ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

Candidates should be aware that if a method is specified by the question, then they must use that method for their solution. The use of the words 'Hence' or 'use your…' in the second part of a question is an indication that the method employed should use the result from the previous part. Care should be taken to read the wording of such questions.

Candidates should be aware that if they are requested not to use a calculator it is particularly important to show all steps in their working.

Candidates should read questions carefully and check that they have fully answered the question and have given their answer in the required form. For example, in **Question 4** answers had to be given in terms of π. Answers should usually be given to 3 significant figures and candidates should be particularly careful when expressing an answer to 3 significant figures for a decimal number with a leading zero.

General comments

This paper required candidates to recall and use a range of mathematical techniques, to devise mathematical arguments and present those arguments precisely and logically. Good responses were set out clearly and demonstrated a good understanding of fundamental techniques. They employed carefully chosen and succinct methods and showed that the requirements for each question had been read carefully.

A good range of responses were provided, showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Most candidates attempted all the questions, but there were some who did not answer the final two questions. It is not clear if this was through constraints on time.

There were some topics where candidates appeared to be less familiar with the techniques required and they would benefit from practice in answering questions from all areas of the syllabus as detailed below.

Comments on specific questions

Question 1

(a) Although most candidates knew that the x values –3, 1 and 5 had to be used, few realised that the portion of the curve below the x-axis satisfied the inequality $f(x) < 0$. Some gave either just $-3 < x < 1$ or $x > 5$. Some misread the inequality in the question and gave $x < -3$, $1 < x < 5$ as an answer.

(b) The majority of candidates knew that $f(x) = (x + 3)(x - 1)(x - 5)$ would lead to the given intercepts on the x-axis but most ignored the corrections required to give a graph of the correct orientation and to give a correct y intercept. Those who attempted to solve four equations in four unknowns were not successful and candidates should be advised not to attempt this method. Candidates would benefit from more practice in interpretation and understanding of cubic functions.

Question 2

(a) Candidates did well with this question, showing a good understanding of the various ways of expressing powers of x, y and z and the manipulation required. $\sqrt[3]{xy}$ presented the most difficulty, with an incorrect fraction such as $\frac{3}{2}$ sometimes being used. Most candidates knew when to add and subtract indices but there were sometimes mistakes in handling the fractions. Nearly all candidates gave exact answers as requested, with very few giving their answers as decimals.

(b) Most candidates were able to form an equation in 2^p and correctly solve it. However, not all candidates were able to obtain a value of p from their 2^p and would benefit from practice in the use of logarithms to solve such equations. Some otherwise successful attempts did not give an answer correct to 3 significant figures. Candidates should be aware of how to round numbers with a leading zero to 3 significant figures.

Question 3

(a) Most candidates used the power rule to obtain a^2 and b^4. Most of those then used the subtraction rule to obtain $\frac{a^2}{b^4}$ within a logarithm, but some, who clearly misunderstood the laws of logarithms, used $\frac{\lg a^2}{\lg b^4}$ and could not progress further. Many candidates did not express 3 as $\lg 10^3$ and so were then unable to use the multiplication rule to form a single logarithm. Some candidates did not evaluate 10^3 as 1000 in their final answer.

(b) It was expected that a change of base would be made and either $3\log_a 4$ would be expressed as $\frac{3}{\log_4 a}$ or $2\log_4 a$ would be expressed as $\frac{2}{\log_a 4}$ and that a quadratic equation would be formed. However, candidates tended to use the power rule and addition rule to obtain expressions that did not lend themselves to the formation of equations and so they could not reach a solution. Of those who did form and solve quadratic equations candidates who worked with $\log_4 a$ found obtaining a value of a more straightforward than those who worked with $\log_a 4$.

Question 4

Most candidates obtained $\tan\left(2x+\frac{\pi}{3}\right)=\frac{1}{\sqrt{3}}$ but the work leading to this was not always clearly expressed. Most then obtained $2x+\frac{\pi}{3}=\frac{\pi}{6}$ and correctly manipulated it for a first correct solution. Not all candidates found more than this one solution and few realised that there were, in fact, four solutions in the range. As subtracting $\frac{\pi}{3}$ and dividing by 2 may produce answers within range, candidates should look at further positive and negative angles which satisfy $\tan^{-1}\frac{1}{\sqrt{3}}$. The question required answers in terms of π and most candidates complied with this.

Question 5

Many candidates did not identify a technique to answer this question but successful candidates realised that $y = c$ was a horizontal line and associated this line with maximum and minimum values. Both values of c were found by those using a sketch graph and by those who knew the maximum and minimum values of the sine function. Others who differentiated to find turning points tended to find just one value of c. Candidates should be aware that in the absence of a degree sign, the angle was measured in radians and that the angle calculated for the maximum and minimum should have been in radians.

Question 6

(a) Most candidates showed a good knowledge of the factor and remainder theorems and their unsimplified equations were often correct. Candidates should be aware that the most straightforward approach was to rearrange the first equation to either $a + 4b = -15$ or $2a + 8b = -30$ and then it could easily be solved together with $a + b = -24$. Candidates who continued to work with fractions and decimals tended not to show all of their working.

(b) Candidates who used algebraic long division were usually successful in finding a quadratic factor and then all three factors. Candidates who used synthetic division with $-\frac{1}{2}$ did not always obtain a correct quadratic factor. Those who did manage to obtain all three correct factors following a synthetic division sometimes then lacked clarity and wrote down incorrect statements such as $10x^3 - 27x^2 - 10x + 3 = (2x + 1)(10x^2 - 32x + 6)$ in their working. An understanding of the process would have been shown by $(x + \frac{1}{2})(10x^2 - 32x + 6) = (2x + 1)(5x^2 - 16x + 3)$.

(c) Candidates who calculated f(0) were the most successful. Those who used algebraic long division by x were also successful.

Question 7

(a) (i) Most candidates obtained **b – a**.

(ii) Most candidates obtained **c – b**.

(iii) Most candidates did not use the given ratio to obtain $n\overrightarrow{AB} = m\overrightarrow{BC}$ and were unable to show the given result. Some candidates gave 'correct' proofs that used **a – c** but this did not follow from the previous part as required by the question.

(b) Most candidates considered the x components and y components separately but there were some who tried to combine the two components and could not obtain a pair of equations to solve. Some candidates did not obtain fully correct equations because of errors in expanding $(\mu - 1)(-4)$, $(\mu - 1)(7)$ and $-2(\lambda + 1)$. Further sign errors occurred when like terms were collected to form the simultaneous equations. Most candidates knew how to solve the simultaneous equations that they had obtained, but accuracy was often lost because of earlier errors.

Question 8

(a) Many candidates made a good start and obtained a product involving $^5P_3$ but not all successfully developed a strategy to answer the question. Successful candidates split the problem into two cases such as 'greater than 50 000 and starts with a 6' and 'greater than 50 000 does not start with a 6' but others had not appreciated the relevance of the number 6 and took a less systematic approach.

(b) Most candidates stated the correct equality in terms of factorials, but many went no further. The simplification of the numerical factorials to 72 was more successful than the simplification of the algebraic factorials. The candidates who obtained a correct quadratic equation usually went on to find the correct answer with a few neglecting to reject the negative answer.

Question 9

(a) Nearly all candidates succeeded using one of the many trigonometric methods available but the requirement to show that the value was correct to 3 decimal places was usually missed out. To show the given result an answer of 1.1760 rounded to 1.176 was expected.

(b) Candidates should be aware that a clear plan, derived from careful study of the diagram, is required in perimeter questions. In this case the major arc, lengths ND and MA and the lengths of three sides of the rectangle had to be added together. There was some confusion between major and minor arcs. Rather than using $r\theta$ with $r = 12$ and $\theta = 2\pi - 1.176$, some candidates made the calculation of the length of the major arc too complicated, and prone to error, by attempting to use fractions of the full perimeter or by using the angles AOB and DOC. Candidates should be aware that use of degrees in circular measure questions is almost certainly likely to lead to inaccuracies. Calculations of *ND* and *MA* were not always correct. Two lengths equal to *ND* were not always included and sometimes the sides of the rectangle were not added.

(c) Candidates were more successful in this part than in **part (b)** but some did not form a correct plan. Again, there was some confusion between major and minor sectors. Successful candidates used $\frac{1}{2}r^2(2\pi - 1.176)$ to calculate the area of the major sector rather than more complicated routes. Calculation of the areas of the triangles to be subtracted was sometimes made unnecessarily complicated as the height and length of these triangles could easily be calculated from the lengths 4 cm and 6 cm given on the diagram without the use of trigonometry.

Question 10

(a) Most candidates knew what to do to find the *y*-coordinates. Some made errors in the calculation to find the *y*-coordinate at *B* but the most usual reason for loss of marks was ignoring the request for an exact answer and expressing $\frac{13}{16}$ as a rounded decimal.

(b) This was a complex question requiring candidates to plan carefully and use several different techniques. It also required candidates to work with exact figures and not rounded decimals. The most successful method was to find the area below the curve using integration and then subtract it from the area of the trapezium, found using $\frac{1}{2}h(a+b)$. Finding the equation of the line *AB* and integrating 'equation of line minus equation of curve' presented too many chances for error. Most candidates knew how integrate $\frac{3}{x+2}$ but the integration of $\frac{1}{(x+2)^2}$ seemed to be less familiar and a common misconception was that this expression could also be integrated to give a ln. Some candidates tried to combine the two terms before integration to obtain an expression that could not be integrated. There were some sign errors but candidates with correct integrals usually went on to apply limits correctly. However, some did not give the final answer in the form required by the question.

Question 11

(a) (i) Many correct answers were seen but candidates did not always appreciate that the graph was a velocity–time graph and so did not find the gradient at $t = 12$. Some read off a velocity from the graph and divided 30 by 12.

(ii) Again, candidates did not always appreciate that the graph was a velocity–time graph and so did not find the gradient at $t = 50$.

(iii) Many good solutions were seen. Some candidates made slips with reading from the graph and in their calculations but nearly all were attempting to calculate the area below the graph.

(b) (i) Some candidates attempted differentiation of the expression for *v* but most candidates knew they had to substitute the value of *t* into the given expression for *v*. Answers other than –2 came from incorrect order of operations in the use of a calculator. The relationship between velocity and speed was not fully understood as many candidates who obtained $v = -2$ did not go on to give an answer of 2 for the speed.

(ii) Candidates knew that differentiation was required and many differentiated correctly but some had the wrong sign, some did not multiply by 3 and some did not eliminate 4. Most equated the acceleration to zero but looked no further than $3t = 0$, not appreciating that a positive value of *t* was required. Candidates should be aware that *t* was measured in radians.

(iii) Most candidates attempted integration but although good attempts were made to integrate $4\cos 3t$ some candidates did not integrate –4 and some integrated –4 to obtain $-4x$ rather than $-4t$. Some candidates who had integrated correctly left a constant of integration in their final answer.

ADDITIONAL MATHEMATICS

Paper 4037/13
Paper 13

Key messages

Candidates are reminded of the importance of familiarising themselves with the rubric on the front cover of the examination paper. It would be advantageous if this could be done before the examination using a past paper. It is essential that each question is read carefully to ensure that the demands of the question are met fully and that the answer is in the correct form. It was apparent that few candidates check their work especially when a given answer is not obtained.

General comments

The paper covered a wide range of topics from the syllabus. Questions were designed so that candidates would be able to show what they had learned and apply mathematical techniques correctly and appropriately. There appeared to be no timing issues and most candidates had sufficient space in which to answer the questions. Should extra space be needed, it is quite acceptable to use any additional blank pages in the examination booklet before using additional paper. It was pleasing to see that many candidates indicated where a question was continued when extra space was needed. It was also pleasing to see that very few candidates wrote a solution in pencil first and then went over it in pen afterwards which causes problems for examiners as it can be difficult to read.

Comments on specific questions

Question 1

Most candidates were able to sketch a cubic curve. Errors occurred when the negative sign was not considered, thus giving an incorrect orientation. Some errors also occurred with the identification of the intercepts with the coordinate axes. It was noted that many candidates drew curves with a maximum point very close to or on the *y*-axis, rather than further into the first quadrant.

Question 2

Many candidates are still not familiar with the difference between velocity and speed, with the most common answer to this question being $-4.91\,\text{ms}^{-1}$ rather than the correct $4.91\,\text{ms}^{-1}$. Candidates should be guided by the mark allocation of a question which usually indicates the approximate number of steps needed in the solution.

Question 3

Many candidates had difficulty obtaining the given equation in a form that could be solved easily. It was intended that $\cot^2\left(2x-\frac{\pi}{3}\right)=\frac{1}{3}$ be rewritten as $\tan\left(2x-\frac{\pi}{3}\right)=\pm\sqrt{3}$ in order to find the solutions. Use of other trigonometric identities was perfectly acceptable although involved slightly more work. Many candidates who obtained a form which enabled solution, usually obtained two solutions only as the negative cases were not considered. Of those candidates that did consider both positive and negative cases, many omitted the solution $x=0$ as no negative angles were considered when working through the solution.

Question 4

(a) Many correct solutions were seen. Occasional errors in the simplification of the coefficients of x^2 and x^4 occurred.

(b) Too many candidates were unable to expand out and simplify $\left(2x+\frac{1}{x}\right)^2$ correctly. If a term involving *x* was included in this expansion, then candidates were unable to make any correct progress. For those candidates with a correct expansion or an expansion in a correct form, most made use of their answer to **part (a)** correctly and were able to gain a method mark. Of these solutions, many were fully correct, but most errors involved the omission of the negative sign in the expansion from **part (a)**.

Question 5

(a) Some candidates were not overly familiar with the topic of geometric progressions, which is a recent addition to the syllabus. It was essential that the question was read carefully so that a correct equation be formed. Many candidates obtained a correct simplified equation of $1-r^4=17\left(1-r^2\right)$. Unfortunately, many candidates were unable to solve this equation correctly with the answer of $r=4$ usually appearing after a statement of $r^2=-16$. This should have prompted candidates to check their work for errors.

(b) Most candidates were able to make use of their common ratio and attempt to find the first term of the progression.

(c) Of the candidates who obtained a positive common ratio greater than 1, most made a correct statement about the condition needed for a progression to have a sum to infinity.

Question 6

(a) There were many correct solutions to this question. There were several ways of obtaining the correct answer of 4368, with the most common way to consider the case of numbers starting with 7 or 9 and the case of numbers starting with 8. Some candidates did attempt this method but thought that only seven numbers needed to be considered after 'fixing' the first and last digits. The error was to not consider the 0 correctly.

(b) Many correct solutions were seen although some candidates still have problems with simplification of factorials involving unknowns within an equation.

Question 7

(a) Most candidates were able to gain a method mark by using a correct approach to finding the required angle. Solutions needed to include work using an accuracy greater than 3 decimal places, so that a conclusion of angle *POA* = 2.366 radians, correct to 3 decimal places, could be justified, to obtain both marks. Some candidates still use degrees and then convert to radians. Although this method is not penalised, it should be discouraged.

(b) Many correct solutions were seen as most candidates are familiar with the use of areas of sectors of circles and triangles, to find complex areas. Some incorrect methods using the area of a triangle were seen.

(c) Many correct solutions were seen as most candidates are familiar with the use of arc lengths of sectors of circles and straight lines to find complex perimeters. Some errors were made but these usually involved considering an incorrect number of radii.

Question 8

(a) Although the coordinates of the mid-point were given, most candidates produced a completely correct solution showing sufficient detail for this 'show that' question.

(b) Many correct solutions were seen, with most candidates finding the equation of the normal to the line *AB*, passing through the point (2, 9). Most candidates then showed sufficient working to show that the point (12, 7) lies on this normal. Other correct methods were equally acceptable.

(c) It was intended that the displacement vectors be used to find the possible coordinates of the point *D*. Those candidates that attempted this approach were more successful than those who chose to form an equation using the distance of the point *D* from the line *AB* and the distance of the point *C* from the line *AB*. The use of the equation of the normal obtained in **part (b)** was also needed for this method. Candidates who used this approach were given credit for obtaining an equation of the form $(x-2)^2+\left(-\frac{1}{5}x+\frac{47}{5}-9\right)^2=416$ or equivalent. Correct solutions of this equation were seen although this method was much lengthier with the scope for errors being much greater. Some fortuitous solutions were seen due to incorrect solving of an equation involving terms in both *x* and *y*.

Question 9

(a) A variety of responses was seen. Many candidates obtained the correct equation of $y=\frac{1}{2}\ln\left(2.1x^2-0.44\right)$ but then went on to use the laws of logarithms incorrectly so that an incorrect final answer was obtained. It was important that candidates use brackets correctly. A correct final answer needed to be seen to obtain the final accuracy mark. Most candidates realised that the form of the equation they need to consider first was $e^{2y}=mx^2+c$ and then went on to find the value of *m* by finding the gradient of the line. Errors in finding *c* usually involved the incorrect use of the given coordinates, although this sometimes involved finding an incorrect value for *m* as well.

(b) It was essential that candidates made use of an equation of the form $y=k\ln\left(px^2\pm q\right)$, $p\neq 1$, $q\neq 0$ or $e^{2y}=mx^2+c$ to find the required value of *y*. Too many candidates had simplified their answer to **part (a)** incorrectly and so were unable to gain credit for this part.

(c) Very few correct solutions were seen. The first step needed was to recognise that to have a logarithm of a quantity, that quantity must be greater then zero. The resulting inequality which needed to be using the form $y=k\ln\left(px^2-q\right)$, $p\neq 1$, $q>0$ then needed to be solved. Credit was given for the consideration of an equation equal to zero. Too many candidates thought that the logarithmic term itself needed to be greater than zero. Some incorrect solving of an incorrect basic equation gave fortuitously correct answers. These were not given any credit. Candidates with a correct solution needed to discount the negative value of *x* as required.

Question 10

(a) There were many correct solutions for this part, with sufficient detail being shown. Most candidates were able to integrate the given function correctly although there were occasional slips with signs and coefficients. Some candidates did not include an arbitrary constant in their initial integration. The inclusion of the requirement to show that when *x* = 11, $\frac{dy}{dx}=52$ was meant to alert candidates that if they did not obtain this result, there was an error in their working, either a sign or coefficient error, or the omission of the arbitrary constant. Too few candidates did not check their work when the required result was not obtained.

(b) Many correct solutions for this part were also seen although not as many as for **part (a)**. Again, errors involved slips with signs and coefficients and the omission of a second arbitrary constant which needed to be found. Candidates appeared to be well practised at this type of question.

Question 11

(a) Most candidates recognised the need for differentiation of a quotient and did this correctly. Few errors were seen in the differentiation although the differentiation of $\left(x^2-5\right)^{\frac{1}{3}}$ caused the occasional error in the term involving $\frac{2x}{3}$ in $\frac{2x}{3}\left(x^2-5\right)^{-\frac{2}{3}}$. Many candidates were unable to use correct algebraic processes to obtain the form as given in the question. Simplification of algebraic fractions of this type should be practised as it is a skill which is essential in mathematics. As a result, many candidates were unable to gain the last three marks in this part.

(b) Candidates were only able to score marks in this part if they had managed to obtain a quadratic numerator in **part (a)**. Of those that obtained a correct quadratic numerator, most solved it correctly, but some omitted to discard the negative value of x as it was given that $x > -1$.

(c) Most candidates attempted to describe a method using the second derivative. Although most stated that $\frac{d^2y}{dx^2}$ needed to be found, many then omitted to state that the value of x at the stationary point then needed to be substituted in. This was an important and necessary point in the explanation and without it, candidates could not gain any marks. Some candidates attempted to describe a method using consideration of the gradient either side of the stationary point. Provided sufficient detail was provided, this was usually successful. It should be noted that this part of the question could have been attempted even if **parts (a)** and **(b)** had been unsuccessful.

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

It is important that candidates:

- are familiar with the requirements of the rubric on the front page prior to the examination, particularly with reference to the degree of accuracy required
- are familiar with the mathematical formulae provided on the second page of the examination paper
- show sufficiently clear and logical steps in their working
- understand the requirements of the question by taking note of the key words and phrases in a question
- check that the requirements of the question have been met by re-reading it before moving on
- ensure they have a thorough coverage of the syllabus to access the whole paper.

General comments

Good responses were well-structure with clear and logical steps. Those with minimal or missing steps, and disorganised responses were more likely to lose a greater number of marks in the event of minor errors. Showing all method steps was especially important in 'Show that…' questions and non-calculator questions, in which the key to accessing marks was to show key steps in the response. The need for this was seen in **Questions 3(a)**, **6** and **9(b)** in this examination.

Many candidates gave their final answers to the required degree of accuracy as stated in the rubric on the front page. Those that wrote down correct answers to a greater degree of accuracy did not lose marks when rounding errors were subsequently made. A lack of familiarity with the level of accuracy required was evident in **Questions 3(b)**, **5(b)** and **8(c)** in this examination. Errors in the final answer resulting from prematurely rounding answers in intermediate work were seen in **Questions 3(b), 5(b) and 10(b)(ii)**.

When making a substitution to simplify their working, some candidates would have benefitted from choosing a more appropriate letter. Some confusion and error resulted from an inappropriate choice of letter in **Questions 5(a)**, **5(b)** and **8** in this examination.

The omission of brackets was a cause of errors and loss of marks for some candidates, which was more evident in **Questions 6, 7, 8, and 10a**.

When an answer space provided insufficient space for a candidate's response, it was helpful to see additional sheets annotated with the appropriate question number, whilst also annotating the main response space with an instruction to see the additional sheet. When candidates did not annotate the question number nor referenced extra work in the main answer booklet, it was sometimes difficult to decipher which question their response related to.

Where candidates wish to delete work, it should be advised that this be done with a single line, so that work beneath can be read, as there are occasions when discarded work with credit within it can gain marks. Candidates are also advised not to delete work until they have replaced it.

Most candidates appeared to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

The best responses demonstrated an understanding that the two parts of this question were connected, which was indicated in the wording used for the second part of the question 'Using your graphs…'.

(a) This part required an accurate ruled plot of the graphs to score full marks. This ought to have been apparent from the use of 'Draw…' with a scaled grid provided rather than 'Sketch…' and a set of axes without a scale. Many candidates were able to correctly draw the line, although the modulus function posed more of a challenge, with many candidates not able to locate the vertex of the graph correctly. An effective way to locate the vertex demonstrated by some candidates was to plot the two halves of the modulus function separately which resulted in two intersecting lines, although this scored no marks when candidates left their response as an X-shape graph and did not erase the section below the intersection. Some candidates seemed unaware that the modulus graph was a V-shape, with W-shapes and curves seen.

(b) The intention of this question was to use accurately plotted graphs in the previous part to solve the inequality. Candidates who were unable to draw accurate graphs were able to access this part of the question using the lengthier, algebraic route. Many candidates drew graphs with two intersections. However, not all of these candidates were able to make the connection between the intersections and the critical values they needed to answer this part. Of those candidates correctly finding the critical values, either graphically or algebraically, a significant number of responses were spoilt by giving the final answer as an incorrect inequality. This often followed a correct response. For example, $x < -2$ or $x > 2$ was often spoilt by being followed with $2 < x < -2$. Algebraic solutions all too often resulted in incorrect critical values.

Question 2

Most candidates found this question accessible. They realised that the two parts of the question were connected and worked with their expansion from the first part to answer the second part.

(a) Most candidates answered this question competently, applying the binomial theorem correctly and simplifying terms without error. The negative $3x$ term was dealt with well and relatively few sign errors were seen. Some candidates had the first or last term missing, and some coefficients were incorrectly evaluated by a small number of candidates.

(b) The most efficient responses only listed the terms that were necessary to solve the problem. However, many candidates gave more terms than was necessary and were still able to complete the solution successfully. A few candidates omitted terms that were necessary, and this was often because they had not used sufficient terms from their answer to the previous part of the question.

Most candidates correctly arrived at $a = 2$. Errors usually involved the $\frac{a}{x}$ term, for example a missing in one of the terms, or incorrect cancelling of x. This was an expensive error unless enough steps had been shown so that marks could be awarded for the expansion before the cancelling down errors were made. Despite many correct expansions having been seen, there were a significant number of responses that only equated one term, instead of two, to find b and/or c. An incorrect expansion in the first part could still gain partial credit in this part for those correctly following through. The occasional miscopying of a term from a correct expansion resulted in a loss of marks.

Question 3

(a) Many exemplary solutions were seen to this question, using the expected method of working from the left-hand side expression in a clear and logical manner to arrive at the right-hand side expression. Writing the two fractions as a single fraction and using the given identity relating sec*x* and tan*x* seemed to be a more popular route than multiplying by cos*x* first and then writing as a single fraction, although the latter proved to be a shorter route if done correctly. Candidates should be encouraged to be clear and correct in setting out of their solutions by: ensuring all necessary brackets are shown; abbreviations are not used for the trigonometric expressions, for example 'c' for cos*x* and 's' for sin*x*; arguments are included in all expressions; and tan*x* written as $\frac{\sin x}{\cos x}$

rather than $\frac{\sin}{\cos}x$. Additionally, fractions within fractions can be ambiguous and should be avoided where possible; when not possible to avoid, then re-writing them in a clearer form at the next step should be encouraged, rather than continuing to work at length with fractions within fractions. A very small number of candidates thought that sec$x$ was the reciprocal of sin$x$ rather than cos$x$.

(b) Many candidates were able to reach a correct value for $\sin^2 x$ or $\cos^2 x$, or more commonly for $\tan^2 x$. In square rooting their value, many omitted the negative square root and lost two solutions; or their square root sign was not fully over the fraction, leading to incorrect angles. A few candidates did not take the square root, thinking that taking the inverse tangent function twice was the way to deal with the squared tangent function. Others missed solutions by not realising how to deal with –39.2 to obtain an answer in the correct range. Candidates would do well to remember that 'hence' indicates the need to use the answer to the previous part of the question. Those that did not use the identity given in the previous part generally struggled with the algebra in their attempts with working in sec$x$. Angles in degrees are expected to 1 decimal place, so those candidates working to 3 significant figures did not score the accuracy marks unless they had written more accurate answers prior to their rounding. There were relatively few cases of calculators being used in a mode other than the degrees mode.

Question 4

(a) This question was very well answered. Most candidates were able to correctly differentiate $3\ln x$. Most of these candidates were able to complete the solution correctly. Solution of the correct quadratic equation was generally done by factorising. A few responses rearranged $3\ln x$ to $\ln x^3$ first, thus differentiating to $\frac{3x^2}{x^3}$ before simplifying to $\frac{3}{x}$, although more errors were seen by candidates taking this route.

(b) Most candidates used the second derivative test to correctly identify the nature of the stationary points. Some candidates would have improved if they had taken more care when substituting values into the second derivative and interpreting the result. Weaker responses often suggested that a negative value represented a minimum and a positive value a maximum, for example. Fewer candidates examined the sign of $\frac{dy}{dx}$, and often only gave evidence for one stationary point, or sometimes chose x-values which were outside the domain of the function. It could not be assumed in this question that if one stationary point was a maximum then the second one was a minimum so evidence for both was required. Some candidates used the two corresponding y-coordinates to make their conclusions and scored no marks.

Question 5

(a) Many candidates successfully used a substitution such as $a = e^x$ and $b = e^y$ to turn the given equations into a more recognisable and manageable linear form. Use of different letters to those used in the original question was very sensible. Those candidates replacing e^x by x and e^y by y without stating this generally could not be credited. This should be discouraged. Those candidates stating the substitutions $x = e^x$ and $y = e^y$ often scored marks but were at risk of not replacing the substitution at the end. Evidence of poor understanding of laws of indices was common, for example $e^x + e^y = e^{xy}$. Similarly, some candidates had a poor understanding of the laws of logarithms, for example $\ln(e^x + e^y) = \ln e^x + \ln e^y$ resulting in $x + y = \ln 5$. Candidates should be reminded of the requirement to give non-exact numerical answers to 3 significant figures, as an answer of –0.92 did not gain the accuracy mark for y, unless it had been seen as $\ln\frac{2}{5}$ or a more accurate answer before rounded to 2 significant figures. Some candidates successfully arriving at correct values for e^x and e^y did not take the question to completion to find x and y.

(b) Those candidates choosing to work with indices first were often more successful than those choosing to work with logarithms. Another approach used with some success was to make a substitution for e^t and simplify the expression before taking logarithms. Taking logarithms at the outset proved challenging for many candidates, with many using laws of logarithms incorrectly at the very first step, for example writing $\ln(5e^{5t-3}) = 5(5t-3)$ or $\ln(5e^{5t-3}) = (5t-3)\ln 5$. Dealing with the indices first was often done correctly, except when necessary brackets were omitted in the powers. Some candidates managed to correctly navigate the indices and/or logarithms to arrive at a correct unsimplified expression only to make arithmetic slips in collecting terms and making *t* the subject.

Question 6

This question was designed to test a candidate's skills in a non-calculator context. Therefore, sufficiently detailed and correct working needed to be shown to gain full credit. Many realised this, although some candidates may have fared better if they had reminded themselves of this key instruction for each part. Omission of necessary brackets was an issue in all three parts, resulting in errors and loss of marks for some candidates.

(a) Most candidates recognised that this part required the cosine rule, and correctly wrote down the formula, as given at the front of the paper. It was essential that the expansions of the three products $\left(\sqrt{6}+\sqrt{2}\right)^2$, $\left(\sqrt{6}-\sqrt{2}\right)^2$ and $\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)$, and subsequent simplification, be shown in sufficient detail to gain credit. Many candidates substituted into the cosine rule correctly, following with only two-term expansions of the squared brackets and therefore did not gain full credit. Other errors commonly seen included writing $\left(\sqrt{2}\right)^2 = 4$, and not taking the square root of 12 at the end. A significant number of candidates did their calculations on the side, or around the diagram which made it difficult to interpret and therefore credit.

(b) Most candidates were able to apply the sine rule correctly, as given at the front of the paper. Those with an incorrect answer to the previous part were still able to access the method mark. As this was a 'Show that…' question within a non-calculator context, it was essential that the product $\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{12}}=\frac{1}{4}$ was clearly justified with all steps shown. The demand is to show a particular result and even if the step appears to be trivial, such as $\frac{1}{\sqrt{12}}=\frac{1}{2\sqrt{3}}$, it should be included as well as the cancelling down of the roots and the product of 2 × 2. Those with a simplified answer of $2\sqrt{3}$ in **part (a)** found it easier to show their working and were more likely to be successful in this part. Many responses were disorganised, and difficult to work through.

(c) Two correct approaches were often seen when solving this part of the question. Most commonly candidates equated expressions for sin*ACB*: $\frac{\sqrt{6}+\sqrt{2}}{4}=\frac{x}{\sqrt{6}-\sqrt{2}}$. The second method was by calculating the area of the triangle *ABC* using two formulae and comparing them: $\frac{1}{2}\times BC\times x=\frac{1}{2}\times AC\times AB\times\sin 60$. Some candidates successfully used the sine rule using the angle of 90° between the perpendicular line and the side *BC*. Candidates caused themselves less confusion by introducing a letter for the intersection of the perpendicular with *BC*, e.g. *D*, and then using *AD*, or another letter, e.g. *h* or *x*, for the perpendicular distance. Again, the use of a calculator was not allowed in this question, so it was essential that every step in a calculation be shown, even if considered trivial. Some candidates used the sine or tangent function with no success. A significant number of candidates made no attempt to answer this part.

Question 7

Most candidates recognised that integration was needed in both parts. Those confident in integration techniques gained full marks in both parts, presenting clear and logical solutions in arriving at the particular solution each time. It was not uncommon to see $(x + 1)$ misread as $(x + 2)$ throughout this question.

(a) Many candidates were able to correctly integrate e^{2x}. Fewer candidates were able to deal with integration of $\frac{1}{(x+1)^2}$ correctly. These candidates may have improved if they had a better understanding of integration as the reverse process of differentiation. Common errors included $+\frac{1}{x+1}$ rather than $-\frac{1}{x+1}$, the loss of the arbitrary constant required with indefinite integration and multiplying by 2 rather than dividing by 2 when integrating the exponential term.

(b) A good number of candidates with the previous part fully correct were able to gain full credit in this part too. A common mistake was to think that the integral of $-\frac{1}{x+1}$ was 0 or 1, instead of –ln(x + 1). Omission of brackets caused problems for some candidates.

Question 8

(a) Many good responses were seen for this part, with almost all candidates able to find the correct gradient. Many were able to continue to find the correct value for the *y*-intercept, although putting it into the correct final form proved to be more of a challenge for some. A common error was to take y rather than $\sqrt{y}$ as 10.4 (or 15.4), and x rather than $\log_2(x+1)$ as 2 (or 4). Omitting necessary brackets caused issues for a small number of candidates in this question.

(b) Many candidates with the previous part correct were able to correctly answer this part. The most common error seen was to square root instead of square to arrive at their final answer.

(c) A good number of candidates were able to correctly complete this part following a correct answer in **part (a)**. The most common route was to divide their –0.4 by their $\frac{5}{2}$ and then anti-log before subtracting 1; although a good number of candidates offered $-0.4=\log_2(x+1)^{\frac{5}{2}}$, taking anti-logs, then dealing with the power and subtracting 1. Giving answers to less accuracy than required was a common error, with an answer to a greater level of accuracy not often seen. Errors were also seen resulting from the premature rounding of working values. It was not uncommon to see the negative square root considered, with an extra answer of –0.944 given at the end not gaining full credit. The wrong order of operations was sometimes evident, occasionally because of poor bracketing of their expression for $\sqrt{y}$. Arithmetic slips accounted for some errors, and others were because of a poor understanding of the laws of logarithms, with some candidates giving $\log_2(x+1)$ as $\log_2 x+\log_2 1$.

Question 9

(a) This part of the question was very well answered. Most candidates recognised that differentiation was involved and attempted this successfully, followed by substituting x = 1 to find the correct gradient of the tangent. Some made a numerical error when evaluating the gradient of the tangent and followed through correctly. The use of the gradient m = 1 for their normal was a common mistake as well as mixing up the *x*- and *y*-coordinates and making arithmetic errors when finding the equation of the normal. A relatively small number of candidates equated their derivative to zero, or substituted x = 4 instead of x = 1.

(b) Most candidates were successful in arriving at the correct cubic equation following a correct **part (a)**. Many then realised that (x – 1) was a factor and found the correct quadratic, most commonly through inspection or algebraic long division. Many candidates then understood that the instruction to not use a calculator and to give the exact *x*-coordinates indicated that they should solve the quadratic by using the quadratic formula or by completing the square. Most candidates used the formula, and gave enough detail in their working, showing full substitution. However, some candidates had either not understood the implication of the instruction, or had overlooked this by the time they began to solve their quadratic, and answers were arrived at through use of a calculator and often given as rounded decimals. A relatively small number of candidates were not clear on the process necessary to answer this part, and worked with their differentiated function, or on arriving at the correct cubic, decided to differentiate it.

Question 10

This question on arithmetic and geometric progressions was done well by many candidates. Very few mixed up the definitions of the two progressions, although those candidates lacking confidence with their understanding sometimes chose to use the same method for both parts, for example treating both as geometric progressions and giving the same solution and answers for **parts (a)** and **(b)(i)**.

(a) Candidates were generally well prepared for this arithmetic progression question. Solving two simultaneous equations in d and x was commonly seen, as was equating two expressions for d. Some candidates used the sum of the three terms. Some candidates appeared to have been taught a general method linking the three terms, stating for example $T_1+T_3 = 2\times T_2$. A common error was to omit brackets when finding d from the second and third terms, so $d = 8x + 2 - 5x - 4$ was commonly seen instead of $d = 8x + 2 - (5x - 4)$. Another common error was to write down the third term as $x + 3d$ instead of $x + 2d$. It was not uncommon to see arithmetic slips made, following a correct equation in terms of x. A small number of candidates attempted to guess values, although usually with no success.

(b) (i) Candidates equating two expressions for r were generally successful in completing this question correctly. The best responses included showing the method for solution of their quadratic. The nth term approach was more likely to result in errors in rearrangement to a quadratic. Some candidates appeared to have been taught a general method linking the three terms, for example, stating $u_1 \times u_3 = (u_2)^2$. The square bracket $(5y - 4)^2$ was sometimes incorrectly expanded to $25y^2 - 16$ or $25y^2 + 16$. Candidates who did this may have improved if they had written the two brackets side by side before expanding. Very few eliminated y first to solve an equation in r, but this made the next part easier for those candidates. Candidates who gave non-exact decimal answers such as 0.471 needed to understand that this was not acceptable as it did not define the geometric progression accurately.

(ii) Most candidates with the previous part correct were able to complete this part correctly too. A common error was to substitute 2 instead of $\frac{8}{17}$ in the denominator of their expression for r. Very few candidates erroneously rejected the negative value of r.

ADDITIONAL MATHEMATICS

Paper 4037/23
Paper 23

Key messages

To do well in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. Candidates need to be aware of instructions in questions, such as 'Show that...'. Such instructions mean that when a solution is incomplete, often through calculator use, a significant loss of marks will result. Candidates need to be reminded of the need to show full method and not rely on their calculators when solving simultaneous equations. Equations solved without clear method shown are unlikely to be given full credit. Candidates should also be aware of the instructions on the front of the examination paper which indicate that all necessary method must be shown and that no marks will be given for unsupported answers from a calculator. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions. Candidates are also reminded of the formulae information on page 2 of the examination paper. Some candidates used incorrect trigonometric relationships or incorrect formulae for series.

General comments

Many candidates demonstrated knowledge and understanding of mathematical techniques. This was particularly the case in **Questions 2**, **3**, **4** and **5**. Some candidates were able to formulate problems into mathematical form although this was more challenging. This was seen in **Questions 6** and **7**, for example. Some candidates may have improved if they had a better understanding of the necessity to use bracketing to ensure correct, unambiguous mathematical form. For example, in **Question 8(b)** brackets were needed around the argument of the logarithm as it was not a single term.

Candidates who wrote answers in pencil and then overwrote them in pen should be aware that this made their work difficult to interpret. Candidates who wrote answers elsewhere usually added a note in their script to indicate that their answer was written, or continued, on another page. This was very helpful. The presentation of work was often clear and good.

Showing clear and complete method for every step in a solution was essential for questions where candidates were asked to 'Show that…' a result was of a particular form. This instruction indicated that the marks would be awarded for the method as the end result had been given. Candidates needed to understand that, when showing these results to be true, they were supposed to generate the mathematics to arrive at each result and not use the information given as an assumed part of their solution. The need for this was highlighted in **Questions 5(a)** and **9(c)** in this examination. When proving a trigonometric identity, candidates should be aware that they are expected to show that the left-hand side is equal to the right-hand side.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

(a) This question required candidates to make accurate drawings. A good number of candidates drew neat and accurate diagrams, as required. The graphs drawn needed to intersect with the y-axis and show the two points of intersection to be awarded full marks. Many candidates found drawing the graph of $y = 6 - |2x - 7|$ to be too challenging. These candidates may have improved if they had plotted and joined coordinates, as many tried to draw sketches. Many drew either $y = |13 - 2x|$ or $y = |1 - 2x|$. Some candidates left solid working lines and drew graphs that were X shaped. Other candidates thought that the graph could not pass below the x-axis and drew graphs that were W shaped. These were not accepted. A good number of candidates correctly drew the graph of $y = |x - 5|$. Some candidates, again, made sketches rather than accurate drawings and so could not be awarded marks. Some graphs were curved or flat at the vertex. This was especially the case when attempting $y = 6 - |2x - 7|$ and this was not accepted. A few other graphs had been drawn freehand and this was also not accepted. When a graph is essentially linear in nature, the use of a ruler is expected.

(b) A few fully correct answers were seen to this part. Some candidates earned a mark for finding a correct pair of critical values or for a pair of critical values that followed from their graphs. A few candidates did not have two critical values and did not score in this part.

Question 2

Most candidates chose to make a substitution using a rearrangement of $x + y = 3$ as an initial step. Almost all of these candidates did this successfully. A good number of candidates then went on and solved the simultaneous equations correctly, showing full method. Showing how to rationalise the denominator was a necessary step in the method and needed to be shown. Candidates who omitted to show this step and simply used their calculator to write down $y = \frac{1}{2+\sqrt{3}} = 2 - \sqrt{3}$, for example, were penalised. A few candidates found $y = \frac{1}{2+\sqrt{3}}$ and then used this to find $x = \frac{5+3\sqrt{3}}{2+\sqrt{3}}$ and rationalised the denominator of this value. Many candidates gave a fully correct solution using this method but a few candidates omitted to state the value of y in the correct form and were penalised. Sometimes candidates rationalised the denominator when finding each of y and x. This was not necessary as the second value was more easily found by substitution. A few candidates did not understand how to simplify $2y + \sqrt{3}y = 1$ and these commonly performed incorrect operations such as $2y + y = \frac{1}{\sqrt{3}}$. A few candidates attempted to square both sides of the equation they had formed. This was a valid approach when done correctly, although it resulted in extraneous solutions which needed to be discarded. As candidates often squared term by term, however, this approach was much less successful.

Question 3

(a) A good proportion of candidates stated all three values correctly. A few candidates struggled with finding b. A small number of candidates made little progress, commonly stating $a = 6$ and b as a multiple of π and c as 2, –4 or 3.

(b) Many candidates gave correct answers to both parts of this question. In **part (ii)**, a few candidates gave an answer in degrees only, which was not accepted. Weaker responses suggested an answer of $\frac{\pi}{3}$ or an integer value.

Question 4

Some good solutions were seen to all three parts of this question.

(a) A correct initial step of $2x - 3 = \sqrt{6}$ often resulted in a fully correct solution. The answer was required in exact form and it was essential that candidates did not resort to decimals at an early stage. Candidates who did not state an exact value for x were not awarded the accuracy mark. A few candidates doubled both sides of the given equation and formed $(2x-3)^2 = 6$. This was acceptable. However, those who then formed and solved the equation $4x^2 - 12x + 3 = 0$ often did not discard the extraneous solution that came from this.

(b) Again, a correct initial step often resulted in a correct solution, although a few candidates found the change of subject needed in the final step to be too challenging or did not think it necessary. Some candidates used a base of 10 rather than e when removing the logarithms. Once again, the answer was required in exact form and it was essential that candidates did not resort to decimals at an early stage. Candidates who did not state an exact value for u were not awarded the accuracy

mark. Candidates who made little or no progress usually started by writing, for example,
$\ln 2u - \ln u + \ln 4 = 1$ or $\ln 2u - \ln u - \ln 4 = 1$ or $e^{\ln 2u} - e^{\ln (u-4)} = e^1$ or $\dfrac{\ln 2u}{\ln (u-4)} = 1$.

(c) Candidates who earned the first two marks almost always went on and earned the accuracy mark.

A good number of candidates were able to correctly write all terms of the equation in powers of 3, which was the most common method of solution. Some candidates made sign or arithmetic slips in the first step, although this was not common. Other candidates made sign errors or arithmetic errors when combining and equating powers. This was much more common. Weaker responses offered initial equations with unacceptable method errors such as $3^v = 243^{2v-5}$ or $\dfrac{1^v}{9^{2v-5}} = 9$. Other weak responses tended to suggest a second step of $\dfrac{v}{6v-15} = 2$ or stated $3^v - 3^{6v-15} = 3^2$ $v - 6v + 15 = 2$ which was not accepted.

Question 5

(a) A good number of correct and efficient responses were seen. More candidates seemed to favour an initial step of combining the fractions and then simplifying the denominator to $\operatorname{cosec}^2 x - 1$ before using a correct trigonometric identity to continue and complete the solution. A few candidates needed to take care to check the relationship they were using as, on occasion, slips were made. Some candidates made errors when dealing with fractions whose numerator and denominator were both fractions. This could have been avoided. A small number of candidates earned two marks for obtaining $\dfrac{2\operatorname{cosec} x}{\cot^2 x}$ but were unable to determine a correct next step. The final step in the argument needed to be clearly justified to be credited, so candidates needed to show a step such as $\dfrac{2\sin x}{\cos x} \times \dfrac{1}{\cos x} = 2\tan x \sec x$. Candidates need to understand that the only relationships they can use without justification are the standard relationships given in the syllabus. Most other relationships need to be justified. Drawing a schematic diagram, as was seen on occasion, and leaving the Examiner to deduce the results used from it was not acceptable as this was a 'Show that…' question. A few candidates made little or no progress, undertaking incorrect substitutions and/or invalid operations or incorrectly cancelling terms.

(b) Again, a good number of correct and concise responses were seen. A few candidates earned 3 marks only as they omitted one or two of the possible solutions. This was often because the negative square root had been omitted. A few candidates were penalised for rounding errors. Some candidates struggled with the initial manipulation required to find an equation in a single trigonometric function that was of a form solvable on the calculator. Weaker candidates were unable to deal with the trigonometric function being squared and ignored this, finding $\tan^{-1}(2.5)$, for example. Some candidates made no attempt to answer this part.

Question 6

This question assessed the ability of candidates to solve a problem by applying several different skills. A few candidates were able to earn all the marks available and clearly understood what was required. Responses from these candidates were often neat and concise. Other candidates found the chains of reasoning needed to make the necessary connections to be beyond their capabilities. Presentation of work was often poor when candidates were unable to recognise the appropriate mathematical procedures.

(a) A few candidates gave a correct form of the equation that expressed y as a function of x. There were many different forms that candidates could offer, but the most useful for further work in the question was an equation such as $y = 4 + (x - 2)^2$. Very many candidates gave an answer that was either in terms of θ only, or in terms of x and θ. Neither of these was accepted.

(b) Candidates could only be credited in this part if their solution to **part (a)** was fully correct. Many offerings continued to be stated either in terms of θ or in terms of x and θ.

(c) Some candidates were able to earn all the marks available in this part, as follow through marks were available for a gradient from a suitable form and for the equation. However, as it was possible to find the value of *x* and the value of *y* using the given information, these values needed to be correct for the final mark to be awarded. Sometimes, *x* was stated as 3 and *y* as 5. These candidates had their calculator in the wrong mode. On other occasions $\frac{\pi}{3}$ was used for *x* and θ in the same solution.

Question 7

(a) A good number of correct solutions were seen, although some candidates did not use all the information given. Other candidates found the correct multipliers 3 and 2 and were unable to deduce what to do beyond this or divided the direction vectors by 3 and 2, instead of multiplying. Possibly the simplest approach, finding the unit vector and then multiplying by the magnitude in each case, was not commonly seen. Some candidates gave their answers as column vectors or in the incorrect forms $\begin{pmatrix} -15\mathbf{i} \\ 36\mathbf{j} \end{pmatrix}$ and $\begin{pmatrix} 30\mathbf{i} \\ -16\mathbf{j} \end{pmatrix}$ which were not accepted. The weakest responses were given by candidates who solved equations and found 'values' for **i** and **j**.

(b) A reasonable number of candidates formed a correct vector **p** + **q** and then used this correctly to find its magnitude and the angle required. The angle made with the positive *x*-axis should have been in the first quadrant in this case. Candidates whose vector **p** + **q** was incorrect were only able to earn follow through marks if their vector **p** + **q** was from the summation of two vectors and the method they had used was clearly shown. Some candidates found other vectors, such as **p** – **q**, or found the magnitude for **p** + **q** but then drew a diagram which represented, for example, **p** – **q** and used that to work out the angle. A few candidates simply summed 39 and 34 and stated 73 as the magnitude, misunderstanding what was needed.

Question 8

(a) A good proportion of fully correct answers were seen. The simplest approach was to rewrite the given function as $y = 5(x - 1)^{-1} + 2x$ and differentiate using the chain rule. Many candidates did this successfully. Some candidates applied the quotient rule to $\frac{5}{x-1}$ and occasionally made errors, such as indicating the derivative of 5 was 1. This was heavily penalised. A few other candidates undertook some unnecessary algebraic manipulation and combined the terms to form $y = \frac{2x^2 - 2x + 5}{x-1}$ and then applied the quotient rule. Again, candidates using this approach sometimes made errors in the initial manipulation, or in the application of the rule, and this was heavily penalised. Some candidates made arithmetic or sign errors in their working when finding the value of *x*. Other candidates omitted to find the value of *y* or made arithmetic or premature approximation errors when finding this value. Weaker responses were often attempts to find the coordinates of the points of intersection of the graphs.

(b) A good number of candidates gave full and correct solutions to this part. Some candidates were penalised for not showing the key method step of the substitution of the limits into the integral. Some candidates understood that the integral, with respect to *x*, of $\frac{5}{x-1}$ was going to be a natural logarithm. A few omitted the brackets around the argument of the logarithm but some of these were able to recover, as they gave evidence of such in correct later working. Most candidates were credited for finding the area of the triangle correctly. Several weaker responses indicated that the integral, with respect to *x*, of $\frac{5}{x-1}$ was either $5x\ln(x - 1)$ or $\frac{5(x-1)^0}{0}$ which they subsequently wrote as 0.

Question 9

Most candidates used formulae appropriate for arithmetic progressions in this question, although, on occasion, candidates selected formulae for geometric progressions. More careful reference to page 2 of the examination paper would likely have helped these candidates.

(a) Many candidates were successful in this part. A good proportion formed and solved a correct pair of equations in a and d. A few candidates drew a schematic diagram and deduced the number of differences using that and hence the common difference and first term. This was acceptable but candidates using this approach were less likely to earn part marks. Some candidates would have improved if they had read the question a little more carefully as, on occasion, 13 and 41 were used as the value of n. A few candidates used the sum to n terms, instead of the nth term, in this part.

(b) Again, a good number of candidates were successful in this part of the question. Most candidates used the sum to n terms formula, with their values for a and d, which were commonly correct, to form and solve an equation. It was essential that candidates showed the quadratic equation they were trying to solve and they needed to write it in the standard form for solving in order to be credited. Candidates who did not write the equation in the form $ax^2 + bx + c = 0$ and simply stated the answer as $n = 35$ had omitted necessary working steps and these candidates were heavily penalised. Similarly, candidates who attempted to solve the initial equation using trial and improvement were also heavily penalised as these candidates had eased the level of difficulty of the solution. Some candidates incorrectly simplified their correct initial equation to form a linear equation, which could not be credited. A few candidates stated an answer of $n = 35$ which clearly came from incorrect working, commonly from rounding incorrect values, and this was not accepted. A few candidates used the nth term, instead of the sum to n terms, in this part. Other candidates used incorrect forms such as $\frac{1}{2}\{2a + (n-1)d\}$. These candidates may have improved if they had checked the formulae given on page 2 of the examination paper more carefully.

(c) Most candidates were able to correctly replace n in the sum to n terms formula with $2k$ and k and used the values of a and d they had found to form a correct difference. Many of these candidates went on to complete the argument successfully. A few candidates made sign errors or bracketing errors and lost one or both of the accuracy marks. Other candidates doubled and later halved their expressions. This was not condoned. Some candidates needed to check their work a little more carefully as writing down the given expression incorrectly as a final statement resulted in the loss of a mark. Some candidates did not link this part of the question with the previous two parts and omitted to use their values of a and d. Other candidates incorrectly indicated that $S_{2k} - S_k = S_k$ or attempted to use the sum to n terms formula $\frac{n}{2}(a + l)$ but misinterpreted l as 1. Other candidates offered expressions that were a mixture of k and n and this was not accepted. A few other candidates only verified that the result was true for particular values of k. This was not acceptable as the question required candidates to show that the statement was generally true.

Question 10

(a) This part of the question was very well answered, with many candidates offering fully correct solutions. A few candidates found the value –11 by substituting 1 into their derivative but then used this value as y and did not calculate $y = 3$. A few candidates made arithmetic slips. A few other candidates found the equation of the tangent rather than the normal, as required. These candidates may have improved if they had reread the question. A few candidates equated the derivative to 0 and solved for x. Sometimes they used these values of x as gradient values. Some candidates made no attempt to differentiate and, instead, found two points on the curve and found the gradient of the chord joining them. This was not credited. A few candidates attempted to factorise the cubic in this part of the question, totally misunderstanding the method needed.

(b) In this part of the question, it was expected that candidates use the factor theorem to find the value of the constant a and then, using the factor $x + 2$, factorise and solve the equation. It should have been quite straightforward and for many candidates it was indeed so. The question required candidates to give their solution without using their calculator. The method used to show that a was 2, therefore, needed to be clearly done without the use of the calculator. Some candidates evidently ignored this instruction and without justification stated $x = \frac{3}{2}$ and $x = -2$ and then used them in the factor theorem. Other candidates simply stated $f(-2) = 0$, without showing any substitution into the cubic equation or just wrote the factors $(x + 2)(2x - 3)$ alongside or underneath the cubic equation. This was not accepted. However, candidates could recover from this if they showed that $x + 2$ was a factor using algebraic division or synthetic division. These were acceptable methods for confirming that a was 2 as long as they were stated accurately and completely. Some candidates omitted to state the value of a. This was necessary as it was a demand of the question. These candidates may have improved if they had reread the question and realised their omission. Many candidates were able to find the correct quadratic factor and a good proportion of these factorised or solved correctly to find the correct pair of values for x. A few candidates omitted to indicate the repeated factor and these were penalised.

Some candidates stated that $x = -a$ and then formed a new version of the cubic equation in a and attempted to use that. Using a in this way was condoned, even though the question clearly stated that a was an integer value. Also, whilst this was sometimes successful, it was much more likely that candidates would make sign errors and it was much more likely that $a = -\frac{3}{2}$ would also be given as a solution in these cases.