Cambridge Assessment International Education

Cambridge International AS & A Level

THINKING SKILLS **9694/31**

Paper 3 Problem Analysis and Solution **October/November 2020**

2 hours

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You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
  Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
  In most questions, full marks will be awarded for a correct answer without any working. In some questions, however, you will not be awarded full marks if working needed to support an answer is not shown.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Blank pages are indicated.

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1 In a country where too much food is grown it was decided to pay farmers to leave 10% of their land unused. This is called a set-aside subsidy. The administrators expected that paying for 10% of the land to be left unused would result in a 10% reduction in production. However, the farmers (unsurprisingly) selected the least productive 10% of the land.

Jacques has two fields, each 500 m by 400 m, separated by a thin hedge and surrounded by a stone wall. Each field contains 20 hectares of land (1 hectare = 10 000 m^2). The annual yield in tonnes for each hectare is shown: the top field can yield 91 tonnes in total, and the bottom field 64 tonnes in total.

4	4	5	6	6
4	4	5	5	6
3	4	4	5	6
2	3	4	5	6
2	3	4	5	6
2	3	4	4	5
1	2	3	4	5
1	1	2	3	4

Jacques stopped using the hectares representing the least productive 10% of each field.

(a) How many more tonnes did Jacques produce than the administrators expected? [2]

If Jacques removed the hedge between the fields he could claim to have only one large field, and so increase his production without loss of subsidy.

(b) How many more tonnes could Jacques grow by doing this? [1]

The set-aside subsidy is calculated as 15% of the total possible yield from a field (i.e. as if no land were set-aside), multiplied by last year's sale price for the crop.
Last year's price for Jacques' crop was $900 per tonne.

(c) How much set-aside subsidy would Jacques get this year? [2]

Jacques hopes to sell his crop for $1000 per tonne this year. He knows that there is also an annual environmental grant for having hedges. However, he has discovered that if he cuts away just 100 m of hedge he would be able to consider all his land to be one field.

(d) What is the highest hedge grant per 100 m that would lead to a higher total income if he does this? [2]

The selling price of crops varies from year to year.

(e) What is the smallest price per tonne for this year that would mean a farmer would make more money overall by not setting aside any of his or her land? [2]

Jacques predicted that next year would be such a high price per tonne that no set-aside would be worthwhile, so he grew a hedge between each hectare of his land so that he could get the subsidy for having them next year.

(f) How many metres of hedge would he now have in total? [1]

[Question 2 begins on the next page]

2 The Goodlen Dancing Society holds a dancing competition every Saturday. Ten couples take part each Saturday, and each couple dances the Waltz and the Jive. Their performances are judged by five experts and by the audience.

For each dance, each expert gives each couple a score out of 10. For each couple, the highest score and the lowest score are ignored and the remaining three scores are added together to give the total score for that dance.

Points are then awarded as follows: 12 for the highest total score, 10 for the second-highest total score, 8 for the third-highest total score, then 7, 6, 5, 4, 3, 2 ending with 1 point for the tenth-highest total score. If two or more couples have the same total score, then each of the couples is given the points corresponding to that score. For example, if the leading four couples have scores 25, 24, 24, 23, then they will receive 12, 10, 10, 7 points respectively.

The scores awarded last Saturday by the experts for the Waltz are shown in the following table.

Couple	A	B	C	D	E	F	G	H	I	J
Expert 1	8	6	5	9	4	8	8	6	7	9
Expert 2	7	7	5	8	5	8	9	7	8	7
Expert 3	8	8	6	8	7	8	6	6	7	6
Expert 4	8	7	8	6	5	8	9	7	8	9
Expert 5	7	6	5	8	8	8	8	6	7	8

(a) Copy and complete the table below to show the total scores and points after the Waltz.

Couple	A	B	C	D	E	F	G	H	I	J
Total score for Waltz	23	20	16		17	24	25		22	24
Points for Waltz	6									

[2]

For the Jive, the points awarded were as follows:

Couple	A	B	C	D	E	F	G	H	I	J
Points for Jive	6	3	2	10	4	7	6	12	1	8

(b) Which couple had the highest total number of points after the two dances, and how many points did they have? [1]

The audience vote is also taken into consideration. The following table shows the percentage of the total audience vote gained by each couple.

Couple	A	B	C	D	E	F	G	H	I	J
% of vote	6	4	5	8	7	10	21	14	9	16

Points are awarded as for the two dances (12, 10, 8, 7 etc.) based on this percentage vote. These points are added to those gained from the experts' scores to give a grand total.

The five couples with the highest grand totals qualify for the final.

(c) Which couple had the highest grand total, and what was this grand total? [2]

It was later discovered that there was an error in the recording of the audience vote. The percentages scored by couples B and H had been switched and in fact, couple B gained 14% and couple H gained 4% of the audience vote.

(d) Couple B said that they should have been in the final. Show that they were incorrect. [2]

This Saturday, the same ten couples competed again in the dancing competition. All the rules for scoring and awarding points were the same as last Saturday. The points gained by each couple as a result of the experts' scores for the Waltz are shown in the following table.

Couple	A	B	C	D	E	F	G	H	I	J
Points for Waltz	2	5	4	12	10	6	8	1	7	3

After the points had been awarded for the Jive, five couples were tied for 1st place with 16 points each and four couples were tied for 6th place. No two couples received the same number of points for the Jive.

(e) Which couple were in 10th place and how many points did they have? [3]

The points from the audience vote were then added to give the grand total for each couple.

(f) Explain why none of the couples who tied for 6th place after the two dances could have the highest grand total when the audience vote is included. [2]

In the audience vote, each couple gained at least 1% of the vote and each percentage was an exact whole number. No couples had equal percentages of the vote. Couple D had the highest percentage of the vote.

(g) What are the least and the greatest percentages of the audience vote that couple D could have received? [3]

3 The Bolandian Photographic Society (BPS) organises an annual competition to encourage young people to take photographs of local wildlife. A businessman donates $400 each year to be awarded as cash prizes for the best photographs, and all $400 must be awarded in prizes.

In 2001, the BPS decides to award four prizes, each of a different amount and each a whole number of dollars, in decreasing value. No prize will be more than $200 and no prize will be less than $20.

(a) **(i)** The lowest possible value of the second prize is $68.

State possible values for the other three prizes. [2]

(ii) Find the greatest possible value of the second prize. [1]

Amos enters two photographs in the competition and he wins both the second and third prizes. His total prize money is $175.

(b) What is the lowest possible value that the first prize could have been? [2]

In 2002, the BPS decides that there will be five cash prizes in the competition. The businessman still donates $400 and the restrictions on the values of the prizes still hold.

In addition, the five prizes will be such that the difference between the values of any two consecutive prizes will be the same, and that this difference will be as large as possible.

(c) What are the values of the five prizes? [2]

In 2003, the BPS decides that, instead of having the difference between any two consecutive prizes as the same, the third prize will be equal to one half of the first prize.

(d) What is the greatest possible value of the second prize? [3]

4 John is a ferryman – he takes people across the River Butley near where he lives. His charges, to carry people from one side of the river to the other, are as follows:

Pedestrian: \$4
Cyclist: \$6

At the beginning of the day, John puts some money in his cashbox so that he is able to give change to his customers. This initial money is called the 'float'.

The local currency has only notes with values of \$1, \$10 and \$25.

(a) What are the different amounts of change that John might need to give to an individual customer (who pays only for herself)? [2]

John decides to have enough of each type of note in his float to be able to give change to the first customers using the smallest number of notes that the local currency allows.

(b) How many of each type of note does John need in his float to be able to give change to the first customer, if she is an individual (who pays only for herself)? [2]

The ferry has space for 12 pedestrian passengers. A cyclist takes up the space of two pedestrians. The ferry is not always full when it departs.

(c) Sometimes one customer pays for a whole group.

What is the minimum float that John would need to be sure that he can give change to such a customer, if they were the first customer of the day? [3]

From experience, John knows that, for individual customers (who pay only for themselves)

- half of them pay their fare with \$1 notes,
- one third pay with a \$10 note,
- the remaining sixth pay with a \$25 note.

(d) If the first 12 customers are all pedestrians paying individually, and who pay as described above,

(i) what is the minimum float that John would need to ensure that he will be able to give them all change, whatever order they arrive in? [2]

(ii) what is the minimum float that John could have and still be able to give them all change? [3]

One morning John forgot to bring his cashbox. The first ferry was full, and all of the customers paid individually. Fortunately, John was able to give change to all of them, by choosing the order in which they paid.

(e) What is the maximum number of \$10 notes that John could have received? Justify your answer. [3]

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