ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

There were too few candidates for a meaningful report to be produced.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates should familiarise themselves with the rubric on the front page of the examination paper. It is essential that when a question involves showing a given result, each step of the solution must be shown in detail, especially when some of the result can be deduced. The same message of showing sufficient detail in a solution applies to all questions. Final answers should be checked to ensure that they are in the form required and to the accuracy specified. Candidates are reminded not to write their solutions in pencil first and then overwrite their solution in ink as this often renders the solution very difficult to read.

General comments

Marks covered the entire range available, with many candidates being able to demonstrate a good knowledge of the syllabus and apply this knowledge appropriately. Some candidates were clearly unprepared for the examination. The new syllabus topics appear to have been covered by most centres, but it did appear that some centres have not yet covered them in detail as evidenced by some of the candidate responses.

Comments on specific questions

Question 1

Most candidates were able to make a reasonable attempt to solve the equation of the curve and the equation of the straight line simultaneously. There were some errors in the simplification to obtain a three-term quadratic equation equated to zero. A mark was given for $2x^2-(k+4)x+(k+4)=0$ seen or implied. Most candidates than considered the discriminant and found two critical values which, provided the original quadratic equation was correct, were usually correct. Fewer candidates selected the correct range using these values. Two separate inequalities were expected as a final result.

Question 2

(a) This was a new syllabus topic with the great majority of candidates being able to gain a mark for realising that the function contained the term $(x+5)(x+1)(x-2)$. Fewer candidates considered the basic shape of the cubic function, which implied that the coefficient of x^3 was negative. Consideration of the value of the intercept on the *y*-axis was also omitted by most candidates.

(b) It was intended that the values of *x* be read straight off the given graph. Responses were varied, with many candidates giving $-5<x<-1$and omitting $x>2$. The use of the word 'Write' implies that that the result can just be written down straight away.

Question 3

(a) A correct amplitude was given by the majority of candidates.

(b) Fewer correct periods were seen, with the coefficient of $\frac{1}{3}$ often being misinterpreted. Answers in either degrees and/or radians were acceptable.

(c) Marks were awarded for the basic shape of the graph, the correct end points and for passing through the points $(0, 1)$ and $(\pi, 0)$ on the positive *x*-axis. Candidates tended to gain more marks if they calculated the *y*-values for each of the *x*-values given on the axes with consideration of their responses to **parts (a)** and **(b)** as well.

Question 4

(a) It was evident that some candidates were not aware of the expression for the *n*th term of an arithmetic progression. Most of these did manage to obtain a correct first term and common difference, although the common difference was sometimes given as 3 rather than -3. Candidate using the *n*th term expression usually formed two correct equations which they then solved simultaneously to give a correct result.

(b) For those candidates who had done the first part of the question by observation, most did not seem to be aware of the sum formula for an arithmetic progression and hence were unable to make much progress. For those candidates who did use a correct sum formula equated to zero or < 0, many were able to obtain the critical value $\frac{355}{3}$. Sometimes the incorrect integer value was chosen and sometimes the final answer was left as the critical value. Some candidates chose to equate their sum formula to a negative value, usually -1, and find a critical value. Solutions of 119 were only awarded full marks if the candidate also checked that the sum to 118 terms was positive. This also applied to those candidates who chose a trial and improvement method.

Question 5

Most candidates completed this question completely correctly, gaining full marks. A good understanding of the binomial expansion was shown with candidates obtaining a correct expansion of $\left(x+\frac{2}{x}\right)^5$ and then considering its product with $\left(x-\frac{3}{x}\right)$. Some candidates chose to write out the complete expansion and identify the required term and hence coefficient, while others just considered the two terms needed to find the required coefficient. Occasional sign errors resulted in the loss of some accuracy marks.

Question 6

(a) Very few correct explanations were seen. It was not enough to state that the function was one-one. The reason it was a one-one was also needed, so a statement that the domain had been restricted also needed to be included.

(b) **Part (a)** had been intended to alert candidates to the fact that the domain was restricted and so the function was one-one. Few candidates made the connection between the two parts and many incorrect curves were seen with values of $x < -1$ included. The candidates that did make the connection between the two parts usually produced correct sketches for at least one of the functions.

Question 7

(a) Most candidates made an attempt to differentiate a product, with the occasional error in the differentiation of the logarithmic term. A correct method was used by many candidates, with the occasional tangent equation found rather than the normal. Use of either exact or rounded values was acceptable. There were quite a few different exact forms that were allowed. Often the last accuracy mark was not given due to a calculation slip or a rounding error. It was pleasing to see that many candidates were able to identify the correct processes needed to solve this problem.

(b) A follow through mark was gained by the majority of candidates for realising that they needed to multiply the value of their first derivative from **part (a)** by *h*.

Question 8

(a) Most candidates realised that the use of combinations was needed. Too many candidates did not take into account that once three people had been chosen from twelve people, then there were only nine people left to choose four people from. That meant that there were then just five people left, so only one way these can be chosen.

(b) (i) Many correct answers were seen using either arrangements or permutations.

(ii) Fewer correct answers were obtained, taking into account that there are two possibilities for the final digit.

(iii) Candidates are advised to set out their solution to questions of this type so that an Examiner can follow the process that is being used. Too many solutions have various numbers with no apparent pattern or clear method written down. There were two approaches each consisting of two or three separate cases that could be taken. Either consider the digits that the number could start with that is, the case when the number starting with a 7 or a 9 and the case when the number starts with an 8. Or consider the digits that the number could end with that is, the case when the number ends with a 3 and the case when the number ends with a 7 or 9.

Question 9

(a) Most candidates attempted differentiation of a product correctly, with the occasional error in the differentiation of the term $\sqrt{4x+3}$. Simplification of this result to the required form was completed with various degrees of success depending upon the algebraic skills of each candidate. It was somewhat disconcerting to find that many candidates simplified $4x-2+8x+6$ to $4(3x-1)$.

(b) Of those candidates who had obtained an incorrect linear numerator, most were able to obtain a follow through mark by equating their numerator to zero and solving.

(c) There are methods of determining the nature of a stationary point other than using the second derivative. It had been hoped that one of these methods be used as the second derivative required differentiation of a quotient. Should either of these quicker (in this case) methods be used in future, it is essential that a clear method is set out so that it can be seen whether or not *y*-values or gradients are being considered.

Question 10

(a) Most candidates were able to make use of the factor theorem and obtain the equation $2a+b+25=0$ or equivalent. For the second equation required many candidates took the value of $-2\text{p}(0)$ to be zero, so obtaining the incorrect equation $8+a+b=0$. No obvious attempt had been made to find $-2\text{p}(0)$ with the assumption that it was zero just made. However, most candidates did obtain values for both *a* and *b* and so were able to gain method marks in the following parts.

(b)(i) A correct approach was used by most candidates gaining at least a method mark for those with incorrect values from **part (a)**.

(ii) It was intended that the remainder of zero from the correct answer for **part (b)(i)** would make candidates realise that they could identify another factor of the polynomial. This was not common, with many choosing to use algebraic long division to obtain a quadratic factor. Attempts at synthetic division were less successful as adjustments to the coefficient of x^2 were seldom made. Too many candidates gave the solution of the polynomial equated to zero, sometimes with no factors given first. It is essential that candidates check that they have answered the demands of the question.

Question 11

(a) It was essential that each step of the solution be shown in detail. The length of the chord *BC* could be deduced by looking at the given answer, so this part of the question was marked strictly. Too many candidates did not show sufficient detail in the calculation of the length of the chord *BC*. Some quoted trigonometric ratios involving an unknown quantity, for example *x* or *o*. These unknown quantities were not defined or indicated on the given diagram or indicated on a separate diagram drawn by the candidate, so no method marks were awarded in these cases.

(b) Few correct solutions were seen although many candidates realised that they needed to subtract the area of a segment from the area of a rectangle. Errors in the calculation of the area of the rectangle were common with many candidates not making use of the length of the chord BC obtained in **part (a)**. Errors in the segment area often involved use of an incorrect angle. Calculation errors were all too common.

Question 12

(a) (i) Full marks were usually obtained by those candidates who realised that they needed to find the area under the velocity–time graph and equate it to 2775. However, too many candidates gave an answer of 30.8 having just divided 2775 by 90. Candidates should be guided by the mark allocation of 3 marks which implies that more than a simple division is required.

(ii) Many correct solutions were seen but some candidates did not realise that they needed to find the gradient of the line on the velocity–time graph at the point where $t = 40$.

(b) (i) Most candidates realised that calculus was involved although not all attempts at integration were successful. Some candidates omitted to find the value of the arbitrary constant which then meant that they would not be able to obtain full marks in **part (ii)**.

(ii) Most candidates attempted integration again, with variable results. Again some candidates did not consider the arbitrary constant.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

This paper required candidates to recall and use a range of mathematical techniques and to devise mathematical arguments; presenting those arguments precisely and logically. Good responses were set out clearly and demonstrated a good understanding of fundamental techniques. These techniques included calculations related to circular measure, appropriate use of the binomial theorem, integration of functions of the form $\frac{1}{ax+b}$ and sketching and using graphs of cubic functions. Successful responses demonstrated a thorough understanding of mathematical language and notation. Confidence in the handling of algebraic expressions was essential.

Shorter questions on this paper required recall of an appropriate technique but in longer questions, such as **Question 10(b)**, a structured solution bringing together various techniques was required.

General comments

A good range of responses were provided, showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues and most candidates attempted all the questions.

There were some topics, particularly those new to the syllabus this year, where candidates appeared to be less familiar with the techniques required and they would benefit from practice in answering questions from all areas of the syllabus.

Candidates should be aware that if a question has a short first part asking them to prove a result then that result is likely to be used in the longer second part. Candidates should also be aware that decimal answers are not accepted if an exact answer has been asked for. Care should also be taken when working with angles in radians that calculators are used in the correct mode. Candidates should be advised that it is never necessary to convert radians to degrees or vice versa to solve trigonometry problems.

Candidates should take care when rounding answers to 3 significant figures. Some marks were lost through premature rounding and candidates should be encouraged to include their unrounded answer in their working and to work with more figures than required.

Comments on specific questions

Question 1

(a) Although many candidates knew how the graph of a cubic function should behave, many responses did not show sufficient attention to detail in the sketching of the curve. The curve had to extend for values before $x = -1$ and beyond $x = 3$. The arms had to be those of cubic curve and not bend in or out. The majority of candidates obtained and marked the *x* intercepts correctly. Many candidates with a correctly orientated graph also identified the *y* intercept correctly.

(b) Candidates were expected to identify the parts of the cubic curve above the *x*-axis and form two inequalities. No manipulation or calculation was required or expected within this part. Not all candidates who had a correct graph in the first part took this approach. Candidates with an incorrectly orientated cubic graph could not score. Candidates should note that strict inequalities

(< and >) were required as indicated in the question and $2 < x < 3$ was required rather than separate inequalities.

Question 2

(a) Nearly all candidates knew they had to use the quotient rule and many good solutions were seen. Most candidates knew how to differentiate the exponential term correctly but those who did not differentiate it correctly usually used the quotient rule correctly. Some candidates handicapped themselves for the next part by incorrectly simplifying their expression, inappropriately cancelling or misreading their own writing, particularly e^{2x-3}. Care was required with the expansion of $\left(x^2+1\right)^2$.

(b) A majority of candidates evaluated their answer to the previous part using $x = 2$. Some lost sight of the fact that $\frac{dx}{dt}$ was required and went no further or calculated $\frac{dt}{dx}$. There was some confusion with small changes with the answer sometimes being labelled as δx. Answers had to be exact, which in this case was in terms of e. The majority of candidates either ignored this instruction and went directly to a decimal answer or had difficulty simplifying the expression in terms of e.

Question 3

(a) (i) Candidates had to realise that the argument of the log had to be greater than zero. Some candidates obtained $\frac{1}{2}$ but did not incorporate it into an inequality using *x*. There was some confusion between domain and range.

(ii) There were many good responses showing that candidates understood how to find an inverse function using a correct order of operations and correct notation. A few candidates did not change the variables to obtain a function of *x*. Not all candidates gave a domain and few correct answers were seen for the domain, which had to be clearly identified as a set of *x* values.

(b) Many good solutions were seen to this question and candidates showed a good understanding of composite functions. Some candidates misread the question and found gh(7). Others were confused by the squaring step and introduced a ±4. There were also some sign and arithmetic errors by candidates who clearly knew what to do.

Question 4

(a) (i) Candidates seemed unfamiliar with what was expected here and many did not appear to calculate the gradients of the line segments on the distance-time graph to find the velocity. Of those who did find the correct values, not all knew how to show these as constant velocities on the graph, resulting in a graph that looked similar to the original. Of those who did show horizontal line segments, the most common error was to show the final segment as positive rather than negative.

(ii) Many candidates tried to find areas from their velocity–time graph but an easier and more successful approach was to use the given distance–time graph.

(b) Although most knew that they had to differentiate, many candidates found this part difficult with some not aware of the derivatives of trigonometric functions and others unaware of the application of the chain rule. Others only differentiated once. A common error when evaluating the final value was to have the calculator set for degrees rather than radians.

Question 5

Most candidates found the appropriate formula in the list but were unable to apply it correctly as they did not evaluate expressions such as 1^n, deal with the combinations or simplify fractions involving factorials. To answer such questions, it is best to learn and use the formula in a simplified form such as $\left(1+b\right)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 \ldots$. Candidates had to be careful with signs and arithmetic and some used $\frac{x}{2}$ rather than $-\frac{x}{2}$ or expanded $\left(-\frac{x}{2}\right)^2$ to $-\frac{x^2}{4}$ or $\frac{x^2}{2}$. It had to be appreciated that the x^2 term came from adding two terms. Some candidates worked with just $-\frac{n}{2}=\frac{25}{4}$ or $\frac{n(n-1)}{8}=\frac{25}{4}$. An equation should be formed using the coefficients (i.e. without the inclusion of x^2). Some candidates recovered from mixed equations such as $-\frac{n}{2}x^2+\frac{n(n-1)}{8}x^2=\frac{25}{4}$ but others did not. Some candidates having correctly combined two terms left the equation in a form that still included combinations and either gave up or continued to a correct answer using trial and improvement. Relatively few candidates correctly formed and solved a quadratic equation in *n*. As candidates found this question difficult, presentation was poor and it was sometimes difficult for markers to identify worthwhile work.

Question 6

(a) Successful candidates usually started with $\lg y=\lg A+bx^2$ and it is important to understand the laws of logarithms that lead to this equation. This equation gave a clear indication of the route to be taken for a correct solution whichever method was used. There were three approaches. Candidates who substituted the given coordinates in $\lg y=\lg A+bx^2$ to form simultaneous equations were least successful as there were errors in manipulation. Candidates who first identified *b* as the gradient were more successful as there was less manipulation to be done. Candidates who started by using the given coordinates to find a straight-line equation of the form $\lg y=mx^2+c$ were also more successful but as these candidates were less likely to have stated $\lg y=\lg A+bx^2$, care had to be taken to associate *c* with $\lg A$ once an equation was found. A common source of error in all methods was assuming that the coordinates were values of *x* and *y* and not x^2 and $\lg y$. Some candidates did not deal with $\lg A$ correctly to obtain *A* with some using powers of e instead powers of 10 and others leaving their answer as a power of 10. Premature rounding errors were evident throughout this question. Candidates should be encouraged to show more figures in their working and to not round until giving their final answer which, as usual, should be to 3 significant figures.

(b) Most candidates knew what had to be done here, starting with $\lg y=\lg A+bx^2$ or $y=A\times 10^{bx^2}$. Candidates sometimes confused *y* and lg *y* in these equations and a common error was to use 2 rather than 2^2.

(c) Most candidates started well, with a few confusing 4 and lg 4. Many found the resulting equation difficult to solve and would benefit from practice in solving these types of equations. Candidates should be aware that answers had to be given to three significant figures and that a positive answer was required.

Question 7

(a) Most candidates answered this question well, but some could have made better use of the space provided and it was not always clearly presented. Most candidates knew they had to form an equation using $x=-4$ but the question required careful reading and some candidates misunderstood what they had to do to form the other equation in *a* and *b*. However, the most common source of error was carelessness in manipulation and misreading of their own figures. Although there were many completely correct solutions for *a* and *b*, some candidates forgot to find the quadratic factor as required.

(b) Candidates who had obtained a correct quadratic factor in the first part were usually successful.

(c) Candidates were expected to differentiate their $\text{p}(x)$ and use the remainder theorem. Most tried to use long division. Many candidates divided by *x* to obtain $18x+38-\frac{19}{x}$ but did not identify the remainder.

Question 8

(a) Most candidates answered this question well.

(b) Most candidates added the area of the triangle *AOC* to the area of the sector *AOB*. Most had a correct expression for the area of the sector and many had a correct expression for the triangle, particularly if they took a straight forward approach and used $\frac{1}{2}ab\sin C$. Incorrect final answers were usually due to errors in evaluating sin1.69 or premature rounding of sin1.69 when finding the area of the triangle. A significant number of candidates misinterpreted the diagram as a sector of radius 1.5*r* including a calculation of the angle at *O* based on that misunderstanding.

(c) Candidates found this part involving perimeter more difficult than the previous part. Most candidates knew that the length of *AC* had to be added to 2.95*r* to obtain 12, but many did not calculate *AC* or had misunderstood the diagram and assumed *AC* was also $1.5r$. Some candidates confused AC^2 with *AC* and forgot to square root before adding to $2.95r$.

Most candidates who made a calculation used the cosine rule, and most initially correctly substituted in the formula given on the paper. Candidates should be aware that care has to be taken when using the cosine rule. Here $0.5r$ had to be squared correctly and the product of 2, $0.5r$ and r had to be correct. In the evaluation of $1.25r^2 - r^2\cos 1.69$ a correct order of operations had to be used, the cosine had to be evaluated using an angle in radians and the negative value of the cosine had to be taken into account. *AC* had to be simplified to single term in r^2 before attempting to find a square root.

Some candidates succeeded in forming a quadratic equation in *r* using $AC^2 = (12 - 2.95r)^2$, but this was not a good use of time when easier routes were available and it produced an extra solution that had to be rejected.

Question 9

(a) This question was answered well. The majority of candidates knew they had to use fractions of a quarter and three quarters. Occasional mistakes in expressing the relevant vectors in terms of **a** and **b** were made that could have been avoided. For example, the use of **a** – **b** rather than **b** – **a** and **a** + $\frac{1}{4}$(**b** – **a**) rather than **a**+ $\frac{3}{4}$(**b** – **a**). More care and effective use of the diagram to determine the directions was required.

(b) Most candidates were correct.

(c) This usually followed through correctly from their answer to the first part. There were sign errors, in particular **a** instead of –**a**, that could have been avoided by making effective use of the diagram.

(d) Candidates were expected to compare coefficients for **a** and for **b**. A good number were able to follow through correctly from previous parts but some candidates miscopied from those parts and some were careless with signs when forming equations in *m* and *h.*

Question 10

(a) Candidates should be aware that if asked to show a result, they have to be particularly careful to show all working and use correct mathematical notation.

(b) Most candidates correctly calculated the area of the triangle but some were confused by the negative value of *x* at *Q*. Those that tried to find the area of the triangle by integration often used incorrect limits or an incorrect straight-line equation. Many candidates did not relate the finding of the area under the curve to the previous part and although they knew that integration was necessary did not use $\frac{1}{x+1}$ and $\frac{2}{3x+10}$ as suggested. Candidates who integrated correctly usually went on to use the limits correctly but very few proceeded successfully from there. Most candidates found it difficult to incorporate ln3 into the final form, not appreciating that it could be written as $\frac{2}{3}\ln 3\sqrt{3}$. As answers had to be exact and in the required form, it was necessary to show the manipulation of the ln expressions. Decimal answers from a calculator did not score and working back from a decimal answer to the required form was not credited.

Question 11

(a) As the question required candidates to show a given answer, care was necessary to show all steps clearly and to use correct mathematical notation. The identities $\tan x = \dfrac{\sin x}{\cos x}$ and $\cos^2 x = 1 - \sin^2 x$ had to be used correctly and clearly.

(b) Candidates usually made some progress with this question, using the quadratic equation in $\sin x$ obtained in the first part. Candidates had to reject the impossible solution of –2 and equate $\sin\left(2\alpha + \dfrac{\pi}{4}\right)$ to $\dfrac{1}{2}$. Most candidates knew that $\sin\dfrac{\pi}{6} = \dfrac{1}{2}$ and were able to use a correct order of operations to obtain α. Many were also aware that a solution in range came from $\dfrac{5\pi}{6}$ but few looked for the second solution within the range. Candidates should be aware that it is best to work with multiples of π throughout. Answers had to be in terms of π so use of decimals was not appropriate.

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

There were too few candidates for a meaningful report to be produced.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

It is always a good strategy to check answers in the original equations in order to establish whether a mistake has been made or whether the solution is appropriate.

General comments

Candidates should always follow instructions given in a question. There are times when there is a request not to use a calculator which is not always obeyed. In these cases there is a desire for the candidate to show a particular technique which will not occur if a calculator is used. Candidates using a calculator in these questions will not be awarded any marks.

Comments on specific questions

Question 1

Nearly all candidates were able to expand and rearrange to form the correct quadratic equation and solve it correctly There were occasional slips and a few candidates tried to solve the given inequality as two linear inequalities formed by separating the given brackets.

Those candidates who drew a sketch of the quadratic were more likely to find the correct final solution. Many gave an incorrect final answer of ‘ $x > 15$ or $x > 3$ ’ or gave a correct pair of inequalities with the word ‘and’ rather than ‘or’ between them.

Question 2

Nearly all candidates converted each term to a power of 2 and correctly used the laws of indices to express each side of the equation as a single power of 2. Many fully correct solutions were seen with the most common errors coming from brackets being multiplied out incorrectly and sign errors.

Question 3

(a) There were many good solutions to this question. Most candidates found the gradient and mid-point of the given line correctly. However, there were a significant number who then found the equation of a perpendicular line through one of the end points or used the gradient of the line rather than the perpendicular. Most answers were given in the requested format.

(b) Many candidates found the two points required by setting *x* and y equal to zero and used the distance formula correctly. There were a number who found the distance between the given points of (12, 1) and (4, 3). A few candidates made errors in not using the distance formula correctly.

Question 4

Many candidates produced nearly perfect solutions by removing logs correctly, substituting to find a quadratic in one variable and solving it. Two pairs of values were invariably quoted but most candidates did not realise that $x = -5$ was inappropriate and did not reject it. The most common error was to incorrectly change the first equation to $x + y = 8$,6 or 2.

Question 5

(a) Most candidates differentiated correctly but the result was not always used appropriately. Some set it equal to zero, others found the gradient but then used the gradient of the normal to find the equation of the line. Most found and used the value of $y = 8$ when $x = 1$.

(b) Candidates who had the correct equation from **part (a)** were generally successful in obtaining the cubic and realising that $x = 1$ was a solution. The correct quadratic was then obtained and factorised to give the correct value of $x = 4$.

However there were a variety of errors including those who did not equate the tangent to the cubic but worked from the original cubic. Some tried to factorise it and others equated the differential from **part (a)** to it. There were several incorrect methods to find $x = 4$ including from $x\left(x^2 - 6x + 9\right) = 4$.

Question 6

This question proved to be the most challenging on the paper with many candidates unsure of how to tackle it. Some candidates expanded the numerator and progressed no further whilst others proceeded to integrate the numerator and denominator separately. Those who did manage to divide the numerator by x^2 usually did so successfully but then only integrated 1 and $\frac{1}{x^2}$ correctly. Those who integrated each term correctly rarely gave an exact answer.

Question 7

(a) There were many very good answers to this part with the common ratio correctly identified and the correct formula quoted and accurately applied. Some confused the geometric progression with an arithmetic progression and found a common difference and then used the geometric progression formula. Others treated the sequence as an arithmetic progression throughout and used the inappropriate formula.

(b) If the candidate had found the common ratio correctly in **part (a)** then this part was invariably correct.

(c) Many candidates had problems setting up the initial equation with 195% often seen and a common error of $3\left(1 - 0.8^n\right)$ becoming $3 - 2.4^n$. Most realised that use of logs was required but many attempted to take logs of negative terms. Inaccurate use of inequalities often resulted in the final answer of 13.4 being rounded down to 13 rather than up to 14.

Question 8

(a) The use of the given appropriate formula was well done by most candidates. Progress beyond the initial step was mixed. Rearranging proved difficult for some with one of the $\frac{1}{2}$s often being lost or poor bracketing resulting in incorrect cancelling. The request not to use a calculator was often ignored with the final answer appearing without the necessary detail of rationalisation of their fraction being seen.

(b) The vast majority used the cosine rule correctly and showed sufficient detail in their expansion of the three terms There were often some arithmetic errors in the expansion which resulted in an incorrect final answer.

Question 9

Many fully correct solutions were seen to **parts (a)**, **(b)** and **(d).** In **part (c)** some candidates found $\overrightarrow{QX}$ but forgot to add $\overrightarrow{OQ}$. In **part (e)** many candidates did not equate the terms in **a** and the terms in **b**, and were therefore not able to form a pair of simultaneous equations to solve. Those candidates who had done

part (e) correctly were not always able to interpret their values of μ and λ correctly and results to **parts (f)** and **(g)** often contained terms in **a** and **b.**

Question 10

(a) A number of candidates found it difficult to interpret the information given and could not deal with the exponential term. The initial condition leading to the equation $P + Q = 500$ was not always obtained resulting in incorrect values of P and Q.

Some candidates incorrectly tried to use logs and others used differentiation presumably looking for an irrelevant rate of change.

(b) There were few correct answers to this part due to the fact that the most accurate available values of P and Q were not used and the final answer was not given as an integer.

(c) Most were able to make e^{2t} the subject and then take logs. However, many rounded down their final answer rather than rounding up to a complete number of weeks.

Question 11

(a) Most candidates made the correct first step of using $\tan x = \frac{\sin x}{\cos x}$ and many then proceeded to replace $\sin^2 x$ by $1 - \cos^2 x$. At this point various attempts were made to justify the result by cancelling a variety of terms by incorrect means rather than factorising the numerator and cancelling a common factor. Some changed the initial expression by multiplying by $(1 - \cos x)$ and as a result were not showing the desired relationship.

(b) Many candidates made the correct first step of writing the expression in terms of $\sin x$ and $\cos x$. Most then multiplied through and used an appropriate identity to form a quadratic equation in $\sin x$ and proceeded correctly. There were some who confused $\tan x$ and $\cot x$ and a number who incorrectly replaced $\sec x$ by $1 + \tan x$ in order to obtain a quadratic in $\tan x$. A few candidates squared the equation and obtained a quartic in $\tan x$ and then got into difficulty.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

When more than one approach to a problem is possible, it is worth pausing to consider which is likely to be the more economical.

When a part of question states that another part of the question should be used in its solution, a connection between the two parts should be found and used.

There should be careful attention to detail in algebraic manipulation so that basic errors are avoided.

If time is available, and if it is possible, a check of answers should be carried out to see if they agree with the given information.

General comments

Once again, the general quality of work varied enormously. Some candidates produced excellent answers, resulting, on a good number of occasions, in very high marks. Responses to questions were clearly set out with easy to follow steps leading to correct solutions. But other candidates displayed a lack of the basic knowledge essential for success in mathematics of this standard.

Topics on which candidates were generally well prepared included modulus equations, simultaneous equations, simple permutations, and arithmetic and geometric series. They were generally less well prepared in applying their knowledge to the analysis of a particular physical situation (see **Question 9** below).

It is necessary to point out to candidates the importance of following an instruction in a question which directs them to a specific way of answering it. Such a direction can ask the candidate to use their answer to another part of the question (see **Question 7(c)** below). This particular instruction was sometimes ignored.

Consideration of the best strategy to be used in solving a mathematical problem is important, but often appears to be absent. This was particularly the case where there was a clear choice between two possible routes to a solution, yet often the more difficult was chosen (see **Question 2** below).

At any level of mathematics, but especially at this level, knowledge of the basics of algebraic manipulation is essential. This includes the strict use of essential brackets (see **Question 10(b)** below) and the very basic knowledge that $(a + b)^n \neq a^n + b^n$ (see **Question 9** below). The latter in particular was frequently not observed.

In general, readability and layout of answers was satisfactory, although in some cases, especially when the answer space was filled completely, the work was not easy to follow.

Comments on specific questions

Question 1

Widespread familiarity was shown of how to deal with a simple linear equation involving a modulus. Few attempts made the error of changing just one of the signs on one side for the second solution. The longer method of squaring both sides was seen occasionally and was usually successful.

Question 2

Almost all candidates knew the principle involved in solving simultaneous equations. However, few seemed to have considered the most economical way of applying it. Choosing to find *y* in terms of *x* from the second equation and substituting in the first meant that the substitution had to be made in just one place. But choosing to find *x* in terms of *y* from the second equation and substitute in the first meant that the substitution had to be made in two places, in one of which moreover a term had to be squared. Many candidates chose the latter approach, and whilst frequently successful in the end, subjected themselves to much unnecessary algebraic manipulation. Also, as this approach ended in a two-term quadratic in *y*, from which the *y* could be cancelled immediately, one pair of solutions was sometimes lost.

Question 3

Most candidates knew that the discriminant of the quadratic equation had to be found and used. Critical values of *k* were usually correct, but the relevant inequalities, often incorrect. In some cases no inequalities whatsoever were presented.

Question 4

Candidates able to differentiate correctly in **part (a)** usually made good progress through the other parts of the question. However, in **part (b)**, whilst mathematically correct answers might be given, they were often not in the required form. In **part (c)**, provided **part (a)** was correct, and $\tan x$ was expressed properly in terms of $\sin x$ and $\cos x$, success almost always followed. Only a few candidates gave their answers, incorrectly, in degrees. Candidates unable to differentiate correctly in **part (a)** obtained few marks.

Question 5

The need to first express the given equations in powers of two and three was well understood. Many good solutions were seen where this was done correctly, then the laws of indices used to form two ordinary linear simultaneous equations, which were solved. The first such equation was obtained more readily, although occasionally 9 was replaced by 3^3. The most common source of error occurred in work on the right-hand side of the second equation, where, instead of $2x + 1 – 2.5$, quite often $(2x + 1) \div 2.5$ was seen.

Question 6

This question was a good source of marks for many candidates. One common error in **part (b)** was to treat 1 as a prime number. There was some use of combinations formulae in a minority of cases.

Question 7

In **part (a)**, whilst many candidates were able to apply the product rule correctly, some answers were spoiled with incorrect development. Provided **part (a)** was correct, full marks usually followed in **part (b)**. When it was not, marks were still available in **part (b)** for finding the *y*-coordinate of the point in question and knowing the relationship between the gradients of tangent and normal.

Some very good answers to **part (c)** were seen, starting with a re-statement of the relationship in **part (a)** in integral form, followed by clear logical steps to make the unknown integral the subject. One limitation of otherwise good answers came at the end, on evaluation, when a decimal answer was given rather than the exact value the question required. However, overall, responses varied in quality. Some candidates seemed to think that in being asked to use their answer to **part (a)** they were being told that this was to be used as the result of the unknown integral. Others ignored this instruction completely and used totally invalid mathematics to produce a result.

Question 8

In **part (a)** the need to first rewrite the given equations as purely algebraic equations without logs was commonly appreciated. This was often done correctly. Following on it was then a fairly simple task to eliminate *y* to prove the given relationship. Quite often crossed out work was seen and replaced with correct work. It is commendable that these candidates abandoned work which did not lead to the given relationship and started again, rather than trying to manipulate incorrect work incorrectly to obtain what had to be shown. For some unknown reason some candidates chose to eliminate *x* instead of *y*.

Many responses to **part (b)** earned three of the four marks available. One factor was easily identified, then the quadratic factor found, and so all three linear factors correctly obtained. But almost always only two roots were given for the cubic equation, rather than formally showing also the repeated root. There was confusion in some cases between roots and factors. In **part (c)**, answers addressing the 'reason' which the question asked for were commonly too vague, making no reference to the original equations of the question.

Question 9

In **part (a)** it was fairly common for just one or two of the three marks available to be earned. Very often, after having expressed AC^2 correctly as $300^2 + x^2$, many candidates then made the very serious error of stating the distance AC to be $300 + x$. Very often also, even when a fully correct expression for the required time was presented (and had earned full marks) it was subsequently 'developed' with incorrect mathematics, including this same error, to a simple linear expression. In such cases it was impossible for the candidate to obtain any marks in **part (b)**. Even candidates who avoided errors, and developed their correct two term expression for T further by taking a common denominator, made things difficult for themselves in **part (b)** by creating an expression that was less straightforward to differentiate, and so more likely to lead to error. Working often seen in **part (b)** was also much more complicated than it needed to be through unnecessary use of the quotient rule. Candidates should be aware of the fact that the rule is not needed for differentiating a fraction where the denominator is a constant.

Nevertheless, in a minority of cases, excellent solutions were seen. These usually retained the two terms in the expression for T from **part (a)**, making the differentiation in **part (b)** simpler, and thus less prone to error. The derivative was set equal to zero and clear algebraic steps resulted in correct answers. Some even presented second derivative work, although this was not required.

Question 10

The questions on series were generally answered well, although full marks were more common in **part (b)** than in **part (a)**. Appropriate formulae were almost always used in both parts, with very little confusion seen between formulae for a particular term in the series, and those for its sum.

A very common error in **part (a)** was to treat 86 as the sum of the first eight terms of the series, instead of the sum of the next four terms, after the first four. The most common error in **part (b)** was a sign error when finding the sum of ten terms, the common ratio being negative, and an essential bracket being omitted. Candidates are reminded that in such questions a simple check of answers can be carried out.

Question 11

Many very good answers were seen, with all the steps clearly set out, showing correct manipulation of the surds. The use of the quadratic formula was the method overwhelmingly adopted, and by far the most appropriate method. As this was a non-calculator question it was essential for the candidate to include a full rationalisation step somewhere in their answer, and this was commonly done. For some unknown reason one of the solutions might be rejected, and occasionally also only one found in the first place. Very few tried a direct factorisation method. It was extremely rare for an attempt using the completion of the square method to earn any marks.