



Cambridge International AS & A Level

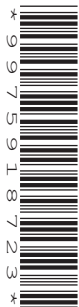
THINKING SKILLS

9694/33

Paper 3 Problem Analysis and Solution

May/June 2021

2 hours



You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
In most questions, full marks will be awarded for a correct answer without any working. In some questions, however, you will not be awarded full marks if working needed to support an answer is not shown.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **8** pages. Any blank pages are indicated.

- 1 A 'tidal river' is the lower section of a river near the coast, where the water flows upstream when the tide is coming in (between low tide and high tide) and flows downstream when the tide is going out (between high tide and low tide).

Heather's motor boat goes at 5 km/h through the water and a full tank has enough fuel to go for 8 hours. She lives in a house next to a tidal river and tried to use a simple model for boat journeys: The water flows at 1 km/h upstream for the 6 hours between low tide and high tide, and then 1 km/h downstream for the following 6 hours until the next low tide. She tries to time her journeys so that, where possible, she saves time and fuel by travelling in the same direction that the water is flowing.

- (a) (i) What is the furthest she could travel in one direction on a continuous 8-hour journey? [2]
(ii) What range of times in relation to low tide could she start such a journey going upstream? [1]

She now considers a better model that still assumes a 12-hour cycle: the water does not flow for 90 minutes before low tide and 90 minutes after low tide, nor for 90 minutes before high tide and 90 minutes after high tide; the rest of the time it flows at 2 km/h.

- (b) What is the furthest she could travel upstream in a continuous 8-hour journey? [1]
(c) What is the furthest point from where she starts that she could travel to and back from in a continuous 8 hour journey? [2]
(d) Heather wants to make a journey to and from a town 26 km upstream, with continuous travel for 4 hours up, a break in the town with the engine off and then 4 hours down.
(i) At what range of times after low tide could she start? [2]
(ii) What would be the shortest possible break? [1]
(e) Unfortunately, her boat was left untied at her house, with the motor turned off. The boat drifted downstream on the tide and was then lost at sea.

Using the model, what is the furthest upstream from the coast that her house could be? [1]

- 2 Meghan runs a candle shop, making and selling decorated candles in three different sizes. Her partner Andy and their daughter Violet also work in the shop.

Meghan makes large candles in batches of 6 and each batch takes her 3 hours to make. No candle in a batch is complete until the end of the 3 hours. She works from 09:00 until 18:00 each day from Monday to Saturday and has Sundays off. She takes 90 minutes each day for refreshment breaks.

Candles that are not complete at the end of a day will be completed on the next working day.

- (a) How many candles does Meghan complete in one week? [2]

Andy makes medium candles in batches of 6 and each batch takes him 2 hours to make. No candle in a batch is complete until the end of the 2 hours. He works the same hours and days as Meghan and takes the same breaks. As well as making candles, Andy serves customers in the shop. This takes between 25% and 50% of his working time, excluding breaks, each day.

- (b) (i) Show that Andy should complete at least 66 candles in a week. [1]
 (ii) Find the greatest number of candles that Andy might complete in one week. [2]
 (iii) Explain how it is possible that on any given day Andy might not finish making his first batch of candles until 16:15. [2]

Violet makes small candles individually and each candle takes her 45 minutes to complete. She works from 09:00 to 13:00 on Mondays to Fridays and does not take any refreshment breaks. She does not work on Saturdays and Sundays.

At 09:00 on Monday 1 May no candles were incomplete from the previous week.

- (c) (i) What is the greatest total number of candles that could be completed by Meghan, Andy and Violet by 13:30 on Tuesday 2 May? [3]
 (ii) What is the least total number of candles that could be completed by Meghan, Andy and Violet by 13:30 on Tuesday 2 May? [2]

Meghan decides to employ Sam to serve customers in the shop so that Andy can devote all his time to making candles.

At 09:00 on Monday 22 May no candles were incomplete from the previous week. Meghan, Andy and Violet all start making candles at 09:00. Meghan and Andy both take their breaks from 12:00 to 13:30 every day.

- (d) At what time and on what day is the 50th candle complete? Who makes this 50th candle? [3]

- 3 Helena and Daisy are thinking about how they can select 6 cards from a box of 100 cards. They wish to have a fair method which involves both of them making decisions that will determine the final set of 6 cards.

Helena begins by choosing a number and taking that many cards from the box. The following process, called a 'division', is then repeated as many times as necessary.

Helena splits the cards into two equal piles, discarding the extra card if necessary.

Daisy chooses one of the two piles to keep and discards the other.

Three cards from the box are added to the pile that Daisy chose to keep.

They repeat this whole process until the end result is a set of 6 cards (after the three cards have been added to Daisy's chosen pile). The cards that are discarded during the process are **not** returned to the box.

- (a) If Helena chooses to begin with 11 cards from the box:
- (i) Show that the method will result in 6 cards after three divisions. [2]
 - (ii) How many cards will be left in the box once the set of 6 cards has been selected? [1]
- (b) How many divisions would be required to result in 6 cards if Helena chooses to begin with 35 cards from the box? [2]
- (c) What is the smallest number of cards Helena could choose to begin with so that four divisions will be required to result in 6 cards? [2]
- (d) What is the largest number of cards that Helena could choose to begin with so that they will not run out of cards in the box before the process results in 6 cards? [3]

[Question 4 begins on the next page]

- 4 A quiz is held on the second Friday of every month at the local village hall. At each quiz, the maximum score that can be achieved is 50 points. The team with the highest score receives a \$40 prize, the team in second place receives \$30 and the team in third place receives \$25. If teams are tied on the same score then additional questions are asked to determine the rank order, but the score for the quiz is not changed.

The organisers want to award a prize to the best team at the end of the year, but they are aware that most of the teams do not compete every month and so they will not simply use the total score. After each quiz a table showing the results of that quiz and some information about the performances so far this year for all teams who have won at least one prize is published on a website. The table that appeared after the October quiz is shown below.

<i>Team Captain</i>	<i>Points scored in October</i>	<i>Quizzes entered</i>	<i>Best score in any quiz</i>	<i>Prize money so far</i>
Ali	41	8	46	\$100
Bianca	31	9	42	\$95
Chen	Did not compete	7	45	\$90
Duong	45	5	48	\$155
Ellen	Did not compete	9	47	\$55
Francesca	38	6	44	\$145
Grigory	37	7	43	\$170
Hari	Did not compete	4	40	\$140

- (a) (i) How many times has Bianca's team won each type of prize? [1]
- (ii) What are the two possible combinations of prizes for Chen's team? [1]
- (b) Which team must have finished in second place in the month that Duong's team scored their highest score? [1]
- Duong's and Hari's teams are the only ones that have won a prize every time they have entered the quiz.
- (c) (i) Explain how it can be deduced that Hari's team has won first prize exactly twice. [2]
- (ii) Explain how it can be deduced that Duong's team has won first prize exactly twice. [3]
- (d) Based solely on the information above, what is the maximum number of teams that could have received third prize on at least two occasions this year? Explain your answer. [2]

The quiz organisers have announced that they will use the following system to determine the overall winning team for the year:

- The team with the most first places will be the winning team.
- If there is a tie between teams then the highest quiz score over the year will be used to determine the winning team.
- If there is still a tie between teams then these teams will be asked additional questions after the December quiz has finished to determine a winner.

The organisers have also announced that:

- There are six teams who could be the overall winning team for the year.
- Francesca's team are currently in the lead as the only team with three first places.

Grigory's team have only managed to finish in first place once, so they must win both of the remaining quizzes to be overall winners.

(e) Draw a table to show how many of each type of prize has been won by each team. [5]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.