Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **October/November 2021**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

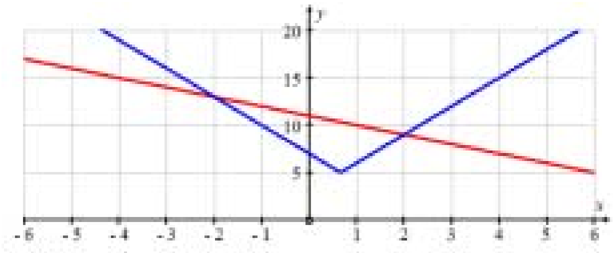
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)		4	**M1** for $\vee$ shape of $y = 5 + \|3x - 2\|$ with vertex at $\left(\frac{2}{3}, 5\right)$ **A1** for correct graph with y-intercept $(0, 7)$ **M1** for correct straight line for $y = 11 - x$ **A1** for correct straight line with y-intercept $(0, 11)$
1(b)	$x > 2$ or $x < -2$	**B2**	Mark final answer for **B2** **B1 FT** for exactly two correct critical values or correct FT critical values soi, FT dependent on at least M1 in **(a)**
2(a)	$16 - 96x + 216x^2 - 216x^3 + 81x^4$	**B4**	Mark final answer for **B4** **B3** for any 4 correct simplified terms in a sum or for all 5 simplified terms listed but not summed or for a correct simplified expansion that is not their final answer or **B2** for any 3 correct simplified terms in a sum or for 4 correct simplified terms listed but not summed or **B1** for any 2 correct simplified terms in a sum or for 3 correct simplified terms listed but not summed or **M1** for correct unsimplified expansion $2^4 + 4 \times 2^3(-3x) + 6 \times 2^2(-3x)^2$ $+ 4 \times 2(-3x)^3 + (-3x)^4$

Question	Answer	Marks	Partial Marks
2(b)	$their\left(16-96x+216x^2 \ldots..\right)\times\left(1+\dfrac{a}{x}\right)$ $= 16-96x+16\dfrac{a}{x}-96a+216ax\ldots$ soi	**B1**	**FT** Expansion using *their* **(a)**
	$a=2$	**B1**	**FT** *their* $16\dfrac{a}{x}$
	$b=-176$	**B1**	
	$c=336$	**B1**	
3(a)	$\dfrac{\cos x}{1-\cos x}+\dfrac{\cos x}{1+\cos x}$ or $\dfrac{\sec x+1+\sec x-1}{\sec^2 x-1}$	**M1**	
	$\dfrac{\cos x+\cos^2 x+\cos x-\cos^2 x}{1-\cos^2 x}$ or $\dfrac{2\sec x}{\tan^2 x}$	**A1**	
	$\dfrac{2\cos x}{\sin^2 x}$ or $\dfrac{2\cos^2 x}{\cos x\sin^2 x}$ oe	**A1**	
	Fully correct justification of given answer: $2\cot x\operatorname{cosec} x$	**A1**	
3(b)	$3\tan^2 x=2$ oe or better, soi or $5\cos^2 x=3$ oe or better, soi or $5\sin^2 x=2$ oe or better, soi	**B1**	
	$\tan x=[\pm]\sqrt{\dfrac{2}{3}}$ oe or [±] 0.816[4…] or $\cos x=[\pm]\sqrt{\dfrac{3}{5}}$ oe or [±] 0.774[5…] or $\sin x=[\pm]\sqrt{\dfrac{2}{5}}$ oe or [±] 0.632[4…]	**M1**	**FT** an equation of the form $a\tan^2 x=b$, $a>0$, $b>0$ or $p\sin^2 x=q$ or $p\cos^2 x=q$ where $p>0$, $q>0$ and $p>q$
	39.2° or 39.2315… rot to 2 or more dp 140.8° or 140.7684… rot to 2 or more dp 219.2° or 219.2315… rot to 2 or more dp 320.8° or 320.7684… rot to 2 or more dp	**A2**	no extras in range **A1** for any two correct answers

Question	Answer	Marks	Partial Marks
4(a)	$\frac{dy}{dx}=\frac{3}{x}+2x-7$	B2	**B1** for the first term correct and one other term correct or for all terms correct with extra terms seen
	Equates *their* $\frac{dy}{dx}$ to zero and rearranges to 3-term quadratic in x	M1	
	Solves *their* 3-term quadratic	M1	**Dep** on previous M1
	$x=0.5,\ 3$ nfww isw	A1	no extra solutions
4(b)	$\frac{d^2y}{dx^2}=-\frac{3}{x^2}+2$	M1	**FT** *their* $\frac{dy}{dx}$ providing B1 earned in **(a)**
	$x=0.5$, $\frac{d^2y}{dx^2}<0\to\text{max}$ or $\frac{d^2y}{dx^2}=-10\to\text{max}$	A1	
	$x=3$, $\frac{d^2y}{dx^2}>0\to\text{min}$ or $\frac{d^2y}{dx^2}=\frac{5}{3}\to\text{min}$	A1	
	Alternative method		
	Considers gradient at $x-h$ and $x+h$ for $x=0.5$ or $x=3$ [where h is small] or Considers y-values at $x-h$ and $x+h$ for $x=0.5$ or $x=3$ [where h is small]	(M1)	**FT** *their* $\frac{dy}{dx}$ providing B1 earned in **(a)**
	Correct conclusion for one turning point max at $x=0.5$ or min at $x=3$	(A1)	
	Correct method and conclusion for second turning point	(A1)	

Question	Answer	Marks	Partial Marks
5(a)	Solves $3e^x + 3e^y = 15$ and $2e^x - 3e^y = 8$ oe by elimination as far as $3e^x + 2e^x = 23$ or substitutes $e^y = 5 - e^x$ into $2e^x - 3e^y = 8$ oe OR Solves $2e^x + 2e^y = 10$ and $2e^x - 3e^y = 8$ oe by elimination as far as $2e^y + 3e^y = 2$ or substitutes $e^x = 5 - e^y$ into $2e^x - 3e^y = 8$ oe	**M1**	
	$e^x = \frac{23}{5}$ or $e^y = \frac{2}{5}$ oe	**A1**	
	$x = \ln 4.6 [= 1.53]$ oe or $y = \ln 0.4 [= -0.916]$ oe	**A1**	If M0 scored **SC1** for using *their* expression of the form $ce^x = d$ to give $x = \ln\frac{d}{c}$ provided $\frac{d}{c} > 0$
	Finds the other value, e^y or e^x, by substituting *their* e^x or e^y	**M1**	**FT** *their* e^x or e^y
	$y = \ln 0.4 [= -0.916]$ oe or $x = \ln 4.6 [= 1.53]$ oe	**A1**	

Question	Answer	Marks	Partial Marks
5(b)	$\mathrm{e}^{2t-1-(5t-3)}=5$ or $\mathrm{e}^{5t-3-(2t-1)}=\frac{1}{5}$ oe	M1	
	$\mathrm{e}^{2-3t}=5$ or $\mathrm{e}^{3t-2}=\frac{1}{5}$	A1	
	$2-3t=\ln 5$ or $3t-2=\ln\frac{1}{5}$	M1	**FT** *their* $\mathrm{e}^{a-bt}=5$ or *their* $\mathrm{e}^{ct-d}=\frac{1}{5}$ where a, b, c and d are positive integers
	$t=\frac{2-\ln 5}{3}$ or $t=\frac{2+\ln 0.2}{3}$ or 0.13[0] oe	A1	
	Alternative method		
	$\ln \mathrm{e}^{2t-1}=\ln 5+\ln \mathrm{e}^{5t-3}$ oe	(M1)	
	$(2t-1)[\ln \mathrm{e}]=\ln 5+(5t-3)[\ln \mathrm{e}]$ oe	(A1)	
	$5t-2t=3-1-\ln 5$ oe	(M1)	**Dep** on one correct log law applied with at most one sign error
	$t=\frac{2-\ln 5}{3}$ or $t=\frac{2+\ln 0.2}{3}$ or 0.13[0] oe	(A1)	
6(a)	$\left(\sqrt{6}-\sqrt{2}\right)^2+\left(\sqrt{6}+\sqrt{2}\right)^2-2\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)\cos 60$	M1	
	$6+2-2\sqrt{12}+6+2+2\sqrt{12}-2\times(6-2)\times\frac{1}{2}$	M1	Condone one error in expansion of brackets
	$[BC=]2\sqrt{3}$ isw	A1	
6(b)	$\frac{their\,2\sqrt{3}}{\sin 60}=\frac{\sqrt{6}+\sqrt{2}}{\sin ACB}$ or $\frac{their\,2\sqrt{3}}{\frac{\sqrt{3}}{2}}=\frac{\sqrt{6}+\sqrt{2}}{\sin ACB}$	M1	Condone other letters for ACB
	$\sin ACB=\left(\sqrt{6}+\sqrt{2}\right)\times\frac{\sqrt{3}}{2}\times\frac{1}{2\sqrt{3}}=\frac{\sqrt{6}+\sqrt{2}}{4}$	A1	**A0** if necessary brackets missing unless clearly recovered

Question	Answer	Marks	Partial Marks
6(c)	$\dfrac{\sqrt{6}+\sqrt{2}}{4}=\dfrac{x}{\sqrt{6}-\sqrt{2}}$ or $\dfrac{1}{2}\times \textit{their}\ 2\sqrt{3}\times x=$ $\dfrac{1}{2}\times\left(\sqrt{6}-\sqrt{2}\right)\times\left(\sqrt{6}+\sqrt{2}\right)\times\sin 60$ [where x is the perpendicular from A to BC]	**M1**	Complete method
	$x=\dfrac{\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)}{4}=\dfrac{6-2}{4}=1$ or $x=\dfrac{(6-2)}{2\sqrt{3}}\times\dfrac{\sqrt{3}}{2}=\dfrac{4}{4}=1$	**A1**	
7(a)	$\left[\dfrac{dy}{dx}=\right]\dfrac{1}{2}e^{2x}-(x+1)^{-1}+\dfrac{5}{2}$ oe	**B3**	**M2** for $\dfrac{1}{2}e^{2x}-(x+1)^{-1}+c$ oe or **M1** for any two terms correct from $\dfrac{1}{2}e^{2x}$, $-(x+1)^{-1}$, $+c$
7(b)	$[y=]\dfrac{1}{4}e^{2x}-\ln(x+1)$	**M1**	
	$+\textit{their}\ \dfrac{5}{2}\times x+d$	**M1**	**FT** *their* c from **(a)**, providing $c\neq 0$
	$[y=]\dfrac{1}{4}e^{2x}-\ln(x+1)+\dfrac{5}{2}x+\dfrac{15}{4}$ oe	**A1**	

Question	Answer	Marks	Partial Marks
8(a)	[Gradient =] $\frac{15.4-10.4}{4-2}$ oe soi	**M1**	
	$10.4 = their2.5 \times 2 + c$ or $15.4 = their2.5 \times 4 + c$ or $\frac{y-10.4}{x-2} = their2.5$ or $\frac{y-15.4}{x-4} = their2.5$	**M1**	**FT** *their* gradient
	[Gradient =] 2.5 soi and [intercept =] 5.4 soi	**A1**	
	$\sqrt{y} = 2.5\log_2(x+1) + 5.4$ oe isw	**A1**	
	Alternative method		
	$10.4 = 2m + c$ and $15.4 = 4m + c$ and solving to find m or c	**(M1)**	
	Use *their* m or c to find *their* c or m	**(M1)**	
	$m = 2.5$ and $c = 5.4$	**(A1)**	
	$\sqrt{y} = 2.5\log_2(x+1) + 5.4$ oe isw	**(A1)**	
8(b)	$\frac{5929}{25}$ or 237.16	**B1**	
8(c)	$5 = their2.5\log_2(x+1) + their5.4$ and rearrange to make $\log_2(x+1)$ the subject	**M1**	**FT** *their* equation from **(a)** of correct form with $m \neq 1$ or 0, and $c \neq 0$ Condone any base
	$-\frac{4}{25} = \log_2(x+1)$ oe	**A1**	Condone any base
	$x = -0.105$ or $-0.1049[74\ldots]$ rot to 4 or more sf	**A1**	
9(a)	$\frac{dy}{dx} = 3x^2 + 2x - 4$	**M2**	**M1** for any two terms correct
	$x = 1 \rightarrow \frac{dy}{dx} = 1$	**A1**	
	$[m_\perp =]\ -1$	**M1**	**FT** $\frac{-1}{their1}$
	$y - 4 = -1(x - 1)$ oe isw	**A1**	**FT** *their* $m_\perp$

Question	Answer	Marks	Partial Marks
9(b)	$x^3 + x^2 - 4x + 6 = their(-x + 5)$ $\rightarrow \quad x^3 + x^2 - 3x + 1\ [= 0]$	**M1**	**FT** *their* linear equation of the form $y = mx + c$ where $m \neq 0$ and $c \neq 0$ from **(a)**
	Correct quadratic factor: $x^2 + 2x - 1$	**B2**	**B1** for any two out of three terms correct Must be from the correct cubic
	Solves *their* $(x^2 + 2x - 1) = 0$ using the formula or by completing the square	**M1**	**dep** on M1 and valid attempt at finding quadratic factor **M0** if *their* quadratic factor does not have real roots
	$\dfrac{-2 \pm \sqrt{8}}{2}$ isw or $\dfrac{-2 \pm 2\sqrt{2}}{2}$ isw	**A1**	
10(a)	Eliminate one unknown using two correct equations e.g. $d = 4x - 4$ oe $d = 3x + 6$ oe and solve as far as $x = \ldots$ or $d = \ldots$	**M2**	**B1** for one correct equation seen, e.g. $d = 4x - 4$ oe or $d = 3x + 6$ oe or $2d = 7x + 2$ oe May come from the sum of terms, e.g. $11x - 3d = 2$
	$x = 10$	**A1**	
	$d = 36$	**A1**	

Question	Answer	Marks	Partial Marks
10(b)(i)	$\frac{5y-4}{y}=\frac{8y+2}{5y-4}$ oe	M1	
	$25y^2-40y+16=8y^2+2y$ $\rightarrow 17y^2-42y+16[=0]$	M1	
	$(17y-8)(y-2)[=0]$	M1	Solves *their* 3-term quadratic
	$\frac{8}{17}$, 2	A1	Both values
	Alternative method		
	Eliminates y from $yr = 5y - 4$ and $yr^2 = 8y + 2$ and simplifies to 3-term quadratic in r $\rightarrow 2r^2+r-21[=0]$	(M1)	
	Solves *their* 3-term quadratic	(M1)	
	Substitutes *their* two r values to find two y values	(M1)	
	$\frac{8}{17}$, 2	(A1)	
10(b)(ii)	$-\frac{7}{2}$, 3	B2	**B1** for one correct