Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9938938839

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC22 11_9709_33/RP

1 Solve the equation $\ln(2x - 1) = 2\ln(x + 1) - \ln x$. Give your answer correct to 3 decimal places. [4]

2 Expand $\sqrt{\dfrac{1+2x}{1-2x}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

3 Find the exact value of $\int_0^{\frac{1}{4}\pi} x \sec^2 x \, dx$. [5]

4 The parametric equations of a curve are

$$x = 2t - \tan t, \qquad y = \ln(\sin 2t),$$

for $0 < t < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot t$. [5]

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5 **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 2| \leqslant 2$ and $\text{Im}\, z \geqslant 1$. [4]

(b) Find the greatest value of $\arg z$ for points in the shaded region. [2]

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6 Solve the quadratic equation $(1 - 3i)z^2 - (2 + i)z + i = 0$, giving your answers in the form $x + iy$, where x and y are real. [6]

7 **(a)** Show that the equation $\sqrt{5}\sec x + \tan x = 4$ can be expressed as $R\cos(x + \alpha) = \sqrt{5}$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [4]

(b) Hence solve the equation $\sqrt{5}\sec 2x + \tan 2x = 4$, for $0° < x < 180°$. [4]

  [Turn over

8 The curve with equation $y = \dfrac{x^3}{e^x - 1}$ has a stationary point at $x = p$, where $p > 0$.

(a) Show that $p = 3(1 - e^{-p})$. [3]

(b) Verify by calculation that p lies between 2.5 and 3. [2]

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(c) Use an iterative formula based on the equation in part **(a)** to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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9 With respect to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC, between B and C, and is such that $BN = 2NC$.

(a) Find the position vectors of M and N. [3]

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(b) Find a vector equation for the line through M and N. [2]

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(c) Find the position vector of the point Q where the line through M and N intersects the line through A and B. [4]

10 A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time t minutes after filling begins the volume of water in the pool is V litres. The pool has a small leak and loses water at a rate of $0.01V$ litres per minute.

The differential equation satisfied by V and t is of the form $\frac{dV}{dt} = a - bV$.

(a) Write down the values of the constants a and b. [1]

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(b) Solve the differential equation and find the value of t when $V = 1000$. [6]

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(c) Obtain an expression for V in terms of t and hence state what happens to V as t becomes large. [2]

11 Let $f(x) = \dfrac{5 - x + 6x^2}{(3 - x)(1 + 3x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Find the exact value of $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$, simplifying your answer. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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