ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

It is essential to read each question carefully and ensure that the demands of the question are met. This includes ensuring that the final answer is given in the form required. Some candidates still appear to be unfamiliar with the form an exact answer should take. Reference to previous mark schemes should clarify any misunderstandings. Candidates should also be familiar with the meanings of the command words used in this syllabus.

General comments

Most candidates were well prepared and able to show their knowledge of the syllabus objectives by producing papers of a reasonable standard. There appeared to be no timing issues and candidates made the correct use of additional paper when the need arose.

Comments on specific questions

Question 1

(a) Most candidates did not give their final answer in the required form, which was to find a factorised form of the equation and give each linear factor with its coefficients as integers. The most common answer was $18\left(x+\frac{1}{3}\right)(x-1)\left(x-\frac{5}{2}\right)$ or similar. It was expected that the answer be given in the form $-3(3x+1)(x-1)(2x-5)$. Some candidates did obtain this form, but others chose to multiply out and give the equation in an unfactorised form.

(b) The use of the phrase 'Write down' implies that the answer can just be written down with no extra working involved. The values of *x* could be obtained immediately from the graph with no calculation required. Whilst many candidates obtained a correct solution, some attempted to manipulate their answer to **part (a)**.

Question 2

(a) The majority of candidates gave the correct amplitude.

(b) Although many candidates obtained the correct period, it was evident that some were unable to calculate the period correctly. This did not appear to cause a problem with **part (c)** as most candidates 'plotted' points.

(c) Many fully correct sketches were seen, with candidates choosing to 'plot' points in order to obtain the correct shape. Some candidates did not have their graphs starting and finishing in the correct places. It was expected that at $x=-180^{\circ}$, $-6<y<-5$, and at $x=180^{\circ}$, $1<y<2$. It was also essential that the maximum point and the minimum point be in the correct position on the grid, with the curve passing through the point $(0, 2)$.

Question 3

Many completely correct solutions were seen, showing that candidates had a good understanding of the syllabus for transformation of straight line graphs. Most candidates were able to find the correct gradient of the straight line and were familiar with the form the equation should take, There were some errors when substituting to find the value of c, with the incorrect substitution of 2.25^2 or 4.75^2 rather than the correct values of 2.25 or 4.47.

Question 4

(a) It was essential that all working be shown clearly. Many candidates wrote down the binomial expansion making use of the formula given. To obtain marks, it was necessary to equate the terms in this expansion to – 8, p and q which too many candidates did not do, making little progress beyond the expansion. Once a value for n was obtained, it was relatively straightforward to obtain the values of p and q, although there were often sign errors and calculation errors when dealing with the powers of $-\frac{x}{2}$.

(b) This part was well done by many candidates who identified and evaluated the correct term. Some candidates chose to expand $\left(\frac{2}{x^2}+\frac{x}{3}\right)^6$ and identify the term independent of x from this.

Question 5

Many candidates were able to identify two of the solutions, but very few were able to obtain all three. There were two possible options, either working with $\cos\left(2\theta+\frac{\pi}{6}\right)=(\pm)\frac{\sqrt{3}}{2}$ or $\tan\left(2\theta+\frac{\pi}{6}\right)=(\pm)\frac{1}{\sqrt{3}}$. It was not essential to see $\pm$ for the first mark, but omission of this resulted in only 2 solutions in the range being found. It was pleasing to see the most candidates appreciated the order of operations and gave their answers in terms of π as required.

Question 6

(a) Candidates who dealt with the information given in the stem of the question in order tended to have greater success with the solution. The information that $\mathrm{p}'(0)=12$ enabled candidates to find the value of c straightaway. Many candidates appeared to ignore this information and just make use of the factor and remainder theorem obtaining two equations with three unknowns which could not be solved. Of those candidates that did obtain the value of c, some made arithmetic slips when either forming their equations using the factor and remainder theorems, or when solving their simultaneous equations. Candidates should again be guided by the information given in the stem of the question which states that a, b and c are integers. Obtaining non-integer solutions should alert candidates to errors having been made and thus giving an opportunity to check their work.

(b) It was expected that candidates make use of their values of a, b and c, together with algebraic long division or inspection, to obtain $\mathrm{p}(x)$ as a product of $(3x-1)$ and a quadratic factor. Candidates were able to obtain a method mark even if their values of a, b and c were incorrect. The correct quadratic factor was $2x^2+3x+15$. Too many candidates, having obtained the correct quadratic factor, then concluded that, as this could not be factorised, there were no further solutions to the equation $\mathrm{p}(x)=0$. It was necessary for candidates to make use of the discriminant to show that the equation $2x^2+3x+15=0$ has no real roots. For candidates using synthetic division, it was essential that they dealt with the factor $(3x-1)$ correctly to find the correct quadratic factor in order to obtain marks,

Question 7

(a) (i) Most candidates were able to obtain the correct answer.

(ii) Fewer candidates were able to obtain the number of 6-digit numbers divisible by 5. This was because some candidates incorrectly thought that a 6-digit number could start with 0 when dealing with their solutions. This highlights the importance of reading the information in the stem of the question carefully.

(b) (i) Most candidates were able to obtain the correct answer.

(ii) Fewer candidates were able to obtain the correct answer of 18. If all 6 doctors are on the committee then there is only one place left to fill. This can be filled by any one of the 10 nurses or 8 dentists.

(iii) The most straightforward solution was to consider the number of committees that could be formed if there were no dentists and subtract this from the answer to **part (i)**, the total number of committees. This method was used by many of those that attempted this part of the question. Others chose a longer method by listing all the 7 possible combinations with no dentists on the committee. This method was fairly successful for those that did attempt it provided all the 7 options were included and no arithmetic slips were made. Candidates should be guided by the mark allocation and, in this case, the amount of working space, both of which imply that not a great deal of work is needed to solve this problem.

Question 8

(a) (i) Many candidates were unable to obtain the correct value of a. For the inverse function to exist, the function has to be a 1 to 1 function. The largest domain is when *x* is greater than or equal to the value of *x* at the stationary point on the curve, As the function is in completed square form, this is when $3x+1 \geqslant 0$.

(ii) More candidates were successful at finding the range of the function, Again, the fact that the function was given in completed square form should have helped candidates write down the answer of – 4. The use of the phrase 'Write down' implies that the answer can just be written straight down with no extra working involved.

(iii) Very few candidates were able to gain full marks for this question part. The first two parts of the question were intended to help candidates with the sketch of the function and its inverse. Most candidates did not take into account that the function had to be a 1 to 1 function, so many produced sketches of quadratic curves. It is essential that candidates take into account work that has been done in previous parts as this often has a bearing on the work to be done subsequently.

(c) Most candidates were able to use the correct order of operations and obtain the correct expression of $3\ln\left(2x^2+5\right)-2$ for the composite function. There was mixed success at the solution of the resulting equation with some candidates being unable to deal with the logarithms correctly and others not giving their answer in exact form. This again highlights the need to ensure that the answer is given in the required form. It was also essential that candidates realised that only the positive root of their solution was valid.

Question 9

Provided the given equation was recognised as a disguised quadratic equation, many candidates were able to make a reasonable attempt at the solution of the equation. It was intended that each term be multiplied by $x^{\frac{2}{3}}$, which resulted in the equation $12x^{\frac{4}{3}}-11x^{\frac{2}{3}}-5=0$. Of those candidates who obtained this equation, some chose to make use of a substitution to solve their equation. It should be noted that some candidates made use of a substitution $x=x^{\frac{2}{3}}$, which is not particularly useful or mathematically sound and often led to original substitution being forgotten after the solution of the quadratic equation. Candidates are advised to use a different letter to represent their substitution. Many solutions of $x^{\frac{2}{3}}=\frac{5}{4}$ were seen, but some candidates were unable to solve this correctly. It was required that the answer be given to one decimal place, but some candidates again demonstrated that they had not read the question fully and given their answer in the required form.

Question 10

(a) To make any progress in this question, it was essential that the angle *AOB* be found. Most candidates did this and obtained a correct value of 2.25 radians, The length of the line *CB* needed to be found and this involved finding the angle *COB*. Unfortunately, many candidates rounded the value of this angle prematurely which resulted in a final perimeter value of either 80.6 or 80.8 rather than the expected answer of 80.7. Candidates should be working to at least four significant figures if a final answer is to be given correct to three significant figures as specified in the rubric on the paper.

(b) Most candidates were able to make use of their work in **part (a)** to successfully find the area of the kite and the area of the sector.

Question 11

(a) Of the candidates that attempted this question part, most were able to obtain the correct vector $\overrightarrow{OZ}$. There were some sign errors involving $\overrightarrow{AX}$ and the vector $\mathbf{a}$, which would unfortunately affect accuracy marks in the later stages of the question.

(b) Again, there were sign errors when finding the vector $\overrightarrow{OZ}$. The most common error was to take $\overrightarrow{OY} = \frac{1}{2}(\mathbf{a} - \mathbf{b})$ rather than $\overrightarrow{OY} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$.

(c) Provided the answers to **parts (a)** and **(b)** were equated and then like vectors equated, most candidates were able to obtain at least two method marks. Sign errors and arithmetic slips meant that few correct solutions were seen.

(d) Very few candidates were able to obtain the accuracy mark associated with this part of the question due to previous errors,

Question 12

(a) A variety of responses were seen. It was expected that some reference be made to the square root of a negative number. Some candidates just stated that the curve was undefined. Candidates should be aware of the meaning of the command word 'Explain'.

(b) Most candidates were able to obtain 2 or 3 marks for the differentiation of a quotient and differentiation using the chain rule. Some candidates made errors with the differentiation of $\sqrt{5x-2}$ but were able to obtain credit for attempting to differentiate a quotient. Very few attempted differentiation of a correct product. Few candidates were able to simplify their quotient correctly, being unable to divide through by a factor of $(5x-2)^{-\frac{1}{2}}$ in order to obtain linear factors in the numerator of the quotient.

ADDITIONAL MATHEMATICS

Paper 4037/13
Paper 13

Key messages

Candidates should be reminded to read each question carefully. They should clearly state the intercepts of the *x* and *y* axis when sketching a graph. They should pay attention to the required form of an answer. An exact answer is written either as a surd and/or a fraction and decimal answers should not be given in these cases. More work needs to be done to ensure candidates' understanding of modulus functions, trigonometric identities and circular measures.

General comments

There did not seem to be any issues with timing in this examination. A few candidates did not attempt the later parts of **Question 11**.

Some candidates would have done better if they had taken more care, especially when manipulating expressions and equations where errors in signs often occurred. This was particularly noted in **Question 4(a)**. Multiplying out the terms prior to differentiation in **Question 6(a)** would also have helped many candidates. Candidates need to take care to write their figures clearly as some mis-read their own figures which meant errors later in the solution; this was especially important in **Question 10**.

Some candidates displayed a very poor understanding of trigonometrical identities, sometimes separating them from their variables e.g. $\sin(3x) = k$ becoming $3x = \frac{k}{\sin}$ or treating $\frac{1}{\tan x}$ as if it is $\tan^{-1}(x)$, which gave rise to many incorrect starts in **Question 12**, although where the poor notation was recovered credit was usually given. Some candidates appear to reject negative results in situations where they are perfectly acceptable. Thus in **Question 7** after finding the correct $\frac{2}{3}$ and $-\frac{1}{2}$, the latter of these was ignored.

Candidates need to appreciate that for questions involving drawing graphs all intercepts need to be clearly labelled – putting a scale on an axis is insufficient.

Comments on specific questions

Question 1

(a) A V-shaped graph was expected for $y = |4x - 3|$ and to be credited it needed to have a vertex on the positive *x*-axis. Although this was usually done, some had the vertex elsewhere and often omitted the labelling of the intercepts. The line $y = 2x + 5$ was usually drawn but sometimes the intercept at –2.5 was offered as 2.5. Frequently this latter line was not drawn in such a way that it would eventually meet $y = |4x - 3|$ for a second time, which is clearly what it would do if their respective gradients had been considered. Even if that intersection would be off the diagram space credit was given to one that showed a suitable slope.

(b) Most candidates could deal with $4x - 3 < 2x + 5$ but obtaining the second limit was fraught with error. Many used $4x - 3 < -2x - 5$ which demonstrated an incorrect understanding of the modulus principle. Frequently those that began this way later did not deal with the signs correctly and stumbled into a correct answer which was not accepted as it came from wrong working. An alternative approach was to square both sides of the inequality and simplify to a quadratic, and this was generally a more successful route, although many only squared the $(4x - 3)$. Where the correct quadratic was produced most found the critical values but not all then provided a range of solutions. For the final mark one of the forms $-\frac{1}{3} < x < 4$ or $-\frac{1}{3} < x$ **and** $x < 4$ were expected.

Question 2

Finding the correct gradient was the norm and credit was given to those that started properly but then did not continue accurately. It was not unusual to see the correct $\dfrac{-\frac{7}{3}-\frac{2}{3}}{6--3}$ being simplified to 1. Many omitted to attempt the mid-point which was essential for finding a bisector and the subsequent attempt at the required equation was often based on one of the given points. The perpendicular gradients were generally found correctly and their use in finding a perpendicular bisector from any incorrect mid-points was credited on a follow through basis, but too often one of the given points was used to establish a 'bisector' and this earned no marks. Use of the correct perpendicular bisector and $x = 2$ usually led to finding $k = \frac{2}{3}$ although a few made errors dealing with their fractions in the latter part of the calculation.

A number of candidates also found the equation of the line connecting the two given points which was unnecessary and potentially confusing to candidates.

Question 3

Many candidates scored full marks on this question. Some candidates lost one or two marks through poor drawing (especially with respect to passing through the point (90,0)) and missing an intercept (usually the x-intercept) by a sufficiently large margin. A significant minority of candidates scored no marks – often because the period was 90 or 180. A small number of candidates omitted the question altogether or drew something which was not recognisable as a sine wave. A few had multiple sine waves as they seemed to have misunderstood how $\frac{x}{3}$ affects the period of the graph.

Question 4

(a) Many completely correct answers were seen. A few candidates slipped up with the algebra, losing the final mark, and 6 was a common misread for –6, resulting in a penalty of 1 mark if all else was correct. Some candidates evaluated $P\left(\frac{1}{2}\right)$ and P(1), thus could not earn any marks. Very few candidates worked with long division rather than the factor/remainder theorem. Poor arithmetic manipulation let down many who had correct equations but did not solve the simultaneous equations.

(b) Strong candidates used long division to find the quadratic factor and went on to find the discriminant correctly, although their reasoning was not always correct. Those who did not score full marks in this part still managed to divide out successfully, but did not know what to do next. Some relied on a simple statement for the final mark but they did not show that the discriminant is less than zero.

Question 5

Many candidates did not seem to know which parts required a combination and which a permutation and it would be helpful for candidates to have more experience of situations where each is applicable. Where candidates offered a variety of ${}^nC_r$ or ${}^nP_r$ solutions it was generally difficult to be certain of the candidate's intention. Throughout this question, evaluated values only were assessed.

(a) (i) This part was usually correct from using ${}^{10}P_5$ or $10 \times 9 \times 8 \times 7 \times 6$.

(ii) The most common route was to find the total number of passwords with no symbols and then subtract this from the result of **part (i)**. A few tried the alternative of finding the four other options. The most common incorrect solution was to offer $4 \times {}^9P_4 = 12\,096$.

(iii) Whilst many found the correct result of 2160 from 6 × 5 × 6 × 4 × 3, or the equivalent, many went astray, although some credit was given to those that showed the 6 × 5 and 4 × 3 calculations as part of their solution.

(b) In this part candidates needed to use combinations as order of selection of the team members was not important but many used permutations and thus produced some very large results. It was expected that 5 + 900 + 600 + 20 = 1525 would be produced and many achieved that.

Question 6

(a) Weaker candidates did not score because they did not attempt the differentiation. Some candidates expanded brackets first, but then differentiated successfully and found two roots from their quadratic, gaining at least two marks. Those who used the product and chain rules sometimes slipped up differentiating the first term: $(2x – 1) –1$ was seen more frequently than expected. Most errors occurred in manipulating the equations, both q(x) and q'(x), incorrectly and some thought that stationary points meant solving $y = 0$, even after expanding the cubic.

(b) Some candidates who completed **part (a)** successfully did not relate their answers to the requested sketch. Some candidates who did not earn marks in **part (a)** produced a perfect sketch in this part. For those who scored part marks, the *y*-intercept was usually correct, but the local maximum was often misplaced in the first quadrant or at (0, 3). Some did not score at all: usually the cubic was the wrong shape, but sometimes the graph was not recognisable as a cubic at all – and the occasional quadratic or straight-line graph was seen. Those that knew what a cubic should look like did generally score at least two marks.

(c) Many candidates started again rather than use their graph, often setting the original expression equal to *k* and being unable to deal with the algebraic manipulation. Some candidates wrote down $k < 0$ and no more. As many had not differentiated in **part (a)**, they struggled to find the maximum and therefore the second inequality in *k*. A few found the *y*-coordinate of the maximum point and then wrote $k = 4.23$ and $k = 0$. Generally, it was only the best candidates who scored well on this question.

Question 7

The substitution of a new variable such as $m = x^{\frac{1}{3}}$ was commonly attempted, but dealing with the second term caused issues where this was often written as $\frac{1}{2m}$ rather than $\frac{2}{m}$. Having achieved the required three term quadratic, most candidates scored the remaining marks. Occasionally dealing with the power of $\frac{1}{3}$ was an issue especially with the negative solution to the quadratic. Some rejected the result for $-\frac{1}{2}$ from their quadratic and did not try to find the corresponding value of *x*, which resulted in the loss of the final mark. Where $x^{\frac{1}{3}}$ was replaced by a dummy variable not all remembered to then compare the results of $\frac{2}{3}$ and $-\frac{1}{2}$ to the original variable, especially those who used *x* as their dummy variable so that once they obtained $x = \frac{2}{3}$ and $-\frac{1}{2}$ that was the end of their solution.

Question 8

Correct solutions to this question were achieved by many candidates. The vast majority wrote out the expansion correctly, generally using the ${}^{n}C_{r}$ notation, although many did not move beyond that as they did not find a value for *n*. The most common error in the expansion was not putting $(2x^2)$ in brackets so that the application of the powers of this term became confused as to whether they applied to the 2 as well as the x^2.
Those that identified $n = 8$ from the first term usually moved successfully into attempting the other terms, although a few did not show that they were working with $-\frac{1}{4x}$ rather than $\frac{1}{4x}$ which led to a subsequent loss

of marks. For the second term, the most common error was to lose the negative sign at some stage such that the value of *a* was given as 256 rather than – 256. Similarly for the third term, some candidates did not lose the negative when they squared $-\frac{1}{4}x$ such that they found *b* to be – 112 rather than 112. Some candidates separated off the coefficients from the $-\frac{1}{4x}$ and often this led to an error in finding *c*.

Question 9

A good majority of candidates earned the first three marks but then struggled to manipulate the fraction successfully. Most worked with the quotient rule, but some were successful with the product rule. Those that tried using the product rule, struggled with differentiating a negative power on the bracket. Those who tried to deal with the factor of (x – 1) first were generally more successful in earning a mark. The factor of 3 was often lost when multiplying top and bottom by $3(5x+2)\times\frac{2}{3}$. A few weaker candidates did not recognise the need to use either the product or the chain rule, and just attempted to differentiate the two terms separately.

Question 10

The main error made by many candidates was that they began by assuming values for sides *AC* and *BC*, usually deciding they were 20, and/or for the angles *BAC*, *ABC*, *BAO* and *ABO* which were generally stated as being $\frac{\pi}{4}$. This meant that they were usually working with a square rather than a kite. The first step should have been to work with the sector perimeter to find θ as 1.25 but many made errors and often the line *AB* was stated as being 25 showing a lack of understanding.

Candidates needed to find the area of *ABC* and either subtract the area of the segment from it or add on the area of triangle *ABO* to it and subtract the area of the sector. Many assumed triangle *ABC* was the same size as triangle *ABO*, which earned little credit.

There were various routes that could be used to correctly find the area of triangle *ABC*. Finding *AB* and the height of the triangle *ABC* from *AB* was one. Alternatively sides $\frac{AC}{BC}$ could be found and used in conjunction with the angle at *C* or along with *AB* and the angle *CAB* or *CBA*. *AC* formed part of triangle *CAO* which is right-angled and equal to half of the area of the kite and so this could provide a very quick route to success. Many found some useful values for sides and angles but often the methods were incomplete.

Many candidates did not make it clear what they were finding as minimal labelling was seen on many scripts. Had candidates thought out a plan for their solution rather than just calculating everything they could, then the overall method for many would have been clearer to them, thus reducing the likelihood of errors.

A few lost accuracy marks by using their rounded values for later calculations, often 14.4 for $\frac{AC}{BC}$. This is an important point in a multi-stage calculation as such amended values can build up errors. Candidates should always return to an unrounded form for later calculations.

Question 11

(a) This was generally well done. A few candidates did not know how to start the question, and some made sign errors where they did not know that *AB* is the negative of *BA* and tried to 'fudge' the result, but a good many scored full marks. Successful answers included clearly written routes, with calculations on the side, where necessary.

(b) Most candidates wrote the correct answer down, although occasionally there were slips such as $\frac{17}{15}$ instead of $\frac{7}{15}$ for the coefficient of *b*, and some tried, not always successfully, to multiply out the bracket of λ.

(c) There were many correct answers to this part, but poor manipulation of the vectors often led to sign errors. The most common mistake was giving the coefficient of *a* as $\mu - \frac{1}{3}$. Use of the basic triangle law for vectors was sometimes a problem: $YZ = ZA + AY$ or $YZ = YA - AZ$.

(d) A significant minority of candidates equated their expressions, but then made no further progress – in spite of several lines of algebra. Those who successfully answered **parts (b)** and **(c)** usually successfully answered this part. Some only scored a single mark due either to an earlier error or to slips in the arithmetic. Most could equate the two equations, but many struggled to compare coefficients.

Question 12

The first step should have been to write the expression in a form involving one of the main trigonometric identities. Using $\dfrac{1}{\operatorname{cosec}\left(\frac{2x}{3}-\frac{\pi}{3}\right)} = \sin\left(\frac{2x}{3}-\frac{\pi}{3}\right)$ was the most obvious but some preferred to re-arrange to a form using $\tan\left(\frac{2x}{3}-\frac{\pi}{3}\right)$ which was perfectly acceptable. After the change of trigonometric function it was expected that they would be equated to the correct value, $\frac{\sqrt{3}}{2}$ in the case of sine or $\sqrt{3}$ in the case of the tangent, although sometimes the square root of the values were ignored. Too often the wrong conversion of trigonometrical identity was attempted and/or the numerical value was incorrect. Some managed to omit the squaring of the function altogether.

Those who reached the equation $\frac{2x}{3}-\frac{\pi}{3}=\frac{\pi}{3}$ as an initial point in finding the solutions usually went on to find all or most of the correct results, although sometimes not all the range was considered. A small number offered solutions in decimals despite it having been requested as a multiple of π which did not gain full marks.

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

In order to do well in this examination, candidates need to demonstrate knowledge and understanding of mathematical techniques and apply these techniques to solving problems.

Candidates should always show clear and full method and not rely on their calculator to avoid the application of algebraic skills. Candidates should be aware that a calculator is an excellent tool to check that they have applied their skills correctly. Candidates should also ensure that, when using their calculator to evaluate trigonometric functions, it is in the correct mode for the argument of the function.

It is important that candidates understand that, to be credited for follow through values, they must show their method of calculation, as a correct method will not be assumed from incorrect values alone.

When a substitution is used, candidates should always state the substitution they are using before they use it. They should also check that any solutions that arise are valid for the original function.

Candidates should pay attention to key words and phrases.

- When a calculator is not to be used, candidates who resort to inexact decimals will not earn full credit.
- When an exact answer is required, it will generally involve, for example, a surd, logarithm, exponential, π and/or a fraction.
- When the instruction in the question is Show that …, candidates should make sure that they write down all method steps needed to find the answer that has been given.
- When the question demands that an answer is given in a particular form, candidates should check they have stated their final answer in that form.

General comments

Candidates needed to read each question carefully in order to answer the question that had been set. There was some evidence, in **Question 1(a)** for example, that candidates were reading the question at speed and making an incorrect assumption.

Logarithms were often incorrectly simplified in this examination. This was evident in **Question 4**. For example in **Question 4(b)**, candidates often rewrote $\log_9(4y-9)$ as $\log_9 4y - \log_9 9$.

It is important that candidates are aware of the Mathematical Formulae that are listed on page 2 of the examination paper. Some candidates incorrectly stated formulae that were given to them. This was particularly evident in **Question 9**.

The presentation of responses was mostly reasonably good. Candidates who presented their working in clear steps generally offered solutions that were ordered and logical. Candidates whose working was more chaotic and disorganised often made errors, such as misreading their own work. Some candidates crossed out answers in their main script and therefore had reduced space to write any replacements. These candidates should have used the blank page at the end of the examination paper, or additional writing paper, for their solutions. Some of them did this and indicated that their response was written elsewhere. This was very helpful.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

(a) The majority of candidates understood the need to find the equation of the line passing through (4, 23) and (–8, 29). A good number of candidates managed this successfully. A reasonable number of candidates went on to complete the solution without error. Some candidates had a correct unsimplified equation in one unknown, usually *x*, but made slips when attempting to solve it. For example $2x+\frac{1}{2}x$ often became $\frac{3}{2}x$. Candidates who equated expressions for *y* in order to eliminate one unknown were more successful. Those who made other substitutions, or attempted to form a pair of scaled equations in order to eliminate one unknown, often made sign or scaling errors in the first step of this part of their method.

Those candidates who used the form $y = mx + c$ for the equation of the line needed, sometimes made sign or arithmetic slips when finding *c*. A few candidates assumed that the two lines were perpendicular and clearly derived the gradient of the line passing through (4, 23) and (–8, 29) using the gradient of the given line and the product $m_1m_2 = -1$. Other candidates found the gradient of the line passing through (4, 23) and (–8, 29) correctly and then decided that the gradient they needed was perpendicular to this line and therefore 2. As this resulted in parallel lines, no solution was possible. Some candidates found and used the mid-point of the two points given. This was not incorrect but was not necessary and introduced a greater chance of an error being made. A few of these candidates clearly were attempting to find the equation of the perpendicular bisector of the two points given. A small number of candidates inverted the gradient calculation. In the weakest responses, no attempt was made to find the equation of the line required.

(b) Again, a good number of candidates earned both marks for this part. Those who had an incorrect point from **part (a)** were able to earn partial credit for a correct distance calculation and many did this. Some candidates did not seem to know that the origin was (0, 0) and a few used the *y*-intercept of $y = 2x + 5$, for example. Other candidates found the distance from the origin to the mid-point (–2, 26) and others found the distance between (4, 23) and (–8, 29). These candidates may have improved if they had read the question more carefully.

Question 2

A reasonable number of fully correct solutions to this question were seen. Most of these solutions used the discriminant of a suitable quadratic equation in *x* and *k*. Candidates whose equation had coefficients that were all negative were less successful than those whose equation had coefficients that were all positive as sign errors were more likely in this case. Some candidates formed a correct initial equation in *x* and *k* but were not able to simplify this correctly. Occasionally an error arose when expanding $x(x + 2k)$. Omitting terms or making sign errors were also very common at this stage. Some candidates needed to take more care over which terms were in *x* and which were in *k* as some miscopying errors were made. A few candidates made slips when copying $x(x + 2k)$. Rereading the question may have avoided some of these slips. A few candidates formed a correct, simplified quadratic in *x* and *k* but were unable to correctly identify *a*, *b* and/or *c*. A common error was to write *b* as $2(1 + k)$ and then use $b = 1 + k$, for example. Some candidates made slips when expanding $(2k + 2)^2$ and others made sign or expansion errors when dealing with $-4(6k + 1)$. Some candidates needed to take care to copy terms correctly as it was not uncommon for a correct unsimplified expression in *k* to be incorrectly simplified to, for example, $4k^2 - 16 = 0$. A few candidates omitted to discard the solution $k = 0$ and this was not condoned for the final mark. Attempts using calculus were seen but were not common and not as successful as most did not progress beyond finding a correct derivative.

Question 3

The formula for the volume of a cylinder is assumed knowledge for this syllabus. It was evident that many candidates did not know this formula. Candidates were not permitted to use a calculator in this question. It was essential that candidates showed sufficient detail in their method to demonstrate that a calculator had not been used. A good number of candidates did this and offered fully correct solutions. Most candidates chose to square the radius before rationalising. A few candidates omitted essential method and simply wrote, for example, $\frac{(16+9\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})} = 4-\sqrt{3}$. This was sufficient for the first 2 marks only. Some candidates chose to multiply all terms by π, rather than cancelling the π as a common factor in their fraction. This often resulted in errors when multiplying out in order to rationalise the denominator. Candidates could gain partial credit even if the formula being used was not correct. In this case, candidates could gain credit for a correctly expressed statement of the form $\frac{c(16+9\sqrt{3})(a-b\sqrt{3})}{(a+b\sqrt{3})(a-b\sqrt{3})}$ or $\frac{c(16+9\sqrt{3})}{a+b\sqrt{3}}\times\frac{a-b\sqrt{3}}{a-b\sqrt{3}}$. Missing fraction bars and/or brackets were not condoned. In weaker responses, candidates resorted to using decimals at an early stage implying the use of a calculator.

Question 4

(a) The most common approach to solving this equation was to simplify the powers of e on the left-hand side, collect the powers and then take natural logarithms. A good number of candidates were successful with this approach. Some candidates used a substitution and these usually rewrote the left-hand side as $\frac{e^{2x}\times e^2}{(e^x)^{\frac{1}{2}}}$ before stating and using a substitution such as $y = e^x$. This was generally also a successful approach. Some candidates made slips when removing the brackets in the numerator as e^{2x+1} was seen on occasion. Similarly, some candidates made slips when rewriting the denominator with $e^{\frac{1}{2}}$ or $e^{-\frac{x}{2}}$ or $e^{x^{\frac{1}{2}}}$ all seen. A few candidates incorrectly manipulated 10 in the initial step. A good number of candidates were able to combine the algebraic powers of e correctly, although some did incorrectly write, for example, $\frac{2x+2}{\frac{x}{2}} = 10$ or $\frac{2x+2}{\frac{x}{2}} = e^{10}$. Other candidates made errors when collecting the terms in x. This was more commonly the case when the expression they were simplifying was still a power of e and candidates tried to double all the terms. These candidates often offered $e^{4x-x+4} = \ln 10$. Some candidates collected the terms correctly but made slips when trying to deal with the exponential. These candidates often offered $1.5x + 2 = e^{10}$ or $1.5x + 2 = 10$. A few candidates omitted brackets in the final step of working and offered no correct final answer. The natural logarithm of an exact value was acceptable as a final answer. The natural logarithm of an inexact decimal needed to be evaluated in order to be credited as a final answer. In weaker responses, candidates used common logs as if they were natural logs or took logarithms inappropriately, offering $\frac{\ln(e^{x+1})^2}{\ln(\sqrt{e^x})} = \ln 10$ or similar. In the weakest responses, candidates differentiated, applying the quotient rule, showing no real understanding of the process required.

(b) Some fully correct, neat and logical solutions were presented for this question. Candidates who took care to apply the necessary laws of logarithms and correctly derived the quadratic equation often earned full marks. A few candidates went on and used the values of y they had found to generate further values of y, often from solving something such as $\log_9 y = 3$ and $\log_9 y = 9$. This was not condoned.

A few candidates were unable to successfully rearrange $\frac{y^2}{4y-9} = 3$ with the most common errors being sign slips or omitting to multiply –9 by 3. Candidates who at least had a correct initial step could gain further credit for solving their quadratic equation by factorising, using the formula or completing the square. Credit was given for an attempt at appropriate factors or for sight of full substitution into the formula providing the discriminant was not negative. Similar conditions applied to attempts at completing the square.

Candidates needed to take great care not to make an error in the early stages of this question as no recovery was permitted from an incorrect use of laws of logarithms. Statements such as

$\dfrac{\cancel{\log_9} y^2}{\cancel{\log_9}(4y-9)} = \cancel{\log_9} 3$ then $\dfrac{y^2}{4y-9} = 3$ were not accepted. Many seemingly correct solutions were based on incorrect methods such as this.

Other incorrect first steps were

- rewriting $2\log_9 y$ as $\log_9 2y$
- rewriting $\log_9(4y-9)$ as $\log_9 4y - \log_9 9$ or $\log_9 4y \div \log_9 9$
- spurious deletion of logarithms giving equations such as $2y - (4y - 9) = 9^{\frac{1}{2}}$ and
- incorrectly dealing with the $\dfrac{1}{2}$, which sometimes became $9^{\frac{1}{2}}$ at an inappropriate stage.

Question 5

(a) A good proportion of fully correct answers were seen to this part of the question. The majority of candidates who attempted to differentiate did so correctly. Some candidates may have improved if they had reread the question as they found the equation of the tangent and not the equation of the normal. Some candidates did not seem to be aware that by substituting $x = 1$ into their derivative, they were finding the gradient of the tangent to the curve at that point. A few candidates found that the value of the derivative was also 1 and confused this with the y-coordinate. These candidates sometimes equated the derivative to 0 and solved to find values of x. They then attempted to use these values as gradients. Therefore, when candidates found the value of the derivative when $x = 1$, it was necessary for them to indicate, by labelling or use, that they knew this to be the gradient of the tangent and not the y-coordinate. In weaker responses, candidates found the coordinates of a second point and did not differentiate but instead found the gradient of the chord from (1, 1) to their point and used that as the gradient of the tangent. This was not acceptable.

(b) Without the correct equation of the normal, the cubic equation candidates formed to solve for the points of intersection was unlikely to have $x - 1$ as a factor. A reasonable number of candidates who did have the correct equation in the previous part were also successful here. Some candidates would have improved if they had taken more care with signs as some errors were made in the initial stages. Various methods were used to find the quadratic factor, including algebraic long division, which seemed quite popular. Candidates who attempted to factorise had not paid attention to the form required for the answer. This clearly indicated that they needed to use the quadratic formula or complete the square. Other candidates who left their answers as $\dfrac{6 \pm \sqrt{8}}{2}$ may also have improved if they had read the question a little more carefully. Answers of $3 \pm \sqrt{8}$, from incorrect cancelling of the 2 with the 6 only, were also fairly commonly seen. In weaker responses, candidates did not attempt to find the quadratic factor and instead attempted either to apply the quadratic formula to a cubic equation, or to solve the derivative of a cubic polynomial equated to 0 or to solve a quadratic that had been derived, for example, from $x(x^2 - 7x + 13) = 7$. Usually this was $x^2 - 7x + 6 = 0$.

Question 6

To be successful in this question candidates needed to multiply out the brackets and divide each term by x as a first step. A reasonable number of candidates did this, but even so, fully correct solutions were not common. Some candidates, after stating $\dfrac{x^2 + 4x + 4}{x}$, did not simplify to three terms but instead integrated each component of their algebraic fraction. A few candidates did not expand the brackets correctly and common errors were to omit the cross terms or to have either $\dfrac{x^2 + 2x + 4}{x}$ or $\dfrac{x^2 + 4x + 2}{x}$. Some candidates who simplified correctly to $x + 4 + \dfrac{4}{x}$ integrated the first two terms without issue but were unable to integrate the final term correctly. It was either omitted or written as 4 or $-\dfrac{4}{x^2}$ or $\ln(4x)$ for example. Sometimes candidates simplified $\frac{x^2}{x}+\frac{4x}{x}$ to $x+4$ and in the same step rewrote $\frac{4}{x}$ as $4\ln x$, confusing themselves in the process. Candidates who had a reasonably derived expression to evaluate, usually substituted the limits into it correctly. A few candidates omitted brackets around the terms for the lower limit and this was not condoned unless there was evidence of a swift recovery in correct further working. It was important that candidates did not resort to decimals at this stage as the answer was required in exact form. In the weakest solutions, candidates offered, for example, $\left[\frac{(x+2)^3}{3x}\right]_2^3$, $\left[\frac{(x+2)^3}{3}\ln x\right]_2^3$ or $\int \frac{2x+4}{x}\,dx$ as a first step or attempted to use the quotient rule for differentiation.

Question 7

The parts of this question were sequentially linked and candidates needed to take great care in **part (a)** to ensure they had expressions of the correct structure to access marks in **parts (b)** and **(c)**.

(a) Candidates needed to differentiate the expression for the displacement to find an expression for the velocity. Some did this correctly and went on to differentiate again and state the correct expression for the acceleration. A common error was to not differentiate – t as – 1 but for that term to disappear and the velocity to be stated as $3e^{2t}-4e^{-2t}$. These candidates were still able to earn the mark available for the acceleration. In weaker responses, candidates either integrated twice or reduced the powers of e by 1 and/or reduced the powers of e by t and/or multiplied each term by t. In the weakest responses, candidates attempted to use, for example, velocity $=\frac{\text{displacement}}{\text{time}}$ or no attempt was made to answer.

(b) Candidates who had a correct expression for the velocity in **part (a)** usually knew that the correct process was to equate this to 0 and solve for t. Many of these candidates attempted to do this and some were successful. Some candidates stated and used a sensible substitution in this part of the question and it proved to be a useful approach. Those who were not successful often tried to rewrite the equation as, for example, $3e^{2t}-4e^{-2t}=1$ and then take the natural logarithm of each term i.e. $\ln(3e^{2t})-\ln(4e^{-2t})=\ln 1$ or similar. Candidates who did form a correct quadratic in e^{2t} or quartic in e^t mostly factorised correctly, although some sign errors in factors were seen. Those candidates who had correct factors almost always went on to give the correct value of t and discarded the invalid solution. A small number of candidates derived a value of t from $e^{2t}=-1$ and this was not condoned. Candidates who stated the value of t only, without correct or any method, were not credited. Those very few candidates who did this may have improved if they had referenced the instructions on the front of the examination paper that indicate that 'all necessary method must be shown; no marks will be given for unsupported answers from a calculator'. A few candidates found what appeared to be the correct value of t but it was derived from completely incorrect working and was not, therefore, credited. Candidates who had no constant term in their expression for the velocity could not be credited in this part as the solution was not of equivalent difficulty, being somewhat easier.

(c) A reasonable number of candidates earned the method mark for substituting a positive value of t into an expression for the acceleration that was of the correct structure. Some candidates earned both marks, having fully correct work to this point. A few candidates had a correct expression for the acceleration but used $t=0$ and this was not credited as the value of t used needed to be positive.

Question 8

(a) This question should have been straightforward. Candidates needed to apply both the chain rule and the product rule. A reasonable number did both correctly. Some managed a correct chain rule but not a correct product rule and some managed a correct product rule but not a correct chain rule. In weaker responses, candidates offered derivatives such as 2cos. These candidates commonly treated $\sin 2x$ as a product of sin and $2x$.

(b) Candidates needed to have a correct derivative in order to gain full credit. Some candidates started again and stated a correct derivative, even though their answer to **part (a)** had been incorrect, but this was not common. Relatively few candidates earned full marks in this part. Some candidates earned two of the marks available but they either simply stated $y = \frac{\pi}{4}$ as their answer or were unable to simplify their equation to $y = x$ or found the equation of the normal and so were not awarded the third mark. Commonly they made a sign slip resulting in a y-intercept of $-\frac{\pi}{2}$. A small number of candidates earned a mark for finding a correct y-coordinate. This was the only mark available to candidates who did not have a correct derivative. The most common error was to evaluate the trigonometric functions with their calculator in degree mode or to not attempt any evaluation, which was not acceptable. A few candidates evaluated the gradient function correctly but used it as the y-coordinate or used a y-coordinate of 0. A few other candidates evaluated the y-coordinate correctly but used it as the gradient. Candidates who converted $\frac{\pi}{4}$ to 45° needed to understand that this was a poor choice of approach in a calculus question.

(c) Some well-presented and fully correct solutions were seen. These were, however, not very common. A few candidates earned the first three method marks but then omitted to substitute the limits into the whole expression, giving their final answer as $x\sin 2x - \frac{1}{4}$ and so no further credit could be given. A small number of candidates omitted to take note that the answer had to be exact and used inexact decimals. Some of these candidates showed the substitution of the limits into the trigonometric functions before they changed to decimals only. This earned more credit than solutions where candidates wrote inexact decimals as soon as the limits were used. A few candidates made slips when integrating $\sin 2x$. These candidates earned two of the first three marks if their attempt was partially correct. Some candidates made no real progress beyond an initial correct statement such as $\int 2x\cos 2x \, dx = x\sin 2x - \int \sin 2x \, dx$ but this was rewarded with a mark.

Generally, in questions requiring candidates to integrate, poor presentation of answers is condoned. However, in a question such as this, where candidates were working with expressions that both do and do not need to be integrated, it was essential that candidates made it clear which functions they intended to integrate. Some candidates made this clear using a correct first step linking this part of the question to **part (a)** and then substituting the limits into an expression of suitable structure. Other candidates did not make this clear and were not credited. Some candidates made clearly incorrect statements such as $\int 2x\cos 2x + \sin 2x = 2x \times \frac{\sin 2x}{2} + \frac{1}{2}\cos 2x$ and could also not be credited.

Candidates may have improved, therefore, if they had taken more care over the presentation of their work and had only used the integral sign to show the intention to integrate. In some responses, candidates made a correct initial statement but then wrote $\int 2x\cos 2x = x\sin 2x - \sin 2x$, or similar, omitting a necessary integral sign. This also could not be credited.

There were many transcription errors in this part of the question. In many solutions, the equation of the curve became $x\sin x$. This was condoned for method marks. On some occasions, the derivative became $2x\cos 2x + \sin x$. This was not condoned as the derivative used needed to be correct for the solution to be equivalent. The weakest solutions made no attempt to use **part (a)** and offered solutions with initial steps of $\int 2x\cos 2x = \frac{2x^2}{2} \times \frac{\sin 2x}{2}$ or similar.

Question 9

Some candidates were unable to recall the correct formulae for arithmetic and geometric progressions. These candidates may have done better in this question if they had checked the list of Mathematical formulae on page 2 of the examination paper.

(a) A reasonable number of correct solutions were seen. Many candidates earned two marks for finding a correct equation using the sum of the first 3 terms. Those using $a + a + d + a + 2d = -36$

were most successful. Although many candidates correctly used the formula for S_3, a few candidates made slips when manipulating the $\frac{3}{2}$ in the equation which resulted. In good responses, candidates formed a second, correct equation. Again those using $a + 9d + a + 10d + a + 11d = 72$ were most successful. Other successful approaches were to use a first term of $a + 9d$ in the sum formula for 3 terms or to use $S_{12} - S_9 = 72$. Candidates who found a second correct equation usually went on to find correct answers. There were some common incorrect equations found for the sum of the last three terms. These were often formed using $S_{12} = 72$, $S_9 = 72$ or $S_6 = 72$. Some of these equations gave what seemed to be a correct answer, but as the equations used were not fully correct, full credit was not given. In weaker responses, candidates formed a pair of equations using $S_3 = -36$ and $S_3 = 72$. This resulted in a pair of equations that had no solution. In the weakest solutions, candidates used the expression for the nth term rather than the sum formula or they used the sum of a geometric progression rather than an arithmetic progression.

(b) Again, a reasonable number of correct solutions were seen. Most candidates saw that the common ratio was 1.2 and were able to write this down from the terms given. A few candidates attempted unnecessary ratios of terms and made errors. Those who used 1.2 were more successful than those who used ratios to find $\frac{6}{5}$, as use of this in the sum formula sometimes resulted in brackets being omitted in the numerator. Candidates were permitted to form inequalities or equations in order to solve the problem. Candidates who used $\frac{1.2^n - 1}{1.2 - 1} = 500$, for example, usually went on to simplify this correctly. A good number of these then went on to solve correctly also, the majority of these using logarithms as expected. Those who chose to use $\frac{1 - 1.2^n}{1 - 1.2} = 500$ sometimes stopped when they arrived at $-1.2^n = -101$ and seemed unsure of how to progress. Negating both sides before taking logarithms was necessary from this form. These candidates were also more likely to make sign errors and this sometimes resulted in $1.2^n = 99$ or an inequality or equation that had one side negative and one positive.

Some candidates used trials at an early stage. This was not condoned as a method of solution if a suitable equation or inequality had not been stated.

In weaker responses,

- candidates calculated a difference of terms for the ratio, stating $r = 0.2$, or
- the expression for the sum of the first n terms was stated as $\frac{(1 - 1.2)^n}{1 - 1.2} = 500$ or
- the expression used was not for a sum of n terms but for the nth term or
- $\frac{1.2^n - 1}{1.2 - 1} = 501$ was used or
- the sum of n terms of an arithmetic progression was used.

Question 10

(a) In this question, candidates needed to prove a trigonometric identity. Candidates needed to work to show that the left-hand side of the identity was equal to the right-hand side. Most candidates attempted to do this, although a small number still worked as if they were manipulating an equation, moving terms from side to side and assuming the right-hand side was valid, which was not permitted. There were two main approaches used. The following comments relate to these main approaches although a very small number of other successful methods were applied. There were a few fully correct responses seen, although these were not common. The question required candidates to prove the identity by writing $\cot x$ and $\tan x$ in terms of $\cos x$ and $\sin x$. Candidates found this to be quite a challenge even with the clear instruction given. In the first approach, candidates were able to earn the first three marks very concisely, keeping the expression as the sum of two fractions. A reasonable number of those making a good attempt at the question chose this method. In the second approach, candidates formed a single fraction with a common denominator before simplifying terms and this was more likely to result in a sign error at some stage as the resulting fraction was far more complex. However, it was still possible for candidates to earn the first three marks using simple algebraic manipulation techniques on this single fraction and some did so. The fourth mark depended on candidates making a sensible change of sign so that a common denominator or common factor could be established. A small number of candidates did this and some of these concluded the proof correctly.

There was a great deal of incorrect algebraic manipulation seen using both approaches. Sometimes this happened in the early stages of the solution resulting in few, if any marks, being awarded. For example, it was not uncommon to see $\dfrac{\sin x}{1-\dfrac{\cos x}{\sin x}}+\dfrac{\cos x}{1-\dfrac{\sin x}{\cos x}}=\dfrac{\sin^2 x}{1-\cos x}+\dfrac{\cos^2 x}{1-\sin x}$

or $\dfrac{\sin x\left(1-\dfrac{\sin x}{\cos x}\right)+\cos x\left(1-\dfrac{\cos x}{\sin x}\right)}{1-\dfrac{\cos x}{\sin x}-\dfrac{\sin x}{\cos x}+1}=\dfrac{\sin x\left(1-\dfrac{\sin x}{\cos x}\right)+\cos x\left(1-\dfrac{\cos x}{\sin x}\right)}{2-\dfrac{\cos^2 x-\sin^2 x}{\sin x\cos x}}$ or similar.

In weaker responses, candidates formed a single fraction and then performed some spurious cancellation of terms to arrive at the given answer in one step.

For example, $\dfrac{\sin x\,\cancel{(1-\tan x)}+\cos x\,\cancel{(1-\cot x)}}{\cancel{(1-\cot x)}\,\cancel{(1-\tan x)}}=\sin x+\cos x$.

(b) Candidates found this part of the question a little less challenging. More candidates were able to derive a correct quadratic, usually in cos x. Those who had a correct equation often went on to solve it correctly and earned full marks. A few candidates who had a correct equation made a sign slip when factorising, but that was not common. Again, quite a few candidates made good use of a substitution which they stated before using. The handful of candidates who wrote $y=\cos^2 x$ and then factorised as if they had $y=\cos x$ totally confused themselves, however. This was not condoned. Some candidates were able to state a correct pair of angles, but fewer were able to state all four angles. Some candidates rounded to 3 significant figures without offering more accurate angles. This was not condoned. These candidates may have improved if they had paid attention to the instructions on the front of the examination paper regarding accuracy of angles in degrees. In weaker responses, candidates were often unable to carry out the manipulation needed to arrive at the quadratic form in a single trigonometric function. A commonly incorrect first step offered in these cases was $9\cot x+3(1+\cot x)=\tan x$.

ADDITIONAL MATHEMATICS

Paper 4037/23
Paper 23

Key messages

In order to do well in this examination, candidates need to read the questions carefully and take note of key words or phrases such as 'find the **exact** value' or '**hence** show that'. Finding the exact value usually means that the answer should be left in terms of, for example, a fraction, a logarithm, an exponential or in surd form. If a question states 'hence', then the result found in a previous part should be used.

Solutions to questions requiring candidates to **show** a given result should be presented carefully showing all the stages of the working. Similarly, if a question states that calculators should not be used, then it is important to show all the stages of the working clearly, providing evidence that a calculator has not been used.

If a candidate decides to use a substitution to help them solve a problem, then they should clearly state the substitution that they are using.

Having completed a solution, it is sensible to re-read the question to make sure that all parts have been answered, as sometimes a question contains more than one demand, as in **Question 5(b)** of this examination.

General comments

Most candidates provided clearly set-out solutions to problems. This was particularly important in **Questions 5(a)**, **7(a)** and **8(a)** where candidates were required to show a given result. In **Question 7(a)**, for example, some candidates began by writing down an equation with two unknowns, they then realised that they needed to take a different approach to find one of the unknowns before they could proceed. Careful labelling of the work enabled the examiner to follow the logic of the candidate's work.

If a candidate is using a substitution to solve a problem, then they should think carefully about the letter that they choose for the substitution. It was unwise to replace $\log_2 x$ with x in **Question 3**, for example, or to replace r^2 with r in **Question 10(b)**.

Care should always be taken when expanding brackets to check that sign errors have not occurred. In this examination, sign errors or missing essential brackets were quite common in **Questions 2**, **3(b)** and **8**.

Comments on specific questions

Question 1

This question was generally answered very well, and most candidates scored full marks in all three parts.

(a) This part proved to be the most straightforward, with two different approaches seen. In the first approach, candidates evaluated g(0) and used their answer to evaluate fg(0). It was rare to see an error when this approach was adopted, with just a very small number of candidates making an error evaluating $2\sin 0 + 3\cos 0$. In the second approach, candidates began by finding the composite function fg(x) before substituting $x = 0$. This method resulted in a slightly higher risk of errors, with brackets sometimes being omitted in the expression for fg(x). A small number of candidates misunderstood the notation for composite functions and attempted to multiply the functions.

(b) Most candidates were able to find gg(x) correctly. The most common error was the omission of –1 from the end of the expression or the –1 appearing as part of the power. As in **part (a)**, a small number of candidates multiplied the functions.

(c) Most candidates were able to score at least some of the available marks in this part. It was quite common to see brackets missing from the expression for $g^{-1}(x)$, which was the main source of errors. Other errors seen were an incorrect order of operations at the start, leading to $\ln x = \ln\left(e^{3x} - 1\right)$ or the variables not being swapped at any stage, resulting in an answer of $y = 4$. Those that found the correct $g^{-1}(x)$ usually went on to obtain a fully correct solution.

Question 2

Most candidates recognised the need to use the discriminant and many therefore obtained a correct quadratic expression in k. There were sometimes sign errors when expanding the brackets, and a small number of candidates were unable to correctly identify a, b and c from the equation of the curve, with x sometimes appearing in the discriminant. Those that obtained a correct quadratic in k were usually able to solve it correctly, usually by factorising, leading to values for k of 6 and 10. The most common error was for the answer to be left as $k = 6$ and $k = 10$, or for inequalities to be given, but with incorrect inequality symbols. Thus, a mark of 3 out of 4 was quite common for this question. A good number did, however, score all four marks with concise solutions, often benefiting from the use of a diagram.

Question 3

(a) There were two approaches taken to solving the simultaneous equations. The first method was to start by solving the equations for $\log_2 x$ and $\log_2 y$ and then find x and y. Some candidates did this by using a substitution, most wisely using letters other than x and y for the substitution. Some however were seen to state 'let $x = \log_2 x$ and let $y = \log_2 y$'; these candidates sometimes forgot that they had used a substitution and gave the values of $\log_2 x$ and $\log_2 y$ as their final answers.

The second approach was to begin by writing the equations as $x^3y^2 = 2^{24}$ and $\frac{x^5}{y^3} = 2^2$. Some candidates did successfully complete the solution if they started this way, but they were more likely to make errors with this approach and some began with incorrect equations such as $x^3y^2 = 24$ or $x^3 + y^2 = 2^{24}$.

(b) Many candidates were successful with this part, usually by equating powers of 2 to obtain a linear equation in t. A few candidates made sign errors, omitting the brackets in $2^{t+4-(1-2t)}$. Most candidates correctly wrote 512 as 2^9. The most common error was to see $\frac{t+4}{1-2t} = 9$ rather than the correct linear equation in t.

Question 4

This question proved to be one of the most challenging on the paper. Many candidates did not realise that to successfully integrate, they needed to rewrite the integrand as a sum of separate powers of x. Thus, many attempted to integrate the numerator and denominator separately and divide the results. Those that realised the need to expand the numerator and divide each term by x^3 usually did so successfully. Further errors were seen in the integration of each term, particularly in the integration of $\frac{1}{x}$. Those that integrated correctly usually showed all their working correctly for the substitution of the limits. A few candidates did not give an exact answer as instructed in the question. It is important to check instructions of this sort.

Question 5

(a) Most candidates correctly rearranged $1000 = \pi r^2 h$ for h and substituted into the formula for the total surface area $S = 2\pi r^2 + 2\pi rh$. A few efficient candidates rearranged to $\pi rh = \frac{1000}{r}$ and substituted this into the formula. Some candidates chose to find the curved surface area and the area of the top and bottom circles separately and this was fine if their method was clear. Some candidates knew the formula for the volume of a cylinder but were unable to rearrange it correctly. Some candidates interpreted the question incorrectly, assuming $h = 2\pi rh$, and hence gave their volume equation as $1000 = \pi r^2 \times 2\pi rh$. Other incorrect volume equations seen were $1000 = \frac{1}{3}(\pi r^2 h)$ and $1000 = 2\pi r^2 h$.

(b) Some candidates did not attempt this part of the question. For those that did, the differentiation was generally good, and most candidates knew that they needed to equate the first derivative to zero and solve for r, which was usually done well. In weaker responses candidates sometimes found the second derivative and incorrectly equated that to zero or even set the surface area equation equal to zero and solved. The final mark was for showing that the value of r gave a minimum for this area. Very few candidates earned this mark, as many simply found the value of the minimum area without using the second derivative to show that it was a minimum.

Question 6

(a) Most candidates knew that they needed to differentiate the expression for the displacement with respect to t to obtain the velocity and differentiate again to obtain the acceleration. Many candidates did not know how to differentiate a log function and some simply divided the expression for displacement by t. Some who knew how to differentiate the log term omitted the second term, which had consequences in **parts (b)** and **(c)**. Those that differentiated correctly to obtain the velocity usually used the quotient rule correctly to obtain the acceleration. Some used the equivalent product rule, but those candidates seemed more likely to make an error.

(b) Most candidates knew that the particle was at rest when the velocity was zero. Those with a correct velocity formula usually found the correct quadratic equation and solved it correctly. Most candidates who reached this stage also correctly eliminated the value of t less than 2.

(c) Candidates who had made earlier errors in this question often omitted **part (c)**. Those who had correct solutions in **parts (a)** and **(b)** usually found the acceleration correctly, although a few changed the negative answer to a positive answer. Some candidates, who had gained full marks in **part (a)**, went on to simplify the expression for the acceleration, but did so incorrectly. Consequently, these candidates lost marks in **part (c)**.

Question 7

(a) Some candidates found **part (a)** challenging, making little progress, but there were also many good solutions. Most successful candidates began by finding BC using the sine rule and then used the area formula based on the 75° angle. Some candidates omitted to show sufficient detail in rationalising the denominator. The final answer was provided in the question, and candidates were instructed not to use a calculator, so showing this detail in the working was vital. Several candidates opted to find AC first, using the area formula based on the 60° angle, and then used the sine rule to find sin75°. As with the first approach, sufficient detail in the working was sometimes missing. Other methods were also seen, such as dropping perpendiculars from either A or B and working around the diagram, quite often successfully. In some cases, the given answer was used in conjunction with the area formula to find that $BC = \sqrt{3}$, which was then used as in the first method above. No marks were available for this circular argument.

(b) The key word in this part was 'hence', and candidates were therefore expected to use the given value for sin75° in their solution. Quite a few candidates did so successfully, correctly using the sine rule. Others started again and found AC using the area formula, but as they were not following the instruction 'hence', only partial credit was available.

Question 8

(a) Some candidates found this part of the question quite challenging. Most were, however, able to score the first mark, either by rewriting the left-hand side purely in terms of $\sin x$ and $\cos x$ or by writing the terms over a common denominator. Those taking the latter approach sometimes made a sign error when expanding brackets in the numerator and some, who did not show an initial stage of working but went straight to an expression with a sign error, were unable to score any marks. It is therefore very important to show all stages of the working in a question of this sort. For the last three marks, candidates needed to have reached a stage where the left-hand side had been written only in terms of $\sin x$ and $\cos x$, and some candidates were unable to reach this stage successfully, sometimes using incorrect identities such as $\tan^2 x - 1 = \sec^2 x$ or $\tan x = \frac{\cos x}{\sin x}$. Those that correctly reached the final given answer usually did so showing all the necessary stages in the working and using correct notation throughout. The final mark was withheld for incorrect notation seen, such as $\cos x^2$ or $\frac{\sin}{\cos} x$.

(b) Many candidates found **part (b)** challenging with some omitting it. Some candidates who attempted it did not realise that the word 'hence' meant that they should use the result from **part (a)**. Those who reached the correct quadratic in $\cos x$ usually went on to solve it correctly, obtaining the correct three values for x. Sometimes the solution of 180° was omitted or extra incorrect solutions, such as 90° and 270°, were included.

Question 9

(a) Many correct solutions were seen here. Some partially correct solutions were seen where either e^{2x} was differentiated correctly, but no attempt was made at the product rule, or e^{2x} was differentiated incorrectly, but the incorrect result was used in a correct product rule structure. Some weaker responses had answers involving e^{2x-1}.

(b) Most candidates appreciated the need to find the coordinates of a point on the normal and many used their answer to **part (a)** to find the gradient of the tangent. A few candidates found the equation of the tangent rather than the normal, but those that attempted the gradient of the normal usually did so correctly. Those that kept their answers in terms of e avoided rounding errors, which were sometimes seen in answers involving decimals.

(c) Very few candidates were able to gain marks in **part (c)**. Those that integrated correctly usually went on to show sufficient working for substituting the limits, although a few made sign errors and some only substituted limits into $-\frac{1}{2}e^{2x}$, leaving xe^{2x} in terms of x. Some candidates, whose work was otherwise correct, did not give an exact answer, as requested in the question.

Question 10

(a) Many candidates correctly wrote down an expression for the 5th term and equated it to 11. Errors sometimes occurred in the second equation in a and d, with some candidates correctly writing down a second equation but then making an error expanding the brackets to reach $a + 6d = 3a + d$. Those that successfully expanded the brackets usually solved the resulting pair of simultaneous equations correctly. Weaker candidates sometimes used the formula for the sum of an arithmetic progression, rather than the nth term of an arithmetic progression.

(b) Many candidates successfully obtained a pair of equations in r and d, although a few wrote down correct expressions for the 2nd term of the arithmetic progression and the 3rd term of the geometric progression, for example, but did not equate them. Those that formed correct equations usually correctly eliminated either r or d. Those that eliminated d were often not able to give their quartic in r in solvable form, often reaching $3r^2\left(5 - r^2\right) = 12$, say, and then equating each factor on the left to 12. Those who correctly solved the quartic usually correctly eliminated the values of r that were less than or equal to 1, although a few who had made a substitution of $r = r^2$ omitted to take the square root, giving an answer of $r = 4$.