams were clearly not symmetrical, and some candidates had $\sqrt{1}, \sqrt{2}, \sqrt{3}$ … in place of a linear scale on the real axis.

(c) Most candidates understood how to find the modulus of a complex number. It was less common to see two correctly stated arguments, with one often being in the incorrect quadrant despite the candidate having a correct diagram.

(d) The most successful approach here was to demonstrate that all three sides of the triangle are equal in length. Some candidates claimed that all three angles were equal, but often gave no justification for this. There was also a lot of confusion between 'isosceles' and 'equilateral', with several candidates stating that the triangle was equilateral because it had two equal sides.

Question 6

(a) The majority of candidates demonstrated knowledge of how to use the scalar product and a lot of correct work was seen. The question asks for the value of the cosine of the angle, but many candidates stated the size of the angle without ever stating the cosine. The other common error was to use an incorrect pair of vectors. To find the angle *BAC*, candidates should be using $\overrightarrow{AB}$ and $\overrightarrow{AC}$ or $\overrightarrow{BA}$ and $\overrightarrow{CA}$. It was common to find candidates using one vector from each pair, or the vector $\overrightarrow{BC}$. Some candidates used the position vectors rather than vectors representing sides of the triangle.

(b) Although the question asks for an exact answer, many candidates could not see how to obtain this and gave a decimal answer instead. The majority of candidates did try to apply the correct formula for the area, with a few getting as far as an expression involving $\sin\left(\cos^{-1}\frac{1}{3}\right)$. A minority of candidates remembered how to find an exact value for this.
A small number of candidates attempted to use the vector product, but no correct solutions using this method were seen.

Question 7

(a) Although this part of the question is only worth one mark, many candidates looked beyond the simple demand and thought that they needed to start by trying to solve the differential equation. Most candidates who tried to differentiate $\cot^2\theta$ made use of the given information, with a minority losing the factor of 2. Some candidates preferred to differentiate $\frac{\cos^2\theta}{\sin^2\theta}$ as a quotient; there were several errors in this process, but a few correct solutions.

(b) Most candidates who understood that the essential first step was to separate the variables usually scored at least 2 marks. Several candidates recognised how to use the result from **part (a)**, although often with a sign error. Some candidates recognised the form $\frac{\cos\theta}{\sin^3\theta}$ as a multiple of the derivative of $\sin^{-2}\theta$. The integration of $\frac{\tan^2\theta}{\sin^2\theta}$ proved to be more challenging, with relatively few candidates recognising this as $\sec^2\theta$. Some candidates attempted to use $\theta = \frac{1}{6}\pi$ without first using the given information to evaluate a constant of integration.

Question 8

(a) The most common value stated for *a* was zero, although the diagram did clearly show that that is not correct. The incorrect answer 32400 was accepted because it does demonstrate correct thinking. The majority of candidates gave an incorrect answer after writing working; they seemingly did not realise that they were looking for a positive value of $\sqrt{x}$ for which $\sin\sqrt{x} = 0$. Some deduced correctly that $\sqrt{a} = \pi$ but then wrongly concluded that $a = \sqrt{\pi}$.

(b) Those candidates who followed the correct process for integration by substitution often scored several marks here. The most common errors occurred in the course of substituting for d*x*, with several candidates having a factor of $\sqrt{u}$ or $\frac{1}{u}$ in their integral. Candidates using integration by parts to integrate the correct function usually obtained a correct answer and were able to go on to use their answer from **part (a)** to obtain an answer. The question asks for the answer in exact form, but several candidates gave a decimal answer.

Question 9

(a) Those candidates who recognised that the angle $AOC = \pi - 2\theta$ usually gave a correct expression for the shaded area. Some candidates found it challenging when using this to obtain the given answer; some answers contained $\sin(\pi - 2\theta)$, and some claimed that $\sin(\pi - 2\theta) = \sin\pi - \sin 2\theta$.

(b) There were several fully correct solutions, particularly from candidates considering $f(\theta) = 0$. Some of the candidates using $f(\theta) = \theta$ did not appreciate what this method requires, with many expecting to achieve a sign change. There were several errors due to candidates mistyping the formula on their calculator, or working in degrees.

(c) Many of the candidates working in radians gave a fully correct solution. Some did not work to the levels of accuracy required, and some omitted to state a final conclusion. Some did not complete enough iterations to obtain two that were equal to 3 decimal places.

Question 10

(a) The majority of candidates started with a correct form for the partial fractions and there were many fully correct solutions. There were several errors due to candidates omitting brackets from the expected form of $A(2+x^2)+(Bx+C)(1+x)$ and instead working with $A(2+x^2)+Bx+C(1+x)$. There were also a number of arithmetic errors made.

(b) Most candidates obtained $A\ln(1+x)$, although the coefficient sometimes appeared as $\frac{1}{A}$. The second integral proved to be more challenging; many candidates ignored the numerator and assumed that the answer would be of the form $p\tan^{-1}q$, or they recognised that it should be a logarithm but they retained *x* or $\frac{1}{x}$ as a coefficient. There were some errors in combining the terms, but most candidates who integrated correctly did go on to obtain the correct answer.

MATHEMATICS

Paper 9709/33
Paper 3 Pure Mathematics 3

Key messages

Candidates need to:
- know that in preparing for an examination it is vitally important that they fully understand the rubric before the examination and prepare for their examination accordingly, see **Question 6**
- know how to integrate $(a + bx^2)^{-1}$, as in **Question 11(b)**
- know how to apply the chain rule in differentiation, as in **Question 4**
- know what is meant by a position vector, and the origin in relation to other points, as in **Question 9(a)**
- realise that integration involving a negative power **does not** necessarily lead to a ln term
- know that $e^{(a+b)}$ is $e^a e^b$ **not** $e^a + e^b$.

General comments

The standard of work on this paper was variable. Some candidates presented very good responses to all questions, whilst other candidates had difficulty in tackling some of the examination topics. Among the more able candidates there were a few of outstanding ability, for example those able to solve **Question 6** by equating real and imaginary parts following the substitution of $z = x + iy$. However, it was also disappointing to see many candidates in the same question failing to show their working, instead using a calculator, since this has been discussed in detail in several of the recent reports.

Another area that caused problems for many candidates was omitting to answer the question as set, for example giving just a single term (**Question 2**), or a decimal instead of an exact form (**Question 3**).

Unfortunately, some candidates' work remains very untidy and difficult to read. Candidates should write clearly, with letters and symbols of a reasonable size, and work logically down the page to help the reader to understand their solutions.

Comments on specific questions

Question 1

Most candidates were able to manipulate the ln terms and most reached a three-term quadratic which they were then able to solve. There were some algebraic slips along the way, but overall the basic method was sound. The majority of candidates were not awarded the final answer mark since they did not check their answers in the logarithmic expressions and reject the negative root. A significant number also gave the answer to the wrong number of decimal places or left both roots in exact form instead of to three decimal places as specified.

Question 2

Candidates who approached this question by using the product $(1+2x)^{\frac{1}{2}}.(1-2x)^{-\frac{1}{2}}$ and then forming two expansions generally did well. Most showed clear working when multiplying their expansions, although a few chose to add the expansions instead. Some candidates expanded and then either showed the expansions as a fraction or attempted some sort of division. Several other approaches were seen, with varying degrees of success; $(1+2x).\left(1-4x^2\right)^{-\frac{1}{2}}$ worked particularly well. Those candidates who attempted a Maclaurin series expansion had little success. A number of candidates did not read the question carefully enough and thought that they had to give only the coefficient of the x^2 term.

Question 3

This question was answered very well by the vast majority of candidates who integrated by parts effectively. A number of candidates were confused between the integral and derivative of tan *x,* and this limited their progress. Otherwise, the most common error was a sign error in the ln cos *x* term.

Question 4

Most candidates were awarded the first **B1** mark, for differentiating either *x* or *y* correctly, with the vast majority achieving success with $\frac{\mathrm{d}x}{\mathrm{d}t}$. About half of the candidates did not use the chain rule correctly in attempting $\frac{\mathrm{d}y}{\mathrm{d}t}$ and so the majority of these candidates gave $\frac{1}{\sin(2t)}$ as their answer. Nearly all the candidates were able to use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}.\frac{\mathrm{d}t}{\mathrm{d}x}$ correctly, so were awarded the first **M1** even if their incorrect differentiation in $\frac{\mathrm{d}y}{\mathrm{d}t}$ meant they could not achieve the **A1** at this stage. The majority of candidates whose solutions were correct at this point went on to gain **M1** for using trigonometric formulae correctly, and most of these candidates also gained the **A1.** Some did not because they could not simplify the trigonometric expressions after expanding them. Candidates with an error in $\frac{\mathrm{d}y}{\mathrm{d}t}$ were generally unable to proceed further since they could not show an expansion of cos 2*t*. In a question such as this, with the answer given, it is essential to show all steps in the argument in a convincing manner to be awarded all the marks.

Question 5

(a) Many candidates produced fully correct diagrams that were awarded full marks. Of the remaining candidates, most recognised the need for a circle of radius 2. Locating the centre proved more troublesome: it was seen on the axes in all four quadrants as well as in some other positions. Most candidates were successful in identifying and drawing the line $y = 1$ regardless of their performance in other parts of the question. Finding the correct region was rarely problematic.

(b) This proved challenging even for those with the required correct diagram, and many candidates made no attempt, seemingly being unable to locate the point *P* that would give the greatest argument. Successful candidates used a variety of different geometrical approaches. The most common approach was to find the angle that *OP* makes with the negative real axis and then subtract that value from π. In many cases, candidates' solutions were unclear, and it was difficult to determine which angle(s) they were attempting to find. Centres should advise candidates to label diagrams and show working clearly, for example using three-letter notation to describe angles.

Question 6

In this question, the quadratic equation with complex coefficients apparently looked unfamiliar to many candidates. In their approaches to solving it, candidates largely split into two groups, with about 30% attempting to replace *z* with $x + \mathrm{i}y$, expand, then set real and imaginary parts to zero. At this point the majority found the algebra required to complete the working to be too challenging: all but a few exceptional candidates made no further progress.
The remaining 70% of candidates chose the expected route of substituting coefficients into the quadratic formula to find the value of *z*. Although this was generally done well, there were often errors in identifying *b* and sometimes *c*, with sign errors for *b* (and –*b*), or *c* given as 1 instead of i.
Multiplication of terms was generally well done with candidates using $\mathrm{i}^2 = -1$ correctly to find the roots in fraction form successfully. Some candidates then found the roots using a calculator so could not be awarded full marks. The rubric demands that candidates should show all their working, and this is an example of a question where that applies. To be awarded the final **M1**, it was necessary to demonstrate multiplication of the numerator and the denominator by the conjugate of the denominator and full expansion of the brackets. Where this was shown, it was generally accurate, and many candidates achieved full marks.
Note that this question required candidates to demonstrate their knowledge of complex numbers in the following ways: using $\mathrm{i}^2 = -1$ twice, $(-9)^{\frac{1}{2}} = \pm 3\mathrm{i}$, multiplying numerator and denominator by the conjugate of

the denominator, and evaluating them fully. Instead, the calculator was often employed to find the roots at the fraction stage, or even to solve the quadratic equation by inputting coefficients then writing down the two final answers immediately. In the former case, a candidate could be awarded a maximum of 4 marks while in the latter case they could only be awarded 2 marks.

Question 7

(a) Despite the relatively unfamiliar form, most candidates recognised the correct approach and rearranged the equation to $4\cos x - \sin x = \sqrt{5}$. Many successfully found R and α, although errors in comparing the coefficients often led to incorrect working. Using $R\sin\alpha = -1$ instead of 1 when evaluating α meant that the accuracy mark could not be awarded. Occasionally $\tan\alpha = 4$ was seen, but this was less common.

(b) Most candidates who were successful in **(a)** were able to equate correctly their expression from **(a)** to $\sqrt{5}$, rearrange and use arc cos to find a principal value for $2x + 14.04°$. Although many candidates did then find both correct values of x, it was fairly common to see x instead of $2x$, hence no correct angles could be found. In some cases, only the first correct angle was found and then errors were made in calculating the second angle.

Question 8

(a) Most candidates achieved at least two marks on this question and the majority of candidates used the quotient rule to differentiate. Few errors were seen, but occasionally the terms were reversed in the numerator, or the denominator was completely omitted. Those candidates who used the product rule often omitted e^x when attempting to differentiate $(e^x - 1)^{-1}$, while a few candidates had an incorrect sign in the formula. Many candidates succeeded in obtaining the given answer for p, often via an unnecessarily long route. Occasionally this last mark could not be awarded due to incomplete working, or showing insufficient steps. When an answer is given, it is essential to show all steps in a complete argument that leads to the answer.

(b) Despite being given the result in **(a)**, a number of candidates successfully evaluated different relevant expressions. Candidates are reminded of the need for a concluding statement in the sign change method.

(c) Almost all candidates answered this very well, but a few worked only to three decimal places so could not be awarded all three marks.

Question 9

(a) Many candidates did not seem well prepared to tackle this vector question. A large number of marks were lost throughout this question through carelessness and inaccuracies. Common errors arose through using $\overrightarrow{OA} - \overrightarrow{OB}$ for vector $\overrightarrow{AB}$, confusing the position vector of the midpoint M of AC, $\overrightarrow{OM}$, with the vector $\overrightarrow{AM}$, or misunderstanding what the statement $\overrightarrow{BN} = 2\overrightarrow{NC}$ meant about the position of N.

(b) The vast majority of candidates used the correct method, but many of them could not be awarded the second mark as they were working from one or even two errors from **(a)**. Unfortunately, many candidates who had correct vectors $\overrightarrow{OM}$ and $\overrightarrow{ON}$ in **(a)** could only be awarded one mark here because they omitted the **r** = … in their vector equation of the line. A small number of candidates produced 'vector equations' both here and in **(c)** in which there was no parameter, or the parameter was associated with the wrong vector.

(c) A large number of candidates had the vector equation for the line AB correct and managed to find a solution to the simultaneous equations, so could be awarded at least half of the marks available. However, as in **(b)**, many were working from errors in earlier parts of the question so only about a third of candidates achieved all the marks in this section. Candidates who used two line equations with the same parameter could not be awarded the **M1** mark.

Question 10

(a) The majority of candidates gave correct values for a and b.

(b) This question proved very challenging for many candidates who were unable to separate the variables correctly. In most cases this meant that none of the remaining marks were available. A significant number of candidates did not even attempt this part. Of the candidates who were able to gain the first three marks, many did not achieve all the remaining marks for a variety of reasons:

- not realising that a constant had to be evaluated
- having an incorrect sign after integrating $(a - bv)^{-1}$
- using values other than $t = 0$ and $V = 0$ to determine the arbitrary constant c
- not stating a solution in V and t
- not realising that, for an answer in context, an actual time (40.5 minutes) was required as opposed to the exact value, $100\ln\left(\frac{3}{2}\right)$.

(c) Only about 10% of candidates produced a correct expression for V in terms of t since most candidates carried forward errors from **(b)**. Some candidates who did achieve full marks in **(b)** could not rearrange the solution successfully to make V the subject. However, some were able to recover sufficiently to be awarded the FT mark for a correct conclusion about the behaviour of V as t increased.

Question 11

(a) Almost all the candidates knew the required form for the partial fractions and were able to obtain the correct values for their constants using standard techniques. A very small number of candidates either started with $B(1 + 3x^2)^{-1}$ instead of $(Bx + C)(1 + 3x^2)^{-1}$ or had numerical errors in determining the values of the three constants.

(b) The $A(3 - x)^{-1}$ term was usually integrated correctly to $-A\ln(3 - x)$, although sometimes the negative sign was missing. The main difficulty was the placement of the $\sqrt{3}$ within the $\tan^{-1}$ term: it was often seen in the denominator as opposed to the numerator or it was missing completely or even replaced by 3. This standard integral is in the List of Formulae MF19 and only required $1 + 3x^2$ to be manipulated to $3\left(\left(\frac{1}{\sqrt{3}}\right)^2 + x^2\right)$ first. A significant number of candidates believed that $(1 + 3x^2)^{-1}$ integrated to a ln term. When differentiating $\ln(1 + 3x^2)$ using the chain rule, $6x$ would appear in the numerator, and this would have been a helpful check on the integration.

MATHEMATICS

Paper 9709/41
Paper 4 Mechanics

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. This was particularly noticeable in **Questions 3, 4(a)** and **6(a)**.
- In questions such as **Question 5** on this paper, where acceleration is given as a function of time, calculus must be used and it is not possible to apply the equations of constant acceleration.
- Non-exact numerical answers are required correct to 3 significant figures or angles correct to 1 decimal place as stated on the front of the question paper. Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

General comments

The questions were well answered by many candidates. **Questions 1** and **3(a)** were found to be the easiest questions whilst **Questions 5** and **6(b)** proved to be the most challenging.

In **Question 3(b)**, the angle θ was given exactly as $\sin\theta = 0.12$. There is no need to evaluate the angle in this case and problems such as this can often lead to inexact answers and so any approximation of the angle can lead to a loss of accuracy.

Comments on specific questions

Question 1

This proved to be a straightforward question for well-prepared candidates. In order to find P and Q, resolution of the forces in two directions was required. The majority of candidates resolved 'horizontally' and 'vertically'. Resolving horizontally gives P directly and then the value of P is substitute in the other resolved equation to find Q. Some candidates made P the subject of each equation to find Q first.

Question 2

This question involved the collision of two small spheres. The word collision should signify to candidates that this is a conservation of momentum problem, but only a minority of candidates used momentum in their responses. Of those who did, there was confusion about the speed of each particle after the collision, with many having the speed of sphere A being $2\,\text{ms}^{-1}$ greater than the speed of sphere B, which in the context of the scenario would indicate that A had passed through B in the collision, which is not possible. Some thought the difference in the speeds of the spheres after the collision being $2\,\text{ms}^{-1}$ meant that the speed of each sphere changed by $2\,\text{ms}^{-1}$. Most knew the formula to calculate kinetic energy.

Question 3

(a) The majority of candidates who attempted this question knew a force needed to be multiplied by the speed of $28\,\text{ms}^{-1}$. Most used the 1400 N force, with a minority using weight rather than resistance.

(b) The approach to this question was to find a driving force and equate this to $\frac{43500}{v}$. The driving force was attempted well with the occasional sign error seen. Once a driving force was obtained, all used $\frac{43500}{v}$ in some form to correctly find the constant speed.

(c) This question involves two connected bodies, the car and the trailer, moving up a hill. It is necessary to apply Newton's second law to either the car and the trailer and solve simultaneously, or to the system, to find acceleration, and then to the car or trailer to find the tension in the cable. Most candidates made good attempts at these equations with errors arising from the wrong signs being written, or the wrong resolved parts of forces or the 5000 N force being resolved. Some omitted weight components from their equations.

Question 4

(a) This is a typical question on this topic. The approach was to use Newton's second law parallel to the plane and resolve perpendicular to the plane. Then $F = \mu R$ is used to find an equation involving tension only and solving this to find the tension. Only a small number of full correct responses were seen. Typical errors seen were a combination of using Newton's second law perpendicular to the plane, or writing the normal reaction as $80\cos 18$ with no component of T, or omitting the weight component parallel to the plane.

(b) The request was for the distance travelled during the fourth second. This requires finding the difference between the distance travelled in 4 seconds and the distance travelled in 3 seconds. The distance travelled in 4 seconds was the most common answer, which did earn some credit.

Question 5

Candidates should be aware that when integrating, a constant of integration is required and that constant may not be zero.

(a) The question gave a function for the acceleration which needs to be integrated to find a velocity as a function of time. Those who did integrate and had a non-zero constant of integration were successful. Often, the constant of integration was omitted or stated to be zero when the initial conditions should lead to this constant to be –20. Those who obtained a correct expression for the velocity usually found the times when the particle was at rest and drew a correct graph.

(b) The request in **part (a)** should have signalled that for some of the motion the velocity of the particle was negative in the first 12 seconds. Candidates knew they had to integrate velocity to get displacement, but this was usually evaluated between $t = 0$ and $t = 12$ only, which gives the displacement from point O, 12 seconds after leaving O. To find the distance, it is necessary to evaluate the integral for 3 separate time intervals $0 \le t \le 2$, $2 \le t \le 10$ and $10 \le t \le 12$, then finally summing the modulus of these values.

Question 6

(a) This pulley question was well answered by many candidates. Occasionally some thought this was a typical pulley question that required an acceleration to be found even though the request stated that the particles were in limiting equilibrium, implying the particles were not moving.

(b) This part was found to be more challenging, with some candidates omitting it altogether and there were relatively few examples of candidates scoring full marks. The distance moved by each particle when they are at the same horizontal level needs to be found. When particle A descends x m, particle b moves x m up the slope, so it gains a height of $x\sin 30$ m, so they are at the same horizontal level when $1 - x = x\sin 30$ leading to $x = \frac{2}{3}$ m. Once this has been obtained, the method is to use conservation of mechanical energy so that the total loss in potential energy causes the total increase in kinetic energy.

MATHEMATICS

Paper 9709/42
Paper 4 Mechanics

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- When answering questions involving forces in equilibrium or Newton's Second Law or an energy approach, a complete force diagram can be helpful to ensure that all relevant terms are included in the equations formed, e.g. **Question 1**, **Question 2(a), Question 3** and **Question 5(a).**
- In questions with a given answer, where equations have to be solved in order to find that answer, candidates would be advised not to use an equation solver on their calculator since this does not explicitly show the given answer, e.g. **Question 7(b)**.

General comments

The questions in the paper were well attempted by many candidates although a wide range of marks was seen. **Questions 2(a), 4(a)**, **6(a), 6(b)** and **7(a)** were found to be the most accessible questions whilst **Questions 4(b)**, **6(c), 7(c)** and **7(d)** proved to be the most demanding for candidates.

Comments on specific questions

Question 1

In this question candidates first had to find the work done by the cyclist and then use a work energy equation in order to calculate the mass. Although a proportion of candidates did so successfully, rather more assumed that the resistance force was constant, which it was not necessarily the case. Such candidates then used Newton's second law to find the mass and were awarded partial credit.

Question 2

(a) This part was done well by the vast majority of candidates. A few used a decimal approximation for the reaction force or the coefficient of friction. Some candidates thought that the reaction force or the friction force consisted of two terms and some used a negative value for the friction force.

(b) In this part candidates had to use Newton's second law to find the acceleration and then use constant acceleration formulae to find the time taken. Many candidates omitted either the weight component or the friction force or both in their attempt at Newton's second law. Most of these candidates then went on to correctly use their wrong acceleration to try to find the time taken.

Question 3

This question was fairly well attempted by many candidates, almost all of whom resolved horizontally and vertically. A few omitted the weight component, but more common errors were to have a wrong sign in the vertical resolution or to use the wrong components (getting sine and cosine mixed up). A small number of candidates used Lami's method, but this was far more difficult as one of the angles $(210-\alpha)$ was quite awkward to find and, even if found correctly, candidates then had to use the compound angle formula to find the value of α. Very few tried to use a triangle of forces, although this method was fairly straightforward. A very small number of candidates thought that the string attached to the wall had a tension of 4 N, and a few others did not use the 4 N force at all and so had two tensions.

Question 4

(a) In this part candidates had to use Newton's second law to find the driving force and then multiply this by the speed in order to find the power. Most candidates successfully found the power, although a few had a wrong sign in their Newton's second law equation or, more rarely, omitted one of the terms in the equation.

(b) This part was found to be very challenging by many candidates with most trying to use a constant acceleration method to get to the given answer. Such a method was incorrect since the power is constant and so the acceleration would be variable. Some candidates realised that the total work done by the engine was equal to the power multiplied by the time taken, but only some of these realised that the change in kinetic energy needed to be found. Those who found both of these almost always than used a work-energy equation to correctly get to the given answer.

Question 5

(a) This question involved two blocks connected by a rope moving up an inclined plane. It was necessary to apply Newton's second law, either to each of the two blocks and solve simultaneous equations, or to the system as a whole to find the acceleration. There were a reasonable number of fully correct responses, but there were also many only partially correct attempts. Some candidates omitted at least one of the forces, often the 50 N resistance force but occasionally the 500 N force or the weights. Others made sign errors in one or more of their equations or included the tension in the system equation. Candidates who did state two correct equations usually went on to solve them correctly. Candidates who stated a correct system equation almost always found the acceleration correctly, even if their equation for tension was incorrect.

(b) The vast majority of candidates answered this using a correct method, usually by simply dividing the given speed of $1.2\,\mathrm{m\,s^{-1}}$ by their acceleration found in **part (a)** to find the time taken.

Question 6

(a) This part was done well by the vast majority of candidates. A few tried to use kinetic energy rather than conservation of momentum. Some did not set out their working very clearly or gave an equation such as $v = \dfrac{0.3(2-0.6)}{0.4}$, which is correct but candidates would be advised to give the conservation of momentum equation in the conventional form, in this case $0.3\times2 = 0.3\times0.6 + 0.4\times v$.

(b) This part was also done well by the vast majority of candidates. A large number gave a correct equation, usually $0.4\times1.05(+0) = (0.4+m)\times0.5$, but then made an error in solving the equation.

(c) This part was found to be very challenging for the vast majority of candidates. There were quite a number who did not attempt the question at all and many other candidates scored no marks as they were unable to devise a strategy to begin to solve this problem. A common mark was 2 marks out of 5 for using $0.5t$ and $0.6t$ for the distances travelled by A and $\dfrac{B}{C}$ respectively. Many candidates had an incorrect 'gap' between the two at the moment of collision of $\dfrac{B}{C}$, often 2.1 m or 1.2 m. Stronger candidates correctly evaluated the distance between A and $\dfrac{B}{C}$ at the moment of collision and also set up an appropriate equation involving this distance and the displacements of each of A and $\dfrac{B}{C}$. Entirely wrong answers often included acceleration elements with expressions for $\dfrac{1}{2}at^2$ that were not set to zero. Acceleration and velocity changes (other than during the collisions) were common features of wrong answers. Further collisions were another common wrong interpretation here.

Question 7

(a) This question was well answered by most candidates. A few candidates incorrectly used $v = u + at$ to obtain the answer 2.4. In some cases, candidates obtained the correct expression for v but failed to evaluate the speed after 4 seconds and, in some very rare cases, the expression for a was differentiated instead of integration.

(b) Most candidates appreciated the need to integrate the second expression for a. Unfortunately, many candidates failed to include the arbitrary constant, and some stated that the constant was zero. Consequently, they failed to correctly show the given value of k. To find k, it was expected that candidates would use the velocities at the two given times to form equations to be solved simultaneously, or use limits 4 to 16 and equate the definite integral to 0.3 – 1.6. Those who formed the two equations in terms of k and d usually successfully found the value of k and d. Many candidates used only one set of values of v and t to obtain a single equation. They then used the given answer for k to find d, which was not satisfactory. A few candidates showed the given value of k but left their solution incomplete as they missed the final expression for v. In a few cases, candidates used $v = u + at$ as in **(a)**. Very few candidates used the simultaneous solving facility on their calculator, and this was not a satisfactory method, since it did not fully show the given answer.

(c) Those candidates who obtained the correct expression for v in **part (b)** were almost always able to find the correct value of T. In a few cases, the exact value was not given but instead rounded to 27 without first stating 27.04. Many candidates had the equation $v = 5.2t^{-\frac{1}{2}}$ in **part (b)** without a constant and then put $v = 0$ and yet found a non-zero value of T.

(d) This part was found to be fairly challenging although the majority of candidates who found the correct expression for v and the correct value for T successfully attempted this part. Most candidates realised there were two parts to find the total distance travelled and earned at least the first mark by appropriate integration to obtain an expression for displacement in the first phase. Those who had a suitable form for the second phase usually also integrated successfully, but many did not have an appropriate form to integrate from. The process of applying limits was demonstrated by many candidates, but of course many did not have appropriate forms and could not score further marks. Completely correct solutions were presented by the strongest candidates.

MATHEMATICS

Paper 9709/43
Paper 4 Mechanics

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper rather than correct to two significant figures as sometimes seen e.g. **Question 6(a)(ii)**.
- When resolving forces in equilibrium or when forming an equation of motion, a clear and complete force diagram can be helpful in ensuring that all relevant forces have been considered e.g. **Question 3, Question 6(a)(ii).**
- If acceleration is given as a function of time as in **Question 5(a)**, then calculus is needed and constant acceleration formulae are not applicable.

General comments

The standard of work for this paper was variable. Whilst some candidates produced work of a very high standard with clearly presented and accurate solutions, some questions proved to be challenging for many candidates. **Question 1**, **Question 6(a)** and **Question 7(a)** were answered particularly well. Many candidates had difficulty with **Question 3**, **Question 4(a)** and **Questions 7(b)** and **7(c)**.

Comments on specific questions

Question 1

(a) Most candidates applied a constant acceleration formula, usually $v = u + at$, to obtain the correct projection speed of $30\,\text{ms}^{-1}$. A few solutions used $g = 9.8\,\text{ms}^{-2}$ or $g = 9.81\,\text{ms}^{-2}$ rather than the expected $g = 10\,\text{ms}^{-2}$.

(b) The greatest height was also found easily by most candidates using a '*suvat*' formula, often $s = ut + \frac{1}{2}at^2$. Occasionally the use of g instead of $-g$ led to a greatest height of $3 \times 30 + \frac{1}{2}g\,(3)^2$ or 135 m instead of 45 m.

Question 2

(a) Candidates usually attempted to use 'PE = *mgh*' to find the loss in potential energy. Some assumed constant acceleration rather than constant speed when calculating the distance moved along the plane and hence calculated an incorrect increase in height.

(b) The work done by the pulling force could be found by totalling the loss in potential energy and the work done against the frictional force. Alternatively, the work done was found as the product of the total force acting down the plane and the distance travelled. Some candidates found the pulling force correctly but omitted to calculate the work done by this force. Others who used a work/energy equation for the situation occasionally included the frictional force of 40 N rather than the work done against this force, or included a kinetic energy term suggesting acceleration rather than constant speed.

Question 3

A successful solution depended on realising that the normal reaction of the wire on the ring acts in the direction perpendicular to the tangent at the contact point and away from the centre of the ring. The reaction force was frequently either missing or opposite to or perpendicular to the direction of the 4 N weight. A force diagram showing the tension, weight, normal reaction and relevant angles is advisable before attempting to resolve. Resolving horizontally and vertically should have produced two equations in T (the tension) and R (the normal reaction) to be solved simultaneously. It was common to see $T\cos 25 = 40$ from resolving vertically with the R component missing, sometimes followed by $T\sin 25 = R$ with a normal reaction mistakenly in the horizontal direction. Those who did include components of R sometimes calculated or assumed incorrect angles. For those who had the normal reaction in the correct direction, resolving tangentially allowed the tension to be found without simultaneous equations.

Question 4

(a) This question was very frequently either misread or misunderstood. Many candidates oversimplified the problem by assuming that particle P travelled 52 m in the first two seconds ($t = 2$) and 64 m in the first four seconds ($t = 4$). Candidates needed to realise that the 2nd second is between $t = 1$ and $t = 2$ whilst the 4th second is between $t = 3$ and $t = 4$. The actual situation of 52 m in the 2nd second could be represented by $52 = s_2 - s_1$, the difference in displacements at times $t = 2$ and $t = 1$, leading to one equation in u and a after using a constant acceleration formula. For those who set up a second equation in u and a for the 4th second, the simultaneous equations could then be solved. If u was taken to be the speed when $t = 1$, a final adjustment was needed, but not always seen, to obtain the initial speed when $t = 0$. An alternative way to calculate the acceleration ($a\,\text{ms}^{-2}$) was to recognise an increase in velocity of $a\,\text{ms}^{-1}$ per second at the same time as an increase in displacement of $(64 - 52) = 12$ m every two seconds.

(b) Having found the initial speed and acceleration of particle P in **part (a)**, the distance travelled could be found using $s = ut + \frac{1}{2}at^2$ or equivalent constant acceleration formulae with $t = 10$. Since the particle P 'travels in the positive direction' with increasing distances each second, the acceleration used from **part (a)** should have been positive, with distance the same as displacement.

Question 5

(a) This question was well attempted by a majority of the candidates who recognised that integration was needed to obtain velocity and displacement functions from the variable accelerations given. However, a significant number of candidates erroneously attempted to find the velocities and the displacements of particles X and Y using constant acceleration formulae instead of integration. Some used integration to calculate the velocities and then constant acceleration formulae to attempt distances. A correct solution involved equating the velocities of X and Y to find the time of collision and then calculating the distance AB as the difference between the displacements at this time. Common errors were to add the displacements (244 m) or to calculate the displacement of only one particle e.g. $s_X = 104$ m.

(b) A straightforward method to verify the information given was to substitute $t = 3$ in the displacements $s_X(t)$ and $s_Y(t)$ found in **part (a)** and to show that $s_X(3) - s_Y(3) = 36$. Some mistakenly attempted $s_X(3) + s_Y(3)$ instead of $s_X(3) - s_Y(3)$. An alternative method was to assume $AB = 36$ and to form an equation e.g. $-2t^3 + 10t^2 = 36$. In this case it was expected either to solve the equation and select $t = 3$ from the solutions or to substitute $t = 3$ to verify the given result. Some candidates mistakenly selected the solution $t = 3.65$ as the collision time. Sign errors were also seen, for example, when working with $s_X - s_Y = 2t^3 + 6t^2 - (4t^3 - 4t^2)$ as $2t^3 + 6t^2 - 4t^3 - 4t^2$.

Question 6

(a) (i) The formula $P = Fv$ was well known and applied correctly in most cases with $F = 650 + 150$ due to the constant speed. The main error was to use $F = 650$ ignoring the resistance of the caravan to obtain $P = 15600$ W.

(ii) This was straightforward for those who were able to apply Newton's Second law correctly to the situation by writing down two equations from the three options of the system, the car and the caravan. A force diagram would be helpful to show the tension, resistances and driving force applying to each part of the system. Candidates sometimes omitted the tension from the car equation or included it in the system equation. The driving force was occasionally believed to be increased by 40 kW to (40 + 19.2) kW instead of increased to 40 kW. The acceleration was sometimes seen as correct to two significant figures ($0.39\,\text{ms}^{-2}$) instead of the expected three significant figures ($0.385\,\text{ms}^{-2}$).

(b) This question needed a further application of $P=Fv$ with the driving force equal to the total of the resistances and the components of weight down the hill. This was frequently solved accurately. The usual errors seen were to omit g from the components of weight down the hill or to completely omit the components of weight when finding the driving force. Since the angle of inclination of the hill was given as $\sin^{-1} 0.14$ it was not necessary to calculate this angle as $\sin(\sin^{-1} 0.14) = 0.14$ could be used directly. A few attempted to use $\sin 0.14$ rather than 0.14.

Question 7

(a) **Part (a)** was well answered by most candidates who applied $F = \mu R$ with $F = 30\sin\alpha$ and $R = 30\cos\alpha$. Many candidates realised that $\tan\alpha = \frac{3}{4}$ implies $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$ making it possible to evaluate exactly without approximating for the angle α. It was also possible to evaluate μ as $\frac{30\sin\alpha}{30\cos\alpha} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$ without finding the angle α or $\sin\alpha$ and $\cos\alpha$.

(b) Although many candidates found this question challenging, complete and accurate solutions were regularly seen. The solution involved constant acceleration from A to B, the conservation of momentum on collision at B and constant speed from B to C. Some solutions omitted to consider the effect of the collision on the velocity of the particles, assuming the speed before and after collision to be the same. Other errors included a miscalculation of the acceleration on BC to be non-zero, or the use of $s = \frac{1}{2}(u + v)t$ on BC with $u = 0$ or $v = 0$ also suggesting acceleration on this part of the plane.

(c) The required energy loss could be found by calculating the difference between the potential energy loss from A to C and the kinetic energy gain from A to C. Although the potential energy loss for AB could not be found if the distance x m was not found in **part (b)**, the kinetic energy gained on this part of the plane could be used instead. A correct kinetic energy at C depended on calculating the constant speed for the section from B to C in **part (b)**. Candidates who attempted this part of the question usually attempted at least one calculation of kinetic energy, potential energy or the work done against friction. A suitable combination of these was often not calculated.

MATHEMATICS

Paper 9709/51
Paper 5 Probability and Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially if there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively. When errors are corrected, candidates would be well advised to cross through and replace the term, rather than overwriting which can be difficult to interpret accurately.

Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. It is important that candidates work to at least 4 significant figures throughout to justify a 3 significant figures value. Many candidates rounded prematurely in calculations and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates should ensure this is included within their preparation.

General comments

Many well-structured responses were seen, however some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. It was encouraging that the accuracy and labelling of the statistical diagram has improved.

Some questions, for example **4(c)**, require candidates to calculate an initial value which is then used to complete the question. It is good practice to read the question again after completion to ensure all demands of the task have been fulfilled and the answer is reasonable.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. The majority of candidates performed well across a range of topics, however applying the normal approximation in different contexts proved to be more of a challenge. Many good solutions were seen for **Questions 1** and **4**. The context in **Questions 2** and **5** was found to be challenging for many.

Comments on specific questions

Question 1

Candidates were expected to use the information provided in the probability distribution table to form the simultaneous equations required to determine the values for p and q. A number of very clear, accurate solutions were seen which included the algebraic solution of the equations. If non-algebraic methods are used to determine the values, then a minimum expectation is that both expressions are in the same form. Weaker candidates often correctly stated the expression for $E(X)$ and then attempted to solve this, often assuming $p = q$ to form an equation in one unknown.

Question 2

The context of this fairly standard binomial and normal approximation question was more challenging than in previous sessions. Although information was provided for 3 levels of internet service, candidates were required to use this to determine the level of satisfaction referenced in each part of the question.

(a) The majority of solutions recognised that the binomial approximation was appropriate. Good solutions initially identified the percentage of the population who fulfilled the given criteria before forming the binomial expression. The most successful solutions simply added the probabilities for the values in the given range. Candidates who attempted the complement approach often struggled to interpret accurately the success criteria, with many not subtracting P(2) and some not subtracting P(8). Candidates would be well advised to practise interpreting this type of success criteria accurately.
A number of binomial terms were seen where the probabilities did not sum to 1, and some candidates evaluated the 'poor' and 'satisfactory' probabilities separately within each term.
Candidates should use their calculators efficiently when evaluating these expressions as premature approximation caused answers to be outside the acceptable accuracy range.

(b) Although over 25% of candidates made no attempt at this routine normal approximation question, many excellent solutions were seen. The most successful showed clear calculations to support the mean and variance and then provided the unsimplified normal approximation formula with the values substituted appropriately. A simple sketch of the normal curve was often helpful in identifying the required probability area.

Question 3

The improvement in the overall quality of stem-and-leaf diagrams was encouraging.

(a) Many excellent back-to-back stem-and-leaf diagrams were seen, which included a clear key with units and good alignment of values within the leaves to represent accurately the distribution of the heights. Weaker solutions often omitted the units from the key.
Candidates should be encouraged not to use values for keys which are symmetrical, e.g. 0 **|** 18 **|** 0, as these do not identify clearly the teams.
A few candidates did not follow the direction that the Lions needed to be on the left.

(b) The median was found accurately by most candidates. There was less consistency for finding the interquartile range, although the majority of candidates determined the upper and lower quartiles as anticipated as the mid-value to the median term. The weakest solutions did not attempt to evaluate the difference as required.

(c) Many candidates found this question challenging. General comparisons in context for spread and central tendency are required rather than comparing specific values. Candidates can state values as evidence in support of a general statement such as 'Tigers are generally taller because their median height is 190 cm'.

Question 4

(a) Many good solutions to this standard normal approximation question were seen. Candidates should be aware that the unevaluated normal standardisation formula is expected in supporting work. A few candidates did not appreciate from the context that the data was continuous.

(b) Although many candidates found this part challenging, many solutions did try to form an appropriate equation. The use of a simple sketch of the normal curve to identify to required probability area was seen, this appeared to clarify whether the *z*-value was positive or negative. A common misconception was that the equation should be formed with the value given in the question, or treating the 88 per cent (0.88) as a *z*-value and using the tables to find the linked probability.

(c) A significant number of candidates did not attempt this part. Successful solutions often had a simple sketch of the normal curve to clarify the criteria and used the symmetry properties of the normal distribution to evaluate the probability area. Some solutions calculated the SBP value with a *z*-value of 1.5 and then substituted back into the normal standardisation formula to complete the solution. Candidates should be aware that statements such as 'within 1.5 standard deviations of the mean' are equivalent to the statement '$-1.5 < z < 1.5$'. A large number of solutions did not continue to find the probability that the three adults each met the given criteria. Candidates are well advised to reread the question when they have concluded their solution to ensure that they have followed all the requirements of the task.

Question 5

The context of this question appeared challenging to many candidates. It is essential to read the information given carefully to understand the success criteria.

(a) Some good tree diagrams were seen. The best used the answer space efficiently to allow the different possible outcomes to be clearly shown. The tree diagram needed to identify each of the possible outcomes of the first throw separately and then continue where a second throw was awarded. The clearest diagrams had a total score recorded at the end of each branch.
Many diagrams simplified the context so that the outcomes of the first throw were 'no further throw' or 'further throw', but this was insufficient detail to support the later work as well as not being a clear communication of the possible outcomes for the dice.

(b) As a 'show' question, candidates should be aware that they need to communicate fully the argument and processes they are presenting to support the initial statement. Good solutions stated the possible outcomes that fulfilled the criteria for event *A*, then linked the probabilities, including calculations where appropriate, for each outcome before summing to the given value of $\frac{1}{3}$. Weaker solutions either simply stated the probabilities with no indication as to which outcome was being considered, or omitted some terms and did not arrive accurately at the given answer.

(c) Many candidates found this question challenging. Good solutions used the relationship $P(A) \times P(B) = P(A \cap B)$, supporting each value with calculation and then stating their conclusion. Weaker solutions often stated values with little clear communication as to how they were related. A number of candidates presented a 'logical argument' as to why the events were not independent, which gains no credit at this level.

(d) This question was omitted by a significant number of candidates. The best solutions used values that had been calculated in **5(b)** and **5(c)** to find the values required to calculate the conditional probability.

Question 6

The context of this question appeared accessible for many candidates. Most solutions used combinations and permutations appropriately.

(a) The most common approach was to list the possible scenarios that fulfilled the given criteria and then linking the combination calculation required to determine the possible outcomes for each. Weaker solutions often omitted the committee of 5 Men and 0 Women. In some solutions there was a lack of explanation as to which scenario was being evaluated. Some candidates did not separate the group into men and women before selecting and so were calculating from the incorrect size pool.

(b) Many candidates found this part challenging. Good solutions often had a simple diagram to clarify the condition given. Where attempted, many used a 'replacement' concept, so that each group was always selected from all 15 members.

(c) The criteria presented in this question was found challenging by many. The most successful solutions considered the possible arrangements with Abel and Betty being a single group with 3 other members. The places where Freya and Gino could then be inserted into the line so that they were not next to each other were considered. Some solutions using this approach failed to multiply by 2 to allow that Abel and Betty could exchange places in the line.
Solutions which used the alternative approach of calculating the number of arrangements with 'Abel and Betty' standing together and then subtracting the number of arrangements when both 'Abel and Betty' and 'Freya and Gino' were standing were often less successful because of the failure to allow each pair to exchange places in the line.

MATHEMATICS

Paper 9709/52
Paper 5 Probability and Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially if there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level, and enables candidates to review their method effectively and is an essential tool when showing given statements are true. When errors are corrected, candidates would be well advised to cross through and replace the term. It is extremely difficult to accurately interpret terms that are overwritten.

Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. In particular there should be a clear understanding of how significant figures work for decimal values less than 1. It is important that candidates realise the need to work to at least 4 significant figures throughout to justify a 3 significant figures value. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Candidates should be aware that cumulative frequency graphs are constructed with a curve, and that this needs to be reasonably accurately drawn. It was encouraging that the labelling of the statistical diagram has improved.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. The vast majority of candidates were well prepared, however candidates found it more challenging when more than one technique was required within a solution. Many good solutions were seen for **Questions 1, 4** and **6**. The context in **Questions 2, 5** and **7** was found to be challenging for many.

Comments on specific questions

Question 1

Many candidates who were successful in this question used a tree diagram to help clarify the conditions and probability relationships stated.

(a) Almost all solutions accurately created the required equation, identifying the different conditions where Kino is late. These were solved appropriately, although occasional arithmetic errors leading to an answer of 0.015 were noted. Weaker solutions ignored the probabilities for the different modes of transport and formed an equation simply involving the 'late probabilities'. A few solutions equated to 1 rather than the 0.235 stated in the question.

(b) Candidates found calculating the required conditional probability more challenging. Some candidates did not include a calculation to support their numerator value. The majority of candidates realised that the denominator was the complement of the probability stated in **part (a)**, although a significant number of full calculations were noted. Many candidates converted their accurate probability fraction into a decimal, which is not required. Some candidates stated their final probability value to 2 significant sigures, possibly confusing 3 decimal places with 3 significant figures.

Question 2

Almost all candidates recognised that the normal approximation was appropriate throughout the question. Premature approximation or incorrect rounding of mid-process values were noted in weaker solutions. A simple sketch of the normal curve was present in the best solutions, which often clarified the information presented in the question.

(a) Almost all solutions used the normal standardisation formula correctly. Very few continuity corrections were noted, but when present were often ±0.5 rather than the ±0.05 that would have been appropriate. Presenting the normal standardisation formula with the values substituted is expected supporting work for this topic.

A significant number of candidates presented their probability as their final answer rather than use it to calculate the expected number of rods in the sample that fulfilled the given criteria. Candidates should remember that to justify a 3 significant figure answer, calculations should be shown to a greater degree of accuracy. Candidates are well advised to read the question again after they have finished to ensure that they have answered the question fully. A single integer value is expected to be stated as a conclusion of the work which is not determined by rounding or approximating.

(b) Even though a significant number of candidates made little or no attempt at this part, the overall standard of solutions was much improved. Good solutions utilised the symmetrical properties of the normal distribution, recognised that ‘half a standard deviation from the mean’ provided a z-value of ±0.5 without further calculation and used $2\Phi\left(\frac{1}{2}\right)-1$ to obtain the probability. Weaker solutions identified that 56.2 and 55.0 were the appropriate X values and used the normal standardisation formula in the usual manner. A common misconception was that 56.1 and 55.1 were the linked X values found by calculating 55.6 ± 0.5.

Question 3

Almost all candidates identified that the geometric approximation was the appropriate approach for the question. However, a significant number of solutions did not interpret the success criteria accurately, either not including 17 as a success value or not evaluating accurately the probabilities using 3 fair dice.

(a) The anticipated geometric approximation formula was used in most solutions. Some solutions misinterpreted the success criteria and calculated a probability of success in fewer than 6 or 7 throws. A small number of candidates attempted to use a binomial approximation approach without success.

(b) Where attempted, most solutions used the standard geometric approximation approach. There was uncertainty with interpreting the success criteria, with the most common error the use of $1 - p^8$. Candidates would benefit from having a secure understanding of how to interpret success criteria within ‘practical’ contexts.

Question 4

There was an encouraging improvement in the overall quality histograms. Candidates should be aware that at this level, graphical statistical representations are expected to be accurate, so rulers should be used to construct the histogram.

(a) The best solutions calculated the frequency density, clearly stating the individual terms. Many candidates used the data table effectively, although some values were difficult to read if they encroached the printed text. Almost all candidates determined that the data was continuous and so did not require a continuity correction on the time axis. The inclusion of units on the time axis was seen in the majority of solutions. Some candidates plotted a frequency graph, but could still gain some credit by using the appropriate class boundaries. Candidates should ensure that the column lines do extend to the axis and that their lines can be identified if drawn on the gridlines.

(b) Many good solutions were noted. These often stated the mid-points initially at the data table in **part (a)** and then used these values accurately in an unevaluated variance formula. A number of correct final values were seen without sufficient supporting evidence. Candidates should be reminded that answers without supporting evidence will not gain credit. A few candidates unnecessarily recalculated the given mean value. Again, a number of candidates did not fully complete the question by using the variance as their final answer.

Question 5

Many candidates found this probability question challenging. **Part (c)** was omitted by a significant number of candidates, but was often completed successfully when attempted as it used a similar process to **part (a)** in a less complex scenario.

(a) As this is a 'show' question, candidates should be aware that a full justification of their method needs to be within their working. The most efficient method was to consider the complement of 'Tails on both biased coins', with successful solutions showing clearly both the calculation and stating the scenario being evaluated. The majority of candidates used the simpler approach of identifying all the possible outcomes and calculating the required probabilities. Weaker solutions did not link the probabilities consistently with the coin scenario, or failed to state what scenarios were being considered. Many successful solutions used a tree diagram to help clarify the problem.

(b) The conditional probability was found challenging by many candidates. The most common misunderstanding was assuming $P(A \cap B) \equiv P(A)$ for the numerator. A number of solutions did not use the value given in **part (a)** as the denominator for $P(B)$ and recalculated this, not always successfully.

(c) When attempted, almost all solutions correctly identified the correct outcomes for X needed to construct a probability distribution table. Most candidates then included at least one probability. The most successful solutions included some supporting calculations for the individual outcomes – often linked with a tree diagram to clarify the context. A common misunderstanding was that Eric was using different coins which were now fair. Candidates would benefit from understanding the structure of questions is such that conditions remain constant unless specifically stated. A number of solutions included probability distribution tables which did not sum to 1.

Question 6

This was a familiar binomial and normal approximation question. The number of candidates who did not attempt **part (b)** was higher than anticipated.

(a) Many good attempts at this standard binomial approximation question were seen. The best clearly stated the individual unevaluated terms within the expression and then stated the required answer. Most candidates used the more efficient 1 – P(10, 11, 12) method, although the alternative approach was also seen used successfully. Again, the most common error was misinterpreting the success criteria and evaluating the probability of '10 and less' rather than 'fewer than 10'.

(b) Again, many good attempts at this standard normal approximation question were seen. The best calculated the mean and variance initially and then substituted these values appropriately in the normal approximation formula. Most candidates recognised that the data was discrete and so required a continuity correction, although a few used the lower rather than the upper bound. The most common error was to use the incorrect probability area, which may have been avoided if a simple sketch of the normal curve was used to clarify the success criteria.

(c) Very few correct solutions were presented here, and the part was not attempted by over 25 per cent of candidates. The best solutions often referenced the calculations used in **part (b)** when identifying the appropriate approximation to use. It is anticipated that a check should always be made to ensure that the use of the normal approximation is appropriate as a substitute for the binomial approximation, rather than just assuming that it must be. The most common error was considering that $npq > 5$ as a requirement. Some candidates discussed the use of a continuity correction.

Question 7

Most candidates used an appropriate combinations approach to this question. The context was often found challenging, and solutions with simple 'diagrams' illustrating possible scenarios were often more successful.

(a) The required conditions appeared to be understood by the majority of candidates. The best solutions used a simple diagram to illustrate an outcome to support the calculation. Some solutions often divided by $2! \times 2!$ to eliminate the repeats from the As and Ls, even though they had treated them each as a single unit in their approach. The lowest scoring solutions simply calculated the number of ways that the letters could be arranged, incorrectly assuming that dividing by $2! \times 2!$ achieved the criteria.

(b) This question was found to be challenging by most candidates. The requirement to find the probability was not always fulfilled, with many solutions simply calculating the number of ways the letters could be arranged with the given criteria. Solutions with simple diagrams of possible scenarios were often more successful. Solutions which identified how the As could be placed initially and then calculated the number of arrangements for the remaining letters often made good progress. The most common error was to not remember that the Ls were still together. Where the probability was attempted, the denominator was often correct. The most common error was to use the value from **part (a)**.

(c) This question was more standard in context, with less complex selection criteria. Again, the most successful solutions used a simple diagram to identify the possible scenarios and then simply stated the single combination expression to evaluate each selection. A common error was to introduce additional terms for the selection of the As and L, rather than recognising that this had already been determined. 85 was a common incorrect final value.

MATHEMATICS

Paper 9709/53
Paper 5 Probability and Statistics 1

Key messages

Candidates should ensure their working is clear and legible and, most importantly of all, they should explain their working and their strategy. In **Questions 2**, **3c** and **5** full credit could not be obtained if the binomial expression or standardisation or variance formula were not shown in full with the correct substituted values. In **Questions 6** and **7** many candidates would have benefited by explaining their strategy for their partially correct solutions.

General comments

Premature approximation and a loss of accuracy was present in many answers. This was particularly apparent in **Question 2** when 4.17 was often used for the standard deviation and resulted in an inaccurate final answer or in **Question 5b** where 1.17 was used for the z-value. If an answer is to be correct to 3 significant figures, the input numbers should be correct to at least 4 figures. There was also confusion about giving decimals correct to 3 significant figures, particularly when the first or third decimal figure is a zero. In **Question 4c**, the answer 0.059 was seen many times as was 0.93 in **Question 5a**.

Comments on specific questions

Question 1

Most candidates understood what was required and calculated $\sum x$ correctly before they substituted this value into the correct variance formula, knowing that they could not mix $\frac{35}{50}$, the mean of (x – 20), with 25 036, the sum of x squared. Only a few confused the standard deviation with the variance and almost all knew to give an exact answer in full.

Question 2

This style of question was familiar to most candidates and it was answered well. Only a few did not follow the instruction to use an approximation and tried to apply the binomial distribution multiple times. As in the previous question, a few confused the variance with the standard deviation but most knew how to standardise, remembered to apply a continuity correction and subtracted their probability from 1 to find the appropriate area. Prematurely approximating the standard deviation to 4.17 resulted in an inaccurate final answer.

Question 3

(a) Careful candidates knew to use a sharp pencil when drawing a graph, plot points with a neat cross, label the horizontal axis (time in minutes), the vertical axis (cumulative frequency) and to scale both axes from zero. Most used a sensible scale on the vertical axis, either 20 or 25 to 1cm. Those who only marked the given cumulative frequencies (32, 66, 112 etc.) on the vertical axis, instead of regular values, rarely plotted the points correctly. Candidates should draw a smooth curve passing through the plotted points and not ruled line segments or a curve that missed the plotted points.

(b) The question instructed candidates to ‘use the graph’ to estimate the 60th percentile and, as such, we needed to see evidence of this on the graph, ideally a line across from 150 to meet their curve. The most common error was to confuse 60th percentile with a cumulative frequency of 60 and to draw a line from 60.

(c) Strong responses involved listing the frequencies and mid-points before substitution into the variance formula, showing calculations in full. Most candidates did appreciate the importance of showing their working in full and there were very few processing errors. Those who produced the correct answer without producing any evidence of their working could not be awarded full marks. The most common errors were to use incorrect values in the variance formula e.g. cumulative frequencies, upper bounds or class widths. As in previous questions there was some confusion between variance and standard deviation with a number of candidates giving 151.24, the variance, as their final answer.

Question 4

(a) When an answer is given in the question, it is particularly important that every stage of the calculation is explained. For this question we needed a justification for both the numerator (7) and the denominator (64) with reference to the context of the question.
The most straightforward method was to list the seven sets of values that would result in a score of 2 and to show that the total number of possible outcomes was 4 × 4 × 4 = 64.
Some looked at the three possible combinations of values (1,1,2; 2,2,1; 2,2,2) but to gain full marks they needed to explain how there were three ways of obtaining 1,1,2 and 2,2,1. We also needed to see ${}^{3}C_{1}$ and/or ${}^{3}C_{2}$ in their explanation.

Very few used our third method of subtracting the probability of three 1s, $\left(\frac{1}{4}\right)^3$, from the probability of all the values being 1 or 2, $\left(\frac{1}{2}\right)^3$. If they chose this method they needed to explain what they were doing to gain full marks.

A few candidates attempted a 3-dimensional diagram to show the seven outcomes but these were rarely clear enough to understand and be acceptable as a proof.
Many candidates did not show their method in appropriate detail for this question and very often those using methods 1 or 2 would hide their sets of values up above the question or down the side.

(b) This question was answered well. Most candidates realised that there was only one way to have a highest number of 1 and then subtracted the sum of the three known probabilities from one to find the probability of a score of 4. Very few of those who tried to find the P(4) from first principles were successful.

(c) Those who appreciated the meaning of the spinner being 'fair' generally answered this question correctly using the geometric distribution with $p = \frac{1}{4}$. Those who thought this was a follow-on question from the probability distribution in the previous part mistakenly used $p = \frac{19}{64}$. A number of candidates confused 3 significant figures with 3 decimal places and rounded their answer to 0.059 and did not show the required answer.

(d) Candidates who realised that the answer required was q^4 were most likely to arrive at the correct answer as long as they were not using $p = \frac{19}{64}$ as in **part (c)**. Many used the longer method of subtracting the probabilities of 1, 2, 3 and 4 from 1, and a significant number using this method missed out the probability of 4.

Question 5

(a) This question was answered well with most applying the binomial distribution correctly, knowing that they needed to sum the probabilities of 0, 1 and 2. Only a few omitted the ${}^{10}C_{x}$ coefficients in their binomial. A number of candidates did not appreciate that when the 3rd significant figure in a non-exact decimal is a zero it is incorrect to omit the zero from the answer and give it as 0.93.

(b) Most candidates knew how to standardise using the given mean and standard deviation and very few omitted the next stage of subtracting from 1 to find the required answer. The *z*-number of 1.1667 caused a few issues. Some rounded it prematurely to 1.17, which resulted in an inaccurate final answer, while a noticeable number changed it to 1.667 before finding the probability.

(c) This question was answered well with very few using the tables the wrong way round and producing a probability instead of a *z*-value.

Question 6

(a) This was the question that candidates found most accessible and nearly everyone divided the factorial of the number of letters in 'Activated' by the product of the factorials of the two repeated letters. The majority of answers were also written exactly and in full.

(b) Stronger candidates realised that there could be 5, 6 or 7 letters between the two As and that there were three positions for the first A when there were 5 letters, two positions when 6 letters and only one position when 7 letters. They also realised that the number of ways of positioning all the letters that were not A was $\frac{7!}{2!}$ for each of these six scenarios. We had hoped that candidates would explain their approach but many gave sufficient answers by choosing a clear approach and having a correct answer. However, those who did not give a fully correct solution and omitted to explain their strategy rarely were awarded any marks.
A significant number of candidates complicated their solution by unnecessarily considering the distribution of the Ts. Very few of these responses resulted in the correct final answer. A few also made the question even more challenging by subtracting the number of ways with 4,3,2,1 or 0 letters between the As from 90 720, the total number of ways calculated in the previous part.

(c) This question proved to be the most challenging on the paper and only a small number obtained the correct final answer. However, as long as the strategy was clearly explained, we could award credit for the method used. A few candidates seemed to be confused by the wording and did not appreciate that there could be an equal number of Ts and As, including none.
A significant number assumed this question was just asking for the number of selections that could be made that did not contain more Ts than As. If this were the case, each selection does not have to be equally likely and the repeats are dealt with in a different way from in a probability question. Others who did appreciate that they needed to find a probability still found the number of possible selections (41) and either divided by the total number of possible selections using the same approach (66) or confused their thinking and divided by ${}^{9}C_{5}$.
Almost all of those who appreciated that, for a probability, each selection had to be equally likely made their approach very clear and, as long as they realised there were six possible outcomes, they usually obtained the correct final answer.
A shorter method that was seen less often was to subtract the number of ways with more Ts than As from the total. There were the same issues with the repeats seen here but this approach was often successful.

Question 7

(a) Those who produced a tree diagram usually formed the correct sum of products that represented Tom removing a red disc on his first turn. Without a tree diagram, a common error was to take no notice of Sam in the calculations and give $\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$ as the final answer. Although $\frac{3}{8}$ is the correct answer we did require some working or explanation to award both marks. We did not accept the correct answer when the candidate clearly worked 'with replacement' and 8 was the denominator of all their probabilities.

(b) As in the previous part, a tree diagram helped candidates to produce the three correct four-factor products which needed to be added. Alternative, often incorrect, approaches still led to the correct answer and we required clear working and explanation if any marks were to be gained. Ignoring Sam and finding the probability of Tom removing two reds $\left(\frac{3}{8}\times\frac{2}{7}\right)$ was a legitimate approach but required a full explanation.

(c) Most candidates appreciated the need for a conditional probability but, unless their answers were correct in the previous parts, they could only gain the method mark if they clearly explained what they were doing. It needed to be very clear that they were intending to divide the probability of RRWR by the probability of Tom winning the game on his second turn, the answer to the previous part. A number of candidates correctly calculated the probability of RRWR but did not go on to divide it by the answer to **part (b)**.

MATHEMATICS

Paper 9709/61
Paper 6 Probability and Statistics 2

Key messages

For questions that require a sum of binomial terms or a sum of Poisson terms all of the terms should be shown.

General comments

Many candidates presented their work clearly and their explanations in detail.

Comments on specific questions

Question 1

Many candidates found the correct unbiased estimates for the population mean and variance of the heights of the trees. Some candidates incorrectly substituted 62.1 or 6.21 in their chosen formula for the unbiased variance. Other candidates found only the biased sample variance.

Question 2

(a) To carry out the test it was necessary to state the hypotheses. Then the relevant tail of the binomial distribution B(40, 0.2) was required, namely $\mathrm{P}(\leq 4 \text{ reds})$. In order to demonstrate the method it was necessary to write down all 5 terms as well as the sum of these (0.0759). This value should then have been compared to the significance level (0.05) in order to show that the result was not significant. The conclusion needed to be written in the context of the question, not in a definite form and without any contradictions.

(b) In **part (a)** the result of 4 reds did not lead to rejection of the null hypothesis H_0. The next possible result would be landing on 3 reds.

To test this possibility required the calculation of $\mathrm{P}(\leq 3 \text{ reds})$. This value was 0.0285 which would be significant. Hence the largest value of *r* was 3. It was necessary to find the probability in order to score the marks.

Question 3

(a) The information given suggested a Poisson distribution $P_0(5.2)$ for a time interval of one minute. Then for a time interval of 30-seconds the distribution was $P_0(2.6)$. Most candidates used this distribution to find the probability of at least 3 drops of water by finding $1-\mathrm{P}(\leq 2 \text{ drops})$ successfully.

(b) For a 2-hour period the Poisson parameter was 624. For this large value a suitable approximating distribution was the normal distribution N(624, 624). Many candidates correctly used this and standardised to find the probability. It was necessary to use the continuity correction 649.5. Many candidates did use this. Some candidates used an incorrect value such as 650.5 or did not apply a factor in their standardisation.

Question 4

(a) For the total amount of sugar the mean could be found by summing the given means and multiplying by 3 and the variance could be found by summing the given variances and also multiplying by 3 in order to allow for the 3 months. The standard deviation could be found by taking the square root of this new variance. Many candidates did this correctly. Some candidates used 3^2 instead of 3. Other candidates omitted to find the standard deviation.

(b) As the company made a profit by selling amounts of brown sugar but made a loss by selling amounts of white sugar it was necessary to select the correct signs when calculating the new expectation (3010) and the new variance (33 076) for the total profit. These values were to be used to standardise the stated $3 000 or less profit. A diagram could be helpful when choosing the correct area for the probability. Some candidates followed these steps successfully. Other candidates added the expected profit and loss instead of subtracting them. Some candidates subtracted the variances instead of adding them. Other candidates used 1.50 and 0.20 without squaring them.

Question 5

(a) A 95 per cent confidence interval was required. For this the critical value for z was 1.96. The normal distribution was N(45, $\frac{36}{200}$). The answer needed to be in the form of an interval.

(b) The given information allowed the critical value of z to be found (2.571). The corresponding Φ value was 0.9949 with tail probability 0.0051. These values needed to be combined to find the central area and hence the confidence interval. It was necessary to show the working for this process. Several different steps were valid. A diagram could be helpful for this.

Question 6

(a) The given diagram and information indicated that E(X) = 1 and so E(S) = 2. The variances of X and S were to be equal. These properties indicated that the sketch for S should show a quadratic graph symmetrical about the line $x = 2$ and of a shape similar to the sketch for X from $x = 0$ to $x = 4$ with highest point (2, 0.375).

(b) The given diagram and information indicated that E(X) = 1 and so E(T) = 1. The variance of T was to be $\frac{1}{4}$ of the variance of X. These properties indicated that the sketch for T should show a quadratic graph symmetrical about the line $x = 1$ and of a shape similar to the sketch for X from $x = 0$ to $x = 2$ with highest point (1, 0.75).

(c) In order to use the information given in the question it was necessary to integrate f(x) with the limits $1 – a$ and $1 + a$ and to equate this to 0.5. Other pairs of limits could be used provided that the right hand side matched the chosen limits. For example limits $1 + a$ to 3 with right hand side equal to 0.25 formed a correct pairing. Candidates then had to show all of the steps leading to the given answer.

(d) Substituting $a = 0.69$ and $a = 0.70$ into the cubic expression gave a positive value and a negative value indicating that there was a root for a between 0.69 and 0.70.

Question 7

(a) The hypothesis test required the hypotheses to be stated and the normal distribution N(32.5, $\frac{3.1^2}{50}$) to be used. Standardisation gave the z value as –1.597 which was then to be compared to the critical value –1.406 for the 8 per cent significance level for this one tail test. This comparison indicated a significant result. A comparison between the corresponding probability values (0.0551 < 0.08) was also acceptable. The conclusion was required to be stated in context, not definite and with no contradictions.

(b) The first stage required the critical value for accepting/rejecting the null hypothesis to be found. Use of the normal distribution N(32.5, $\frac{3.1^2}{50}$) and the critical value of z (–1.406) gave this value as 31.88. The second stage involved standardising this value in the normal distribution N(31.5, $\frac{3.1^2}{50}$) and choosing the correct tail for the probability. Some candidates followed these steps successfully. Other candidates lost accuracy by working with 31.9 instead of 31.88. Some candidates omitted the first stage but could gain some marks for their work.

MATHEMATICS

Paper 9709/62
Paper 6 Probability and Statistics 2

Key messages

- Candidates must show all relevant working.
- It is important when stating the conclusion to a hypothesis test that the language used is not definite and that it is in the context of the question.
- When answering questions involving probability density functions, candidates should not always rely on set formulae but should be able to find and use information given on the graph of a pdf.

General comments

In general, this was a well attempted paper; a couple of questions proved difficult for some candidates thus giving a good spread of marks. There was no indication of candidates having time pressures and presentation was mainly good. However, there were times when full working was not shown; candidates need to be aware that, for example, when calculating cumulative Poisson or Binomial probabilities all individual terms must be shown in the working and they should not just give a final unsupported answer. Similarly, it is expected when using a Normal distribution, that the standardising equation is clearly shown. Questions that were well attempted were **Questions 1(a)**, **3(a)** and **Question 6**, whilst **Questions 4** and **7** proved demanding.

The following comments on specific questions highlights many of the common errors seen, but it should be noted there were also cases of fully correct, complete solutions too.

Comments on specific questions

Question 1

(a) This question was reasonably well attempted, though the usual confusion between alternative formulae for the unbiased estimate of the population variance was seen. A few candidates made the mistake of calculating the biased rather than the unbiased estimate for the variance.

(b) Some candidates were able to set up a correct equation using the information given, and correct values for z (1.748 or 1.747) were reached. However, finding the correct value for α caused problems. Many candidates, having found the correct z value, merely left their answer as 96 per cent, whilst others tried to apply a formula rather than use their understanding; this was not always successful and a diagram may have helped candidates here.

Question 2

This question was reasonably well attempted, with most candidates able to correctly declare their hypotheses (though some incorrectly used p rather than μ). Calculating the z value was well attempted, though the usual confusion between standard deviation and variance was seen, along with the omission of $\sqrt{100}$. A valid comparison (i.e., comparing their z value with 2.054 or 2.055 or comparing their area with 0.02) was then required. It is important that this comparison is clearly made either as an inequality statement or on a clearly labelled diagram. It is not sufficient to make reference to acceptance regions or rejection regions unless these regions are clearly defined in this context. Candidates should note that the final conclusion must be made using language with a level of uncertainty (i.e. phrases such as 'there is evidence to suggest…' are expected whereas phrases such as 'this proves that...' are not appropriate). The conclusion must also be in context. Many candidates, despite reaching the correct conclusion, did not always state it in the required manner.

Question 3

(a) This was a well attempted question with many candidates realising that the suitable approximating distribution was a Poisson distribution with mean 3.2 . Common errors included omitting a term in the Poisson expression for ‘more than 3’: incorrectly stating that P(>3) = 1 – P(0,1,2) was commonly seen. Some candidates chose an incorrect approximating distribution (often a Normal Distribution) or did not use an approximating distribution at all.

(b) It was important that when justifying the approximating distribution, the justification related specifically to this question. So when saying n should be greater than 50 it should have been made clear that n was 200 in this question. Similarly, np here was 3.2 which is less than 5 making the Poisson distribution a suitable approximation.

Question 4

Other than **part (a)**, this question, in general, was challenging for candidates.

(a) Most candidates were able to give the correct hypotheses here. Although it should be noted that if candidates state ‘population mean’ rather than μ, then ‘population’ must be used; just to say mean = 7.2 (or 2.4) would not be acceptable.

(b) This part was challenging for candidates. Some correctly used $\lambda = 7.2$, but did not find both the probabilities $P(X \le 2)$ and $P(X \le 3)$ in order to justify their answer. It was important that candidates showed all working out when reaching these two values. Many candidates tried to work with a Normal distribution rather than with a Poisson distribution.

(c) To carry out the test a valid comparison was needed. So, $P(X \le 3)$ compared with 0.05 was required or alternatively a comparison with the critical region was acceptable (3 > 2). A large number of candidates found P(X = 3) and incorrectly compared that with 0.05 . The conclusion should then have been given in context and as a non-definite statement.

(d) It was important that all relevant working was shown. Errors included an incorrect value of λ being used and some candidates wrongly continued with their use of a Normal distribution.

Question 5

This question was well answered by most candidates and proved to be problematic for some.

(a) Many candidates realised that the distribution of the sample means was Normal, but the value of the mean and the variance was not always correctly given. Many appeared to work with a distribution B(160, 0.2) to find the parameters rather than 160 samples of B(10, 0.2), usually incorrectly stating N(32, 25.6)

(b) Some candidates were able to use the correct values for the parameters in this part. Others gained ‘follow through’ marks for using their values stated in **part (a)**. Answers either with no continuity correction or with a correct continuity correction were accepted.

Question 6

This question was particularly well attempted.

(a) Most candidates successfully found the value for the mean of the total mass of the six bags of flour, but mistakes were made in finding the variance. The most common error seen was to calculate $4^2 \times 100 + 2^2 \times 324$ rather than $4 \times 100 + 2 \times 324$. Another common error was to find an incorrect probability area.

(b) As in **part (a)** common errors here included using an incorrect variance or finding the wrong probability area. In general, attempts at this part were good and, as in **part (a)**, there were many fully correct answers.

Question 7

This question, particularly **parts (a)** and **(b)(ii)** proved demanding for many candidates.

(a) The best approach to this part was to use the diagram to label the given probabilities and to use the given symmetry to find the area under the curve between –0.5 and 0, or to use the fact that the area under the curve between 0 and 2 is the same as the area between –3 and –1. Many candidates did not know how to start the question and seemed unfamiliar with finding probabilities using the graph of a pdf.

(b) (i) Many candidates realised they needed to integrate f(x) between –3 and 2 and equate this to 1. However, there were many incorrect attempts at the integration. The function $a - b(x^2 + x)$ was often interpreted as $(a - b)(x^2 + x)$ or the term $(a - b)$ was taken outside the function prior to integration. Some candidates incorrectly took just a outside the integral. Those who were able to integrate correctly usually went on to obtain the correct expression. As this was a 'show that' question, all necessary working was required to be seen.

(ii) The method required to find the values of a and b was indicated in the question. However, as in **part (a)** candidates were not always able to use information from the diagram. Here the graph showed that both $f(-3) = 0$ and $f(2) = 0$ and either of these could then be used to find $a = 6b$. Some candidates did not substitute a suitable x value into $f(x)$. The majority of candidates did not use the method indicated in the question at all and tried further integration attempts. As such, they were unable to find a second equation in a and b.

MATHEMATICS

Paper 9709/63
Paper 6 Probability and Statistics 2

Key messages

For questions that require a sum of binomial terms or a sum of Poisson terms all of the terms should be shown.

General comments

Many candidates presented their work clearly and their explanations in detail.

Comments on specific questions

Question 1

Many candidates found the correct unbiased estimates for the population mean and variance of the heights of the trees. Some candidates incorrectly substituted 62.1 or 6.21 in their chosen formula for the unbiased variance. Other candidates found only the biased sample variance.

Question 2

(a) To carry out the test it was necessary to state the hypotheses. Then the relevant tail of the binomial distribution B(40, 0.2) was required, namely $\mathrm{P}(\leq 4 \text{ reds})$. In order to demonstrate the method it was necessary to write down all 5 terms as well as the sum of these (0.0759). This value should then have been compared to the significance level (0.05) in order to show that the result was not significant. The conclusion needed to be written in the context of the question, not in a definite form and without any contradictions.

(b) In **part (a)** the result of 4 reds did not lead to rejection of the null hypothesis H_0. The next possible result would be landing on 3 reds.

To test this possibility required the calculation of $\mathrm{P}(\leq 3 \text{ reds})$. This value was 0.0285 which would be significant. Hence the largest value of *r* was 3. It was necessary to find the probability in order to score the marks.

Question 3

(a) The information given suggested a Poisson distribution $P_0(5.2)$ for a time interval of one minute. Then for a time interval of 30-seconds the distribution was $P_0(2.6)$. Most candidates used this distribution to find the probability of at least 3 drops of water by finding $1-\mathrm{P}(\leq 2 \text{ drops})$ successfully.

(b) For a 2-hour period the Poisson parameter was 624. For this large value a suitable approximating distribution was the normal distribution N(624, 624). Many candidates correctly used this and standardised to find the probability. It was necessary to use the continuity correction 649.5. Many candidates did use this. Some candidates used an incorrect value such as 650.5 or did not apply a factor in their standardisation.

Question 4

(a) For the total amount of sugar the mean could be found by summing the given means and multiplying by 3 and the variance could be found by summing the given variances and also multiplying by 3 in order to allow for the 3 months. The standard deviation could be found by taking the square root of this new variance. Many candidates did this correctly. Some candidates used 3^2 instead of 3. Other candidates omitted to find the standard deviation.

(b) As the company made a profit by selling amounts of brown sugar but made a loss by selling amounts of white sugar it was necessary to select the correct signs when calculating the new expectation (3010) and the new variance (33 076) for the total profit. These values were to be used to standardise the stated \$3 000 or less profit. A diagram could be helpful when choosing the correct area for the probability. Some candidates followed these steps successfully. Other candidates added the expected profit and loss instead of subtracting them. Some candidates subtracted the variances instead of adding them. Other candidates used 1.50 and 0.20 without squaring them.

Question 5

(a) A 95 per cent confidence interval was required. For this the critical value for z was 1.96. The normal distribution was $N(45, \frac{36}{200})$. The answer needed to be in the form of an interval.

(b) The given information allowed the critical value of z to be found (2.571). The corresponding Φ value was 0.9949 with tail probability 0.0051. These values needed to be combined to find the central area and hence the confidence interval. It was necessary to show the working for this process. Several different steps were valid. A diagram could be helpful for this.

Question 6

(a) The given diagram and information indicated that $E(X) = 1$ and so $E(S) = 2$. The variances of X and S were to be equal. These properties indicated that the sketch for S should show a quadratic graph symmetrical about the line $x = 2$ and of a shape similar to the sketch for X from $x = 0$ to $x = 4$ with highest point (2, 0.375).

(b) The given diagram and information indicated that $E(X) = 1$ and so $E(T) = 1$. The variance of T was to be $\frac{1}{4}$ of the variance of X. These properties indicated that the sketch for T should show a quadratic graph symmetrical about the line $x = 1$ and of a shape similar to the sketch for X from $x = 0$ to $x = 2$ with highest point (1, 0.75).

(c) In order to use the information given in the question it was necessary to integrate $f(x)$ with the limits $1 – a$ and $1 + a$ and to equate this to 0.5. Other pairs of limits could be used provided that the right hand side matched the chosen limits. For example limits $1 + a$ to 3 with right hand side equal to 0.25 formed a correct pairing. Candidates then had to show all of the steps leading to the given answer.

(d) Substituting $a = 0.69$ and $a = 0.70$ into the cubic expression gave a positive value and a negative value indicating that there was a root for a between 0.69 and 0.70.

Question 7

(a) The hypothesis test required the hypotheses to be stated and the normal distribution $N(32.5, \frac{3.1^2}{50})$ to be used. Standardisation gave the z value as –1.597 which was then to be compared to the critical value –1.406 for the 8 per cent significance level for this one tail test. This comparison indicated a significant result. A comparison between the corresponding probability values ($0.0551 < 0.08$) was also acceptable. The conclusion was required to be stated in context, not definite and with no contradictions.

(b) The first stage required the critical value for accepting/rejecting the null hypothesis to be found. Use of the normal distribution N(32.5, $\frac{3.1^2}{50}$) and the critical value of z (–1.406) gave this value as 31.88. The second stage involved standardising this value in the normal distribution N(31.5, $\frac{3.1^2}{50}$) and choosing the correct tail for the probability. Some candidates followed these steps successfully. Other candidates lost accuracy by working with 31.9 instead of 31.88. Some candidates omitted the first stage but could gain some marks for their work.