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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/51**

Paper 5 (Core) **October/November 2019**

1 hour

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and to communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 24.

Cambridge Assessment
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[Turn over

Answer **all** the questions.

INVESTIGATION   **DECIMAL FORMS**

This investigation looks at the patterns when changing a fraction to its decimal form.

Examples

$\frac{2}{3} = 0.666… = 0.\dot{6}$ This is a repeating decimal.

$\frac{3}{4} = 0.75$ This is a terminating decimal.

The fraction $\frac{5}{8}$ has a numerator of 5 and a denominator of 8.

1   This question is about terminating decimals.

   (a) (i)   Complete these equivalent fractions.

$$\frac{1}{2} = \frac{5}{10} \qquad \frac{1}{5} = \frac{2}{10} \qquad \frac{7}{20} = \frac{}{100} \qquad \frac{1}{25} = \frac{}{100} \qquad \frac{3}{500} = \frac{}{1000}$$

   (ii)   The denominators of the equivalent fractions in **part (i)** are 10, 100 and 1000.

   The smallest prime number is 2.

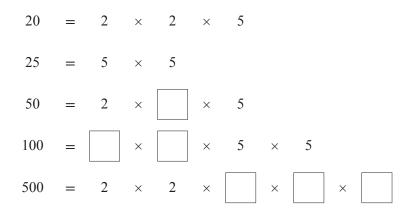   Put a prime number in each box to complete these statements.

   $10 \quad = \qquad\qquad\qquad = \quad 2 \quad \times \quad 5$

   $100 \quad = \quad 10 \quad \times \quad 10 \qquad = \quad 2 \quad \times \quad 5 \quad \times \quad \boxed{} \quad \times \quad \boxed{}$

   $1000 \quad = \quad 10 \quad \times \quad 10 \quad \times \quad 10 \quad = \quad 2 \quad \times \quad 5 \quad \times \quad \boxed{} \quad \times \quad \boxed{} \quad \times \quad \boxed{} \quad \times \quad \boxed{}$

   (iii)   Complete the table.

Fraction	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{1}{25}$	$\frac{3}{500}$
Decimal	0.5	0.2			

   (iv)   Write down a different fraction with a numerator of 1 and a denominator between 30 and 99 which can be written as a terminating decimal.

....

(b) (i) Put a prime number in each box to complete these statements.

$$20 = 2 \times 2 \times 5$$

$$25 = 5 \times 5$$

$$50 = 2 \times \boxed{} \times 5$$

$$100 = \boxed{} \times \boxed{} \times 5 \times 5$$

$$500 = 2 \times 2 \times \boxed{} \times \boxed{} \times \boxed{}$$

(ii) Use your answers to **part (i)** to help you complete the table.

Fraction	Decimal	Number of decimal places	Denominator written as a product of primes using powers	Larger power
$\dfrac{1}{20}$	0.05	2	$2^2 \times 5$	2
$\dfrac{7}{25}$	0.28	2	5^2	2
$\dfrac{9}{50}$	0.18	2		
$\dfrac{19}{100}$	0.19			2
$\dfrac{13}{200}$	0.065	3	$2^3 \times 5^2$	3
$\dfrac{11}{500}$	0.022			
$\dfrac{17}{5000}$	0.0034		$2^3 \times 5^4$	4

(iii) A fraction has a numerator of 1 and a denominator of $2^{14} \times 5^7$.

Write down the number of decimal places in the decimal form of this fraction.

....

(iv) The denominator of a fraction that can be written as a terminating decimal only has one or two possible prime factors.

Write down these prime factors.

.... and ....

2 This question is about repeating decimals.
The number of digits in the repeating pattern is called the *repeat length*.

<u>Example</u>

$\frac{1}{13} = 0.\underline{076923}\,076923\,076923\ldots = 0.\dot{0}76\,92\dot{3}$ This is a repeating decimal with a repeat length of 6.

(a) (i) Complete these equivalent fractions.

$\dfrac{1}{3} = \dfrac{}{9}$ $\dfrac{1}{11} = \dfrac{}{99}$ $\dfrac{1}{37} = \dfrac{}{999}$ $\dfrac{1}{111} = \dfrac{}{999}$ $\dfrac{1}{41} = \dfrac{}{99\,999}$ $\dfrac{1}{7} = \dfrac{}{999\,999}$

(ii) Complete the table.

Fraction	$\frac{1}{3}$	$\frac{1}{11}$	$\frac{1}{37}$	$\frac{1}{111}$	$\frac{1}{41}$	$\frac{1}{7}$
Decimal	$0.\dot{3}$	$0.\dot{0}\dot{9}$	$0.\dot{0}2\dot{7}$			$0.\dot{1}42\,85\dot{7}$
Repeat length	1	2		3	5	6

(iii) Use your answers to **part (i)** and **part (ii)** to help you complete the table.

Fraction	Decimal	Repeat length	Denominator of equivalent fraction
$\frac{1}{3}$	$0.\dot{3}$	1	$9 \;=\; 10^1 - 1$
$\frac{1}{11}$	$0.\dot{0}\dot{9}$	2	$99 \;=\; 10^2 - 1$
$\frac{1}{37}$	$0.\dot{0}2\dot{7}$		$999 \;=$
$\frac{1}{111}$		3	$999 \;=$
$\frac{1}{41}$		5	$99\,999 \;=$
$\frac{1}{7}$	$0.\dot{1}42\,85\dot{7}$	6	$999\,999 \;=$

(iv) Give an example of a fraction with a numerator of 1 which can be written as a repeating decimal with a repeat length of 9.

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(v) A repeating decimal has a repeat length of k.

Write down an expression, in terms of k, for the denominator of this fraction.

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(b) **(i)** $\dfrac{1}{407} = \dfrac{1}{11 \times 37} = \dfrac{1}{11} \times \dfrac{1}{37}$

$\dfrac{1}{407}$ is changed to its decimal form.

Show that this has a repeat length that is equal to the lowest common multiple (LCM) of the repeat lengths of the decimal forms of $\dfrac{1}{11}$ and $\dfrac{1}{37}$.

(ii) Show how the lowest common multiple (LCM) of the repeat lengths of $\dfrac{1}{7}$ and $\dfrac{1}{37}$ gives the repeat length of $\dfrac{1}{259}$.

3 Some decimals have non-repeating decimal parts followed by repeating decimal parts.

<u>Example</u>
$0.6\dot{5} = 0.65555\ldots$ In this decimal, the 6 does not repeat but the 5 does.

(a) Show that adding the decimal forms of $\frac{1}{5}$ and $\frac{1}{3}$ gives a decimal of this type.

(b) Complete the table.

Fraction	Decimal	Number of non-repeating decimal places	Repeat length	Denominator written as a product of primes using powers
$\frac{1}{6}$	$0.1\dot{6}$	1	1	2×3
$\frac{1}{12}$	$0.08\dot{3}$	2	1	
$\frac{7}{75}$				
$\frac{11}{24}$		3		
$\frac{317}{600}$	$0.528\,\dot{3}$			$2^3 \times 5^2 \times 3$
$\frac{1}{1320}$	$0.000\,\dot{7}\dot{5}$	3	2	$2^3 \times 5 \,\times 11 \times 3$
$\frac{50001}{101750}$	$0.491\,\dot{4}10\,319\dot{}$	3	6	$2 \times 5^3 \times 11 \times 37$

(c) A fraction of the form

$$\frac{1}{2^a \times 5^b \times c \times d}$$

where *a* and *b* are positive integers and *c* and *d* are different primes is changed to its decimal form.

Using your answers to **question 1(b)** and **question 2(b)**, explain how to find the number of non-repeating decimal places and the repeat length.

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