ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key messages

For candidates to succeed in this paper, it is essential that they are familiar with the rubric on the front of the paper, taking into account the accuracy required. Careful reading of each question is necessary, together with the checking that the full demands of the question have been met. Setting work out in a clear and concise fashion with all the necessary steps is also essential.

The examination paper tests knowledge, understanding and skills, so familiarity with the syllabus and the assessment objectives is key. The questions cover simple application of standard techniques at a lower level with the expectation that candidates will be able to use these techniques to help with problem solving at a higher level.

General comments

There did not appear to be any timing issues, with the majority of candidates being able to attempt most, if not all, questions. Candidates should also be aware that if they run out of space, or need to make other attempts at a question, it is preferable to use any blank pages at the end of the question paper or ask for additional paper which may then be attached. This helps with the setting out of work in a clear and concise fashion. As in previous sessions, some candidates are still working in pencil first and then overwriting their work in ink. This very often makes the work difficult to read, which can result in marks not being awarded. It is preferable for a candidate to start again and use an additional sheet if necessary.

Many candidates are still unfamiliar with the word 'exact' and continue to give solutions which require an exact final answer in decimal form. Careful reading of a question and check as to what form the final answer should be in is essential.

Past examination papers are often used for examination practice and careful reading of the accompanying mark scheme and Examiner Report will help reinforce the necessity to give final answers in the required form.

Comments on specific questions

Question 1

Many candidates were able to describe the regions in the Venn diagrams correctly. For the first diagram, most chose to describe it as $A' \cap B$. For the second diagram, correct responses varied equally between $X \cap (Y \cup Z)$ and $(X \cap Y) \cup (X \cap Z)$. Candidates found the second of the Venn diagrams harder to deal with, with most errors simply being errors in union and/or intersection signs.

Question 2

Most candidates realised that they needed to equate the equations of the line and the curve to eliminate *y*. Problems arose when it came to the simplification of the resulting equation in *x*, with errors occurring in the coefficient of *x* and the constant term. Use of the discriminant of the resulting quadratic equation was common, but previous simplification errors led to incorrect critical values. For those candidates that did obtain the correct critical values, a correct range usually followed.

Question 3

It was important that the first given equation be written in terms of 7 only or terms of 49 only, and that the second equation be written in terms of 5 or 25 only. If this was not done, then no further relevant progress could be made. The indices of the resulting equations could then be used to form two simultaneous equations in x and y, which could then be solved. Errors in the simplification of the indices meant that some candidates were unable to gain any more than method marks.

Question 4

(i) Most candidates realised that they needed to differentiate a quotient and attempted to do so. Most errors resulted from the incorrect differentiation of $\ln(4x^2+1)$. No simplification of the result was necessary. Some candidates did attempt to write the given equation as a product and differentiate as a product, but this tended to be less successful as attempted simplification often led to errors which then affected **part (ii)**.

(ii) Candidates were expected to make use of small changes. A substitution of $x=2$ was necessary together with a multiplication by h. Some candidates appeared to be unaware of this process choosing to use $x=2+h$ instead.

Question 5

(i) Candidates find questions of this type, involving ranges of functions, difficult. Correct responses for the range of f were less common than the correct responses for the range of g. This was because many candidates did not seem to appreciate the fact that $e^{2x}>0$ and so $3e^{2x}>0$.

(ii) Some candidates did not realise that g^2 means $g(g)$ as far as functions are concerned and use of $(x+1)^2$ was often seen or implied. The most straightforward way of solving this problem was to consider $g(0)$ which is equal to 1, then $g(1)$ which is equal to 2. Then $f(2)$ can be evaluated. The alternative method of finding the composite function as $3e^{2(x+2)}+1$ was equally acceptable, but candidates appeared to make more errors with this approach.

(iii) It was essential that for the graph of $y=f(x)$ the curve extended into the second quadrant from the first quadrant and that the y-axis intercept of 4 was either marked in on the graph or stated as a set of coordinates. It was not necessary for the inverse function to be worked out. A reflection in the line $y=x$ was expected together with the graph extending from the first quadrant to the fourth quadrant and that the x-axis intercept of 4 be marked on the graph or stated as a set of coordinates. Few completely correct solutions were seen.

Question 6

As an unstructured question, it was expected that candidates recognise the correct course of action to take and many candidates showed that they knew what to do. Errors usually occurred in the differentiation of $(8x+5)^{\frac{1}{2}}$, numerical simplification of the derivative and finding the equation of the tangent rather than the normal. This question was an example of candidates not checking that they had answered the question fully, with too many omitting to give their final answer in the required form with integer coefficients.

Question 7

(i) Most candidates recognised that they needed to make use of the equation $\lg y=\lg A+x\lg b$. Of these, most chose to find the gradient of the line joining the two given points and equate it to $\lg b$. For this method, most errors occurred when it came to find the value of $\lg A$, with too many candidates using the given coordinates incorrectly in the equation $\lg y=\lg A+x\lg b$. This error was also common for those candidates who chose to form two simultaneous equations using the given coordinates. It is essential that candidates recognise when to use logarithms in a substitution. For those candidates that did use a correct method and obtain correct values, it was again essential that they gave the answer in the required form. A final answer of $y = 10^{2x-0.8}$ did not get full marks as A and b needed to be identified.

(ii) Although some candidates were able to produce completely correct solutions, the incorrect use of $y = 900$ was also the problem with many solutions

Question 8

It was essential that candidates show sufficient working in each part of this question as the use of a calculator was not permitted. This did not stop the obvious use of a calculator by some candidates.

(i) Use of Pythagoras's theorem to obtain the length of the side *BC* was expected with sufficient evidence of non-calculator use being evident. The minimum acceptable amount of work for this being sight of $54 + 14\sqrt{5} + 54 - 14\sqrt{5}$. The candidates had been asked to show a particular result so each step needed to be shown in sufficient detail. Once *BC* had been found and simplified, finding the perimeter was straightforward.

(ii) Candidates chose either to use the formula for the area of the trapezium or split the shape into a rectangle and a triangle and find the area of each separately. Each method was equally successful provided enough detail was shown. Ideally a candidate should show the four terms obtained from an expansion of two terms each containing two terms to provide sufficient evidence.

Question 9

(i) Most candidates made use of either the cosine rule or basic trigonometry in a right-angled triangle to show that angle $AOB = 1.70$. Fewer candidates justified this answer correct to 2 decimal places. To obtain full marks, it was essential that a statement of angle $AOB = 1.696$ (or greater accuracy) so angle $AOB = 1.70$ to 2 decimal places, or equivalent wording, was seen.

(ii) Most candidates, having stated that, or found that, angle $DOC = \frac{\pi}{3}$, were able to make a reasonable attempt to find the perimeter. There were two possible approaches which were equally successful. The first involved finding angles AOD and BOC, which are equal and then finding the arc lengths AD and BC. An addition to the lengths of 15 and 10 was then needed. Errors were usually concerning the calculation of the angles. The second approach was to subtract the lengths of the arcs subtended by angles AOB and DOC from the circumference of the complete circle and then add the lengths of 15 and 10. Errors in the angles and hence the arc lengths were less common with this approach.

(iii) Similarly, most candidates, having stated that, or found that, angle $DOC = \frac{\pi}{3}$, were able to make a reasonable attempt to find the area of the shaded region shown on the diagram. There were two possible approaches which were equally successful. The first method involved finding the area of each of the sectors *AOD* and *BOC* and adding these areas to the areas of triangles *AOB* and *COD*. The second method involved subtracting the areas of the minor segments formed by the chords *AB* and *CD* and subtracting these areas from the area of the circle. The two possible approaches were equally successful. Those candidates who were unable to obtain angle $DOC = \frac{\pi}{3}$ were often able to obtain method marks for a correct approach.

Question 10

As an unstructured question, candidates were expected to decide what steps needed to be taken to find the area. This is an example of where it is essential that work is set out in a clear and concise fashion. The first step should have been to find the coordinates of the points of intersection between the curve and the straight line. Most candidates attempted this, usually successfully. At this point correct angles given in degrees were accepted. Two approaches to finding the area enclosed by the curve and the straight line were available. Both were attempted fairly equally with varying amounts of success. The first method involved subtraction with candidates attempting to deal with $\int 1.5-(2+\cos 3x)\ \mathrm{d}x$. The integration of these terms was considered before the application of the limits which had to be in radians. The second method involved finding the area of a rectangle and subtraction of the area under the curve so candidates were considering the integral $\int (2+\cos 3x)\ \mathrm{d}x$. Again, the integration of these terms was considered before the application of the limits which had to be in radians.

All too often, candidates resorted to the use of their calculators when evaluating the square brackets using the limits. This is an example of where it is essential that candidates check that their final answer is in the required form. An exact response was needed, and it was expected that once the limits were used, the simplification was fairly straightforward. Few completely correct solutions were seen, with errors involving the process of integration being the most common.

Question 11

(a) (i) Most candidates gave a correct response, with very few incorrect answers seen. It is essential that that factorial or similar notation be evaluated.

(ii) Most candidates realised that they had to consider the number of different possible arrangements of 7 books, with some then not realising that the 2 chemistry books at either end can also be arranged in 2 different ways.

(iii) It was essential that candidates realised that they had to treat the books as 4 separate units considering 2 chemistry books, a block of maths books and a block of physics books. These 4 units could be arranged in 4! ways. Each block of books can also be arranged in different ways amongst themselves, so that the result of $4! \times 4! \times 3!$ was obtained and could then be evaluated. This part of the question was attempted less successfully by candidates, with some not attempting it at all.

(b) (i) Many correct answers were seen. Again, it was important that any notation used was evaluated.

(ii) It was necessary for candidates to consider three separate cases, these being when there were 4 boys, 5 boys and 6 boys. Some completely correct solutions were seen, but some candidates did not attempt this part of the question.

Question 12

As a completely unstructured question, candidates were expected to work out a plan of action. It was expected that integration be used to find an expression for the gradient function together with the use of the given information, to find the value of a first arbitrary constant. Most candidates realised that they had to integrate, and this was done with varying levels of success. The most common error was in the integration of $\cos\left(x+\frac{\pi}{3}\right)$. Some candidates made arithmetic slips when attempting to find the value of the arbitrary constant, others did not consider an arbitrary constant at all. Fewer candidates continued and attempted to integrate a second time, with similar errors occurring. Whilst there were few completely correct solutions, many candidates knew exactly the correct process to take, but made arithmetic slips and sign errors. Unfortunately, there were some candidates who did not attempt this question.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

For candidates to succeed in this paper, it is essential that they are familiar with the rubric on the front of the paper, taking into account the accuracy required. Careful reading of each question is necessary, together with the checking that the full demands of the question have been met. Setting work out in a clear and concise fashion with all the necessary steps is also essential.

The examination paper tests knowledge, understanding and skills, so familiarity with the syllabus and the assessment objectives is key. The questions cover simple application of standard techniques at a lower level with the expectation that candidates will be able to use these techniques to help with problem solving at a higher level.

General comments

There did not appear to be any timing issues, with the majority of candidates being able to attempt most, if not all, questions. Candidates should also be aware that if they run out of space, or need to make other attempts at a question, it is preferable to use any blank pages at the end of the question paper or ask for additional paper which may then be attached. This helps with the setting out of work in a clear and concise fashion. As in previous sessions, some candidates are still working in pencil first and then overwriting their work in ink. This very often makes the work difficult to read, which can result in marks not being awarded. It is preferable for a candidate to start again and use an additional sheet if necessary.

Many candidates are still unfamiliar with the word 'exact' and continue to give solutions which require an exact final answer in decimal form. Careful reading of a question and check as to what form the final answer should be in is essential.

Past examination papers are often used for examination practice and careful reading of the accompanying mark scheme and Examiner Report will help reinforce the necessity to give final answers in the required form.

Comments on specific question

Question 1

(i) Many candidates knew the general form of the graph of a cosine curve, but could not sketch the graph with the required transformation. The majority earned the mark for the *y*-intercept but very often did not gain any further marks as a common error was to start at $(-90°, -1)$ and finish at $(90°, 0)$.

(ii) Most candidates were able to identify the correct amplitude.

(iii) Finding the period of $2\cos 3x - 1$ was more problematic with many candidates not realising that an angle was required. Too many candidates gave 3 as an answer.

Question 2

Most candidates were able to find the gradient of the graph and most went on to find a correct vertical axis intercept of 32, but there were instances of arithmetic slips with 20 being obtained instead. The use of the equation $\lg y^2 = mx + c$ was recognised by most, with many candidates going on to obtain the correct equation $\lg y^2 = -4x + 32$. Some candidates did not attempt to rearrange this equation to obtain the required form. Candidates generally had more success if they wrote $\lg y^2$ as $2\lg y$ and then divided through by 2. Those that chose to write $y^2 = 10^{-4x+32}$ often made errors when attempting to find the square root.

Question 3

Most candidates were able to make reasonable attempts to expand $\left(1-\frac{x}{7}\right)^{14}$ and $(1-2x)^4$. There were occasional errors with signs and with the simplification of the coefficients. The number of correct solutions was unexpectedly low as it appeared that many candidates were unable to successfully multiply their two expansions out and simplify the resulting terms correctly. It was also apparent that some candidates thought that they just had to expand out $\left(1-\frac{x}{7}\right)^{14}$ and $(1-2x)^4$ and left these expansions as a final answer.

Question 4

(i) Most candidates recognised the correct shape of the graph. It was essential that candidates indicated the coordinates of the intercepts of the curve with the coordinate axes as required. Listing the coordinates was acceptable as was marking them in on the axes themselves. Many candidates, having obtained a correct shape and x-intercepts, then omitted to mark in or state the *y*-intercept.

(ii) It was intended that candidates make use of their graph in **part (i)** and identify either the position of the minimum point on the curve $y = 2x^2 - 9x - 5$ or the maximum point on the curve $y = \left|2x^2 - 9x - 5\right|$. Many candidates identified, either by observation, use of the discriminant or by calculus, that the values $\pm\frac{121}{8}$ were significant. Some correct solutions of $k > \frac{121}{8}$ were seen but the other solution of $k = 0$ was seldom obtained correctly.

Question 5

(a) Most candidates were able to identify at least two correct functions. Common errors included the incorrect order in **part (i)** and the erroneous f^2 as an answer to **part (iii)**. Many candidates chose to work out each of the functions listed and then match their responses to the appropriate question part.

(b) (i) Many candidates were under the misapprehension that they had to show that the function was not a one-one function. It was intended that candidates realised that when $x = 0$ the function was not defined and hence the given domain was unsuitable. There were few correct responses.

(ii) It was hoped that candidates would recognise the notation h' being used for the derivative of the function h especially as the notation for the inverse of a function had been used in **part (a)**. Candidates who did not recognise this were unable to gain any marks. Those that did sometimes made errors in the differentiation of $\frac{b}{x^2}$ which often led to a fortuitously correct answer being obtained.

Question 6

(a) Slips in the simplification of indices and sign errors meant that not all candidates were able to gain marks in this part of the question.

(b) Most candidates attempted to change the base of either $\log_7 x$ or $\log_x 7$. The coefficient in $2\log_x 7$ caused problems when it was used to write the term as $\log_x 49$. Having made use of a change of base, most attempted to multiply through by the logarithmic term in the denominator of their equation. Too many, however, were under the misapprehension that $(\log_7 x)^2$ was equal to $\log_7 x^2$

or that $(\log_x 7)^2$ was equal to $\log_x 7^2$. Further progress was not then possible. A quadratic equation in terms of either $\log_7 x$ or $\log_x 7$ was required. Many candidates were able to gain fully correct solutions.

Question 7

(i) Most candidates realised that they had to use the product rule, even if an expansion of the terms was attempted first. Some candidates were unable to gain full marks if they were unable to differentiate the exponential term correctly.

(ii) Errors in simplification of an initially correct derivative in **part (i)** and/or errors in substitution of $x = 0.5$ meant that many candidates were unable to obtain the accuracy mark in this part. There were also candidates who were unable to deal correctly with the idea of small changes.

(iii) Candidates were able to gain follow through marks in this part, making use of the numerical part of their answer to **part (ii)**. There were some solutions when an incorrect rates of change equation was used e.g. 2 being multiplied by, rather than being divided by, the numerical part of the answer to **part (ii)**.

Question 8

(a) (i) It was expected that matrices that were comparable i.e. could be multiplied together, were written down in the correct order. Many candidates were unable to do this. It appeared that many did not really understand the demands of the question.

(ii) Unless a correct pair of matrices had been written down in **part (i)**, it was not possible to gain any marks in this part. Of those candidates that did have a correct matrix pair, some did not answer the question completely, omitting to state which team was awarded the most points. Others made an error, which was all too common, in evaluating the matrix product, obtaining an element of 8 rather than the correct element of 6.

(b) (i) Most candidates were able to write down a completely correct inverse matrix with very few errors seen.

(ii) Many correct solutions to this part were also seen, with most candidates making use of their inverse matrix from **part (i)** and pre-multiplication of the given matrix **B** to obtain the matrix **C**. There were some errors in the evaluation of the matrix product, but most candidates were able to gain marks. There were some who attempted a non-matrix method, but these candidates were unable to gain marks as the question specified 'Hence', meaning that their answer to **part (i)** had to be used.

Question 9

(i) It was essential that each step of working be shown as candidates were working towards a given answer. It appeared that many candidates were unable to recall the formula for the volume of a cylinder. Knowledge of this is assumed to be prior knowledge for this examination. There were slips with terms of π in the simplification of the surface area equation after a substitution had been made. Many candidates were able to produce a completely correct and well set out solution.

(ii) Many candidates realised the process of solution required differentiation and equating the resulting derivative to zero. There were many correct results of $r = 8.43$ or equivalent. There were candidates who, having obtained a correct equation from differentiation and equating to zero, were unable to solve it correctly. As has been common in this type of question in the past, many candidates have not answered the question completely by omitting in this case to find the stationary value of S. Most candidates made use of the second derivative with varying levels of success to determine the nature of the stationary point.

Question 10

(i) Most candidates were able to use either the cosine rule or basic trigonometry involving a suitable right-angled triangle, to show that angle *AOB* was equal to 2.24 radians. Not all of them however, justified this value to 2 decimal places. It was expected that an answer to greater accuracy, in this case 2.2395…, be shown first which then provides justification of 2.24 to 2 decimal places.

(ii) Many candidates were unable to gain many marks as they had not read the question carefully enough. They incorrectly made use of the angle found in **part (i)** as angle *AOC* rather than using the angle found in **part (i)** to help find angle *AOC*. This clearly had an effect on the next part of the question as well. Other errors included the inappropriate use of 18 instead of 10 when attempting to find the arc length *AC*. Most candidates were able to gain method marks for a correct process and many did obtain a correct perimeter.

(iii) Few completely correct solutions were seen, although many candidates were able to obtain method marks when finding the area of the triangle *AOC* and the sector *AOC*. Common errors included the use of 18 instead of 10 for the lengths involved in both calculations and an inaccurate final answer due to premature approximation involving angle *AOC* in **part (ii)**.

Question 11

As a completely unstructured question, candidates were expected to work out a plan of action. It was expected that integration be used to find an expression for the gradient function together with the use of the given information, to find the value of a first arbitrary constant. Most candidates realised that they had to integrate, and this was done with varying levels of success. The most common error was in the coefficient of $(3x-1)^{\frac{1}{3}}$. Some candidates made arithmetic slips when attempting to find the value of the arbitrary constant, others did not consider an arbitrary constant at all. Fewer candidates continued and attempted to integrate a second time, with similar errors occurring. Whilst there were few completely correct solutions, many candidates knew exactly the correct process to take, but made arithmetic slips and sign errors.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

This paper required candidates to recall and use a range of mathematical techniques, to devise mathematical arguments and present those arguments precisely and logically. Good responses needed to be set out clearly and demonstrate a good understanding of fundamental techniques. These techniques include working with logarithms, using Venn diagrams to solve problems, and differentiation and integration of exponential and logarithmic functions. Candidates were also required to demonstrate a thorough understanding of mathematical language and notation. Shorter questions required recall of an appropriate technique but in longer questions a structured solution bringing together various techniques was required.

General comments

A good range of responses were provided, demonstrating candidates' understanding of the syllabus objectives and being able to apply them appropriately. Most candidates attempted all the questions although there were some topics where candidates appeared to be less familiar with the techniques required.

Candidates needed to be aware that if a method was specified by the question then they had to use that method for their solution. The use of the words 'Hence' or 'use your…' in the second part of a question was an indication that the method employed should use the result from the previous part. In these cases, credit was not given for the use of another method. Care was needed in reading the wording of such questions.

Candidates needed to take care when rounding answers to 3 significant figures, particularly those answers with a zero before the decimal point.

Comments on specific questions

Question 1

The question stated that a Venn diagram had to be used in the solution.

(i) Candidates were expected to complete the seven regions in the Venn diagram with expressions in terms of x, sum these expressions, equate to 145 and solve for x. Few correct solutions were seen. Many candidates did not use the information given in the question to produce a suitable Venn diagram. The most common misinterpretation was to use 23, 24 and 28 instead of $23 - x$, $24 - x$ and $28 - x$ in the regions for Chemistry and Physics only, Chemistry and Mathematics only and Mathematics and Physics only.

(ii) Candidates who had obtained $28 + x$ in the appropriate region of their Venn diagram went on to obtain a correct answer.

Question 2

(i) Many responses had a graph that passed through the correct points; (0, 2) and (±90, 2). Some graphs had an incorrect frequency, and some did not go down to –8. However, the primary reason for not gaining full marks was a lack of care when joining the points. Candidates needed to be aware that this was a sine curve and care was needed to draw the correct shape with maximum points implied at 90 and –90.

(ii) There were many correct responses to this question. Candidates appeared to use the form of the equation rather than their graph to answer this part.

(iii) This was generally well answered. Candidates appeared to use the form of the equation rather than their graph to answer this part. Some candidates misunderstood period and gave an answer of 4.

Question 3

(i) This question was well answered with most candidates demonstrating that they understood how to differentiate a composite function. Some candidates incurred difficulties in later parts due to incorrect attempts to simplify their answer.

(ii) Many candidates understood that they had to multiply their derivative by *p*. Not all candidates substituted $\sqrt{3}$ in their derivative and some omitted *p* in their final answer despite having it in their working.

(iii) Most responses successfully calculated that when $x = \sqrt{3},\ y = \frac{1}{2}$ and many used a correct method to find the equation of the normal using $\dfrac{-1}{k\sqrt{3}\left(3\sqrt{3}^2 - 1\right)^{-\frac{4}{3}}}$. A few responses gave the equation of the tangent instead of the normal. Candidates who had incorrectly simplified their expression in **part (i)** and used it here lost marks as did candidates who used prematurely rounded decimal values in their working.

Question 4

(i) Responses to this question were usually correct. Candidates needed to take care when simplifying their final answer and when copying it to use in the next part.

(ii) Ignoring the word 'hence' proved costly for many candidates. Most responses that used a correct matrix method found $\tan x$ and $\tan y$ successfully. Some candidates left their solution incomplete without solving for *x* and *y*. Candidates needed to be aware that the question asked for a solution in radians. Candidates who did not use radian mode in their calculator and instead attempted to convert an answer in degrees were not always successful. Final answers needed to be given correct to 3 significant figures, answers to only 2 significant figures and answers that had been incorrectly rounded did not gain full marks.

Question 5

(i) Most candidates knew that they had to use the product rule and were able to apply it correctly. Some did not differentiate $\ln\left(x^2+3\right)$ correctly; either not recognising it as a composite function or not knowing how to differentiate a ln function. In their final answer candidates needed to take care to write $2x\ln\left(x^2+3\right)$ in an unambiguous form and with brackets around $\left(x^2+3\right)$. Uncancelled answers such as $\left(x^2+3\right)\dfrac{2x}{\left(x^2+3\right)}$ or $\dfrac{2x^3+6x}{\left(x^2+3\right)}$ were condoned. Candidates who did not simplify these to $2x$ and those who simplified the complete expression to $2x(1+\ln(x^2+3))$ made it more difficult to spot the way forward for the second part.

(ii) Candidates needed to use their answer to the first part and the principle that integration is the reverse of differentiation. Of the candidates who used a suitable expression from **part (i)**, few arrived at a correct expression for the required integral. Some made good attempts to rearrange in order to isolate the required integral, but often the $2x$ was not integrated and subtracted.

Question 6

(i) To make progress with this question candidates had to take logs of both sides of the given equation and conclude that either ln*y* or lg*y* had to be plotted against x^2. If this was not done, candidates not only lost marks in this part but were unable to make a successful attempt at the second part. Candidates needed to choose sensible scales as a scale was not provided; responses using difficult scales such as 1:3 cm, 0.6:1 cm were less likely to be accurate. A scale needed to be chosen so that all points were within the area of graph paper.

(ii) Candidates were asked to use their graph for this part and only responses that used a graph of ln*y* or lg*y* against x^2 earned marks. The most straightforward approach was to equate the gradient to ln*b* or lg*b* and the *y* intercept to ln*A* or lg*A* and then use powers of 10 or e to find the constants.

(iii) The most straightforward approach was to read across from 2(lg 100) or 4.6(ln 100) on their graph to obtain a value of x^2 and hence *x*. Candidates who tried to use their values of *A* and *b* in the given equation often misunderstood the order of operations required to find *x*. Candidates using a log equation were more successful, but some confused lg*b* with *b* and lg*A* with *A*.

Question 7

Candidates needed to be aware of when it was appropriate to use combinations and when they should use permutations. More complex questions needed to be broken down into cases.

(a) (i) There were many good solutions but sometimes combinations were used rather than permutations.

(ii) There were many good solutions but sometimes combinations were used rather than permutations.

(iii) The most successful approach was to identify and add two different cases such as: first digit 7 or 9 and first digit 8. A common incorrect answer was 2520 where candidates did not realise that 8 could not be used at both the beginning and the end.

(b) (i) This part was very well answered with candidates being more confident with combinations questions than permutation questions.

(ii) This part was well answered.

(iii) Candidates needed to break the problem down in to two cases; including the husband and wife, and not including the husband and wife. Some candidates considered just one of these cases.

Question 8

This question required a thorough knowledge of the laws of logarithms.

(a) (i) Some candidates did not write $\log_a y^2$ as $2\log_a y$ so were unable to obtain an expression in terms of *p* and *q*. Others did not appreciate that $\log_a a$ equals 1.

(ii) Some candidates successfully simplified $\log_a x^3$ to $3\log_a x$. Many of those obtained either $3p - 1 + q$ or $3p + 1 - q$, not associating the division by *ay* in the argument of the logarithm with subtraction of both $\log_a a$ and $\log_a y$.

(iii) There were many good solutions to this part. Candidates nearly all understood that $\log_x a$ is the reciprocal of $\log_a x$.

(b) Most candidates formed a correct equation and most went on to solve it correctly. Many candidates found *m* but did not go on to find a value of *x*. Candidates who attempted to find *x* usually used a correct method and realised that $m = -1$ would not lead to a solution for *x*. Some candidates did not evaluate their final answer.

Question 9

(i) Candidates did not all understand what was meant by perimeter and some did not attempt to add the relevant parts. Those who added the correct parts and equated to 100 usually rearranged correctly to obtain an expression for *θ.*

(ii) Many candidates made a good start by subtracting the two sector areas. Those using an incorrect expression from the previous part could not obtain a correct expression for *A*. As the answer was given, some other candidates did not obtain the accuracy mark as inconsistencies in powers and signs were evident in their working or they had not shown enough steps in their working.

Candidates who obtained $\frac{5}{2}r^2\theta$ and then substituted their expression for *θ* avoided problems with sign errors.

(iii) Most candidates made a good start by differentiating the given expression for *A* and equating to zero or by completing the square. Many obtained the correct value of *r*. Candidates needed to be aware that *A* had to be evaluated to complete the answer to the question.

(iv) Most candidates substituted into their derivative to obtain 30. From there most candidates showed a good understanding of connected rates of change to obtain a correct answer. A few candidates used *A* rather than $\frac{dA}{dr}$ and some multiplied 3 by 30.

(v) Many candidates correctly realised that they had to multiply their answer to **part (iv)** by the value of $\frac{d\theta}{dr}$ when $r = 10$. Difficulties were encountered with this differentiation, particularly if the quotient rule was attempted. Some candidates used *θ* instead of $\frac{d\theta}{dr}$.

Question 10

(a) (i) Many candidates confused acceleration and deceleration giving an answer of –8 rather than 8.

(ii) This was usually correct.

(iii) Few good responses were seen. Many candidates ignored the position of the *t*-axis and calculated the area of a trapezium of height 40 rather than considering the areas above and below the *t*-axis.

(b) (i) Candidates needed to be aware that 1 had to be integrated to obtain *t* when integrating the expression for *v* with respect to *t.* Most candidates knew that they had to integrate. Many knew how to integrate the exponential expression correctly but not the 1. Some candidates who obtained a correct expression for *x* in terms of *t* either did not include a constant of integration or did not realise that the constant had to be found as part of the full solution to the question.

(ii) Candidates were more successful with this part than **part (i)** and many completely correct solutions were seen. Candidates had to be careful to use the correct order of operations; the value for e^{2t} had to be found before taking logs. Candidates needed to realise that the zero before the decimal point was not significant when rounding their answer.

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

Key messages

The instruction to not use a calculator in certain questions needs to be adhered to as marks are not awarded when there is compelling evidence that a calculator had been used.

General comments

Candidates were well prepared for this paper. There were sound attempts at all questions with most candidates showing a good knowledge of the requirements.

Comments on specific questions

Question 1

(i) A large majority of candidates answered this question very well. A few candidates did not use a ruler and plotted points. As a result the point (1.5, 0) was omitted and the points (1, 1) and (2, 1) were joined by a curve.

(ii) Many completed this entirely successfully. There were a number of candidates who made sign mistakes when attempting to deal with the modulus. Some candidates found only one solution.

Question 2

This was successfully attempted by the vast majority who were awarded all three marks. A few candidates got one value incorrect but very few did not score at all.

Question 3

(a) There were many successful attempts at this question with good knowledge of exponentials displayed. Some candidates were not as proficient in combining exponentials and incorrect equations such as $2x + 1 = 3(4 - 3x)$ were sometimes seen.

(b) There were even more good attempts at this part than **part (a).** Most candidates were comfortable in dealing with log terms. Most recognised that 2 could be replaced by $\lg 100$ and correctly used $\lg a + \lg b = \lg ab$. The resulting quadratic equation was usually solved correctly. There were a number of candidates who were not awarded the final mark as they did not reject $y = -19$ as an inappropriate result.

Question 4

Most were able to eliminate x or y to obtain an appropriate expression. The task of rationalising the denominator by multiplying by $3 - 2\sqrt{2}$ was tackled well by most candidates to give the desired values of x and y.

There were a number of candidates who used their calculators to rationalise the denominator.

Question 5

(i) The vast majority of candidates displayed a sound knowledge of differentiating trigonometric functions and successfully obtained correct expressions for the velocity and acceleration.

(ii) Most candidates realised that their expression for v should be equated to zero and solved. The majority obtained a value for $\tan 2t$ but there were some who changed $\sin 2t$ and $\cos 2t$ into single angles and subsequently struggled to make any further progress. A significant number of candidates were not awarded the final mark as they gave their answer in degrees rather than radians.

(iii) Most candidates inserted their value of t into their expression for a and obtained a value.

Question 6

Most candidates were able to eliminate y from their equations to obtain the correct quadratic in x.

Subsequent use of the quadratic formula gave them the two correct values of $x = \frac{1 \pm \sqrt{21}}{2}$. At this point a number of candidates used their calculators to give these values to 3 significant figures without realising that subsequent work would remove the surd. However, there were a large number who did follow the instructions correctly and provided an elegant and concise solution.

Question 7

(a) Candidates generally tackled this question very well and mostly obtained the correct quadratic factor.

(b) (i) Candidates generally knew how to tackle this part by writing $\tan x$ and $\sec x$ in terms of $\sin x$ and $\cos x$ and then most realised to also use a Pythagorean identity. There were many correct solutions, some succinct and some long-winded. Some candidates did not realise that they needed to use the Pythagorean identity and made only limited progress.

(ii) Most candidates were able to use the previous parts to write the cubic equation in $\sin x$ as the product of the linear and quadratic factors in $\sin x$. Many then solved $2\sin x - 1 = 0$ giving both values. However, many candidates ignored the quadratic equation and did not explain that there were no real roots.

Question 8

(i) The majority of candidates realised that the product rule was required, and many produced the desired result. Those who did not consider the product rule usually obtained some credit for differentiating e^{-2x} correctly.

(ii) Most candidates put their derivative equal to zero and attempted to solve. Many were successful but often the y value was not given in exact form.

(iii) Candidates knew what they were supposed to do and attempted to insert $x = 1$ into their gradient function. There were many completely correct solutions. However, many did not continue working in terms of e (especially one with a negative power) and reverted to the decimal value of the gradient.

(iv) Only the more able candidates could deal with the link between their answer to **part (i)** and the demand of this part, so many attempts were not well managed.

Question 9

(i) and (ii) These were standard matrix calculations that were well understood by all. There were occasional errors in arithmetic in a few candidates' responses.

(iii) The first step in obtaining the correct matrix was to rearrange the given relationship to $\mathbf{C} = \mathbf{B}^2 - \mathbf{BA}$. There were many completely correct solutions. When the relationship was not obtained the most common error was to obtain $\mathbf{C} = \mathbf{B}^2 - \mathbf{A}$.

(iv) This was a case of adapting the given relationship, this time to give $\mathbf{D} = \mathbf{B}^2\mathbf{A}^{-1}$. This was well understood as a concept and there were many correct solutions.

Question 10

This question was answered better than any others on the paper with the vast majority of candidates showing an excellent knowledge of the binomial expansion and subsequent work developing from it. There was the occasional sign error in a few candidates' responses.

Question 11

In order to answer this question well candidates needed to have the correct triangle and then be able to use the cosine and sine rules successfully. There were many completely correct attempts with many others only containing minor errors.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

The instruction to not use a calculator in certain questions needs to be adhered to as marks are not awarded when there is compelling evidence that a calculator has been used.

General comments

There was a wide range of marks achieved on this paper with a number of candidates being awarded full marks. A few candidates seemed to run out of time whilst working on the last question, but most completed as much as they were able.

Comments on specific questions

Question 1

The first diagram proved most difficult with incorrect responses usually shading the area outside both sets. The third diagram was most frequently correct. A fairly common approach was to number all the regions of a diagram and then identify the required set by analysing these numbers. This time-consuming method seemed to work for some but there was no real evidence that it led to fewer mistakes.

Question 2

There were many completely correct solutions to this question. Many candidates were able to correctly differentiate twice and insert their expressions to obtain the required form. A number did not include the expression for $3y$ whilst others made errors in differentiating but realised the need to combine the three expressions. Sign errors were not uncommon and there were some who replaced $3x$ with x. A few considered the given function to be a product and as a result made the question more demanding than necessary.

Question 3

Those who were successful in **part (i)** usually went on to score full marks in the other two parts. A number of candidates did not obtain any marks and there were some who did not attempt the question at all. The greatest source of error was due to candidates believing that the question required combinations rather than permutations. A few candidates managed to miscount the number of symbols in each group and others added correct terms rather than multiply. The simplest successful solution avoided factorials completely and just multiplied the number of options for each choice, that is 14.13.12.11.10 for **part (i)** and similar products for the other two parts.

Question 4

Most candidates were able to eliminate y successfully and obtain a quadratic in x. This was usually correct as was the discriminant that followed and the solution of the quadratic in k. Most were then able to establish a correct range between the two values of –1 and 11. A number of candidates stopped having found the values and others did not express the region correctly. Connecting the two inequalities with ‘or’, a comma or blank space was not accepted. A few gave incorrect inequalities suggesting the outer regions or overlapping regions.

Question 5

This question proved very challenging for all except the most able. The significance of the gradient of the normal being $\frac{1}{3}$ was widely overlooked and the differentiation, when attempted, was demanding. The difficulty with the derivative arose due to unnecessary use of the quotient rule and errors resulting from this by not realising that the derivative of k was 0.

In **part (ii)** candidates obtained credit for finding y using their value of k. Even if a correct derivative had been found in **part(i),** it was often ignored in favour of belatedly finding and using a gradient of –3 from information given in **part (i).**

Question 6

(i) There were a large number of completely correct proofs in this question although some lost credit due to poor notation such as omission of brackets or omission of x from multiple terms. There were a variety of routes to the proof and some were quite long-winded whilst others were succinct. At some point the fractions needed to be combined and $\tan x$ and $\sec x$ needed to be rewritten in terms of $\sin x$ and $\cos x$. Subsequently a correct Pythagorean identity had to be used and correct cancellation performed. Incorrect solutions included errors at various stages usually due to incorrect knowledge of an identity or overcomplicated expressions making progress impossible. As is often the case there were some who jumped to the correct final statement from a totally incorrect preceding statement.

(ii) Some started from scratch not realising the significance of 'hence' and others just solved $1+3\sin x=0$. Those who used the given identity as expected usually obtained a three term quadratic and solved it correctly. The solutions of $\sin x=\frac{2}{3}$ were invariably correct but many candidates quoted a solution of $x = 90°$ from $\sin x = –1$.

Question 7

(a) More able candidates realised that the product of roots was 40 and hence the third root was 5. Finding a and b was then done by multiplying out the factors. The vast majority of candidates solved the simultaneous equations generated from f(2) = 0 and f(4) = 0. Some were successful but many made algebraic mistakes and did not obtain the correct values. The request to find the third root was often overlooked and if attempted was sometimes given as (x – 5). Similar confusion between roots and factors led to some candidates solving equations obtained from f(–2) = 0 and f(–4) = 0.

(b) This was very well attempted, and the vast majority of candidates produced thorough and succinct solutions. However, not all candidates demonstrated that they had found their first root legitimately without using a calculator. There was also an occasional doubt regarding use of a calculator to solve $x^2 – 6x – 40 = 0$ using the quadratic formula. Candidates should ensure that they show sufficient working in such questions to make it clear that they have not used a calculator.

Question 8

The whole of this question seemed unfamiliar to a large number of candidates many of whom did not respond at all.

(i) Many candidates found the magnitude of the given velocity vector but did not know how to link it with the speed of $6.5\,\text{m}\,\text{s}^{-1}$ given in the question.

(ii) This required candidates just to find the modulus but many proceeded to do more work than was required.

(iii) There was a very mixed response here. Many answers did not mention time at all or attached time to the wrong vector. Others ignored the velocity vectors completely.

(iv) Where candidates had vectors in the correct form in **part (iii)** progress was possible. Only the most able candidates achieved complete success.

Question 9

(i) Full marks were obtained by a large number of candidates who recognised how to tackle this question. The mistakes that occurred were mainly due to mishandling of negative coordinates when adding or subtracting for the mid-point or gradient. Some candidates did not write the equation in the requested form.

(ii) Few candidates realised that the answer could be written directly by replacing *x* and *y* by *r* and *s* in their answer to **part (i)**. Many started again and others just left the answer space blank.

(iii) Almost all candidates were able to obtain a correct equation from the information that *PM* was of length 10 units. Many candidates made errors in algebra with the quadratic involving *r* and *s*. One popular choice was to equate the *r* terms to 100 and solve and then do the same for *s*. Other candidates opted to remove the squares leading to $(r-1)+(s-2)=10$.

Some candidates did realise that they had to use the linear expression from **part (ii)** and found that the algebra required was very difficult when substituting into their quadratic expression. Only the more able candidates managed to obtain the correct quadratic equation, solve and reject the negative values of *r* and *s*.

Question 10

(i) There were many good solutions here particularly using the quotient rule which was often quoted correctly prior to substitution. Subsequent simplification was sometimes incorrect and this often prevented progress in **part (ii)**.

(ii) Most equated their answer to **part (i)** to 0 and attempted to solve. Many did get the correct value for *x* but a number did not find *y* or did not do so to the required accuracy. A number still had a factor of *x* in the numerator that had not been cancelled in **part (i)** and this created problems when they were trying to solve.

(iii) There were very few candidates who managed a completely correct answer to this part. Most did not fully grasp the link with **part (i)**. Those that did write down the correct relationship subsequently multiplied or divided by *x* and then attempted to rearrange.

(iv) It was not possible to progress here without a viable attempt at the previous part, so very few marks were awarded. Many used their calculators to obtain an answer.

Question 11

The vast majority knew that it was expected to use the formula but correct simplification of the discriminant was a problem. Some candidates obtained $\sqrt{25}$ but a few then replaced this with $\sqrt{5}$. Errors in the denominator were not as common although $2\sqrt{5}-3$ was seen a number of times. Most candidates knew how to rationalise the denominator and did so quite well. The final problem came when dealing with the minus sign to get the final answers and it was quite common to see $-\frac{1}{4}\sqrt{5}+\frac{3}{4}$ or $\sqrt{5}-3$.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

The mark allocation for a question gives an indication of the amount of work required to answer it. A question carrying only one mark should require very little working if the appropriate method is selected.

In answering a question it is necessary to have a plan of action which is likely to lead to success. In particular, this is essential for problems where trigonometric equations are to be solved.

If a question indicates that a specific method has to be used in answering, full credit can only be given if that specific method is used.

It is basic mathematical practice to use brackets where appropriate and not to omit them.

General comments

The general quality of work showed great variation. Some candidates produced excellent answers, resulting in some cases in full marks, or almost full marks. Their answers were clearly set out with easy to follow steps leading to a solution. Other candidates lacked the essential knowledge of mathematics of this standard, and some obtained only single digit scores.

Topics on which candidates were generally well prepared included modulus equations, completing the square, factor theorem, and composite and inverse functions.

Candidates needed to understand the importance of following an instruction in a question which directed them to use a specific method. Such a direction was indicated by the word 'hence', e.g. **Question 3**, or by a more general direction that the result of an earlier answer was to be used e.g. **Question 4(iii)**. Such instructions were often ignored completely.

Sometimes little notice seems to have been taken of the mark allocation for a question, and the amount of work implied by its value. This was particularly true of questions worth only one mark e.g. **Question 8 (i)** and **Question 9(iv)**. Sometimes the amount of work presented on such occasions was out of all proportion to the one mark available.

The adoption of an appropriate strategy in solving a mathematical problem was not always followed, especially with questions on trigonometry. There are many relationships and identities connecting the basic trigonometric ratios, but candidates often seemed to select from these with no overall plan in mind e.g. **Question 5(b)**.

At any level of mathematics, but especially at this level, essential brackets cannot simply be omitted. Errors occurred frequently as a result of not following this practice e.g. **Question 10(i)**.

Comments on specific questions

Question 1

Most candidates knew that this equation had two solutions, and that to find the second solution a sign change had to be made in the equation. Many did this correctly, but many also made the error of changing only one of the signs on one side of the equation. A few candidates offered only the $x = 1$ solution.

Question 2

In **part (i)**, candidates who expressed the left side of the expression in terms of cos*x* and sin*x*, then cleared the fraction within a fraction, often proved the given relationship very efficiently. A serious error sometimes seen was to replace the denominator with sin*x*. Candidates needed to refer to the correct trigonometric identities on page 2 of the question paper. In **part (ii)**, almost all candidates were successful in using the given relationship to find the two angles.

Question 3

There were some excellent answers to this question which set out the steps to the solution in a very clear, well-ordered, manner: the expression was expanded, coefficients compared, the unknown integer *b* eliminated from the resulting simultaneous equations, and the given quadratic in the unknown integer *a* obtained. Only when the 'show that' element of the question had been completed in such answers were the values of the three unknowns found. Much more limited answers used the given quadratic, without first proving it, even to the extent of substituting the value of the integer *a* so found in the expansion itself. Such answers earned little credit. The question clearly stated that the given quadratic had first to be shown to be true, and then 'hence' the values of *a*, *b* and *c* found.

Quite often, answers which were otherwise very good overlooked the fact that the question had stated that the three unknowns were integers, and presented unacceptable second answers for *a*, *b* and *c*.

Question 4

The process of completing the square was generally well known. Almost all candidates presented an expression of the required form in **part (i)**, and many were correct. The best answers to **part (ii)** used the result from **part (i)** to write down, with no working whatsoever, the two required values. Answers to **part (iii)** were more limited. A good number of candidates were able to see the relationship between *x* and *p*. However, relatively few were able to use the relationship from **part (i)**, as the question directed, without the use of the quadratic formula. Candidates using the latter, even those obtaining the correct numerical answer, were given little credit, as the instruction of the question was ignored.

Question 5

Good answers to **part (a)** were seen where the equation was first rewritten as an expression for $\tan\left(y - \frac{\pi}{4}\right)$ and working throughout was in radians, not expressed as decimals. Many needlessly long attempts were seen, often involving use of the identity connecting $\cot^2 A$ and $\operatorname{cosec}^2 A$, which were usually unsuccessful. There were few answers with working and solutions presented only in degrees.

Answers to **part (b)** which began by rewriting the equation correctly in terms of sin*z* and cos*z* were most likely to achieve success. Many candidates who followed this route produced clear and concise solutions. Those who opted to write cot*z* in terms of tan*z* were less likely to be successful, and some made a very serious error in their first line by replacing cosec*z* with 1 + cot*z*. It was not possible to use the three identities given on page 2 of the question paper with the powers removed.

In this type of problem it was important for the candidate to have a strategy for solving. In this case the strategy had to be to rewrite the equation, which contained three different trigonometric ratios, as an equation containing only one. Quite often candidates were seen using their knowledge of relationships and identities to write the equation in different ways, but made little or no progress and ended with an equation still not reduced to a single ratio. In addition, such long, undirected attempts usually contained an error in the working somewhere along the way.

Question 6

Good work with surds was seen in both parts of the question. The most common error in the question as a whole occurred in **part (i)** where angle *BAC* was occasionally used instead of angle *ACB*. In questions to be solved without the use of a calculator, every step in the working needed to be shown, even though some candidates might find it possible to combine two or more steps mentally.

Question 7

Good concise answers to **part (i)** were seen where, for the differentiation, the second term was treated as $6(3x + 2)^{-2}$. Longer attempts, using the quotient rule on the second term, sometimes made an error in the numerator, and even longer attempts, first rewriting the whole equation as a single quotient, were almost always unsuccessful. It was clear from some answers that a few candidates did not understand the term 'stationary point'.

In **part (ii)**, the first term was usually integrated correctly, the second term less frequently so. Provided limits were shown inserted properly into the integrand, and subtraction carried out in the correct order, credit was given for the method of its evaluation.

Question 8

There was much variation in the amount of work seen in responses to **part (i)**. Some candidates were able to state the correct value for p with no working, by connecting the constant term of the equation with 2, 3 and p. Others multiplied out the three factors in full or used the factor theorem to form three equations in three unknowns. A common incorrect answer was $p = 4$.

The amount of work seen in answers to **part (ii)** also varied greatly. Those which multiplied out the three factors, then compared with the given equation, produced the most efficient solutions. Longer answers used the factor theorem to form and solve simultaneous equations in a and b, but were nevertheless often correct.

A common error in **part (iv)** was to give two pairs of answers. This resulted from ignoring the question's requirement for the integer value of x.

Question 9

Many fully correct answers were seen to **parts (i) and (ii)**. Answers to **part (iii)** were of variable quality. Candidates who knew to equate the **a** coefficients and **b** coefficients on both sides of the equation formed using **parts (i)** and **(ii)** were usually successful. Other candidates were unable to successfully proceed, and much fruitless manipulation was seen, commonly involving the treatment of vectors as algebraic quantities. Success in **part (iii)** usually meant success also in **part (iv)**. Sometimes attempts at **part (iv)** filled the whole of the answer space with working, when only one mark was available.

Question 10

There was scarcely any confusion in **part (i)** between $gf(x)$ and $fg(x)$. Many candidates began their answers well, setting up the equation to be solved, and making good progress in rewriting the equation without the long exponent. Later errors, which were seen frequently, were often caused by the omission of essential brackets. For example $\ln(3x + 2)$ would often appear as $\ln 3x + 2$. Most candidates who used a method leading to a quadratic equation realised that only one of the solutions so generated was acceptable.

Very good knowledge of finding the inverse of a function was shown in **part (ii)**. Answers to **part (iii)** were much more limited. Provided it was recognised that the equation to be solved was essentially a quadratic in e^x, success almost always followed. When it was not so recognised the many lines of subsequent working often consisted of flawed mathematics which did not receive any credit.