ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

It is essential that candidates ensure that they have met the demands of a question and given their response in the required form. The rubric for the paper should be read carefully and answers given to the required level of accuracy of 3 significant figures unless otherwise stated. This means that candidates should be working to a greater level of accuracy in the working of their solution. Where an exact response is required, it is intended that the use of a calculator is not required.

The use of the word 'hence' indicates that work or answers from previous parts of a question should be used.

General comments

Most candidates were able to make an attempt at most questions although it was evident that some were less well prepared than others. Candidates appeared to have sufficient room to answer each question in the allocated space. There appeared to be no timing issues.

Questions 6(ii) and **(iii)**, **9(i)**, **10** and **11(ii)** appeared to cause candidates the greatest difficulty.

It is also essential that candidates are aware of the different notations that may be used in this syllabus.

Comments on specific questions

Question 1

Many candidates still have difficulty when it comes to dealing with negative angles. This is clearly an area of trigonometry that needs more practice. By using the correct order of operations, most candidates were able to obtain at least one correct angle – usually the positive angle.

Some candidates incorrectly assumed that $\sin(x + 50^\circ) = \sin x + \sin 50^\circ$. No marks were available for solutions of this type.

Many candidates gave a correct angle together with other incorrect angles. This was only penalised if both correct angles were present. If in doubt, candidates should check their solutions by substituting them back into the original equation.

Answer: -95°, 175°

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Question 2

Most candidates realised that integration was needed and most were able to obtain method marks provided they had an expression for $\frac{dy}{dx}$ which contained the form $5x + pe^{2x}$, where p is a numerical constant and an expression for y which contained the form $\frac{5x^2}{2} + q\,e^{2x}$, where q is a numerical constant. Many candidates omitted both the arbitrary constants which were needed for a fully correct solution and some only considered an arbitrary constant after the second integration. There was also some misuse of the given conditions required for the calculation of these arbitrary constants. Some candidates were unable to deal with the integration of e^{2x} correctly and others evaluated terms in the form pe^0 as 0 rather than p.

Answer: $y = \frac{5x^2}{2} + \frac{1}{4}e^{2x} + \frac{7x}{2} - \frac{13}{4}$

Question 3

(i) Most candidates showed a good understanding of the graphs of modulus functions and produced a V shaped graph with the vertex at $(2, 0)$ and an intercept at $(0, 6)$. Marks were lost when a minimum point appeared at $(2, 0)$. It was evident that some candidates did not know how to sketch graphs of this type and obtained zero marks in this part, but went on to gain full marks in part **(ii)**.

(ii) Candidates that chose to form 2 linear equations usually had more success than those candidates who chose to form a quadratic equation. When forming a quadratic equation, many candidates omitted to square the '2' on the right hand side, obtaining an incorrect equation $9x^2 - 36x + 34 = 0$ rather than the correct equation $9x^2 - 36x + 32 = 0$. It was intended that candidates would be able to 'check' their solutions by looking at their graph in part **(i)** and recognising that their solutions were in the correct region i.e. both positive.

(iii) The allocation of one mark for this part was an indication that little work needed to be done and that use of both the responses to parts **(i)** and **(ii)** be made use of. There were very few correct responses, with many candidates giving the incorrect solution of $\frac{4}{3} < x < \frac{8}{3}$ rather than the correct response which was expected to be written as two separate statements.

Answer: **(ii)** $\frac{4}{3}$, $\frac{8}{3}$ **(ii)** $x < \frac{4}{3}$, $x > \frac{8}{3}$

Question 4

(i) Most candidates recognised that the differentiation of a product was needed. Most candidates also obtained the correct derivative of $\ln(2x + 1)$ and were able to gain the first 3 marks in this part of the question. There were some candidates who did not know how to differentiate the logarithmic function correctly and others who omitted the '2' from the numerator. Very few candidates did not attempt the differentiation of a product. However, many candidates, who had gained the first 3 marks for correct work, were unable to gain the final accuracy mark for the substitution of $x = 0.3$ into their expression for $\frac{dy}{dx}$. It is essential that candidates are aware of the basic rules of arithmetic when making use of their calculators. It is also essential that solutions are given to the correct level of accuracy. Just because an answer is between -1 and $+1$ does not alter the fact that the answer should be given to 3 significant figures. Too many candidates lost the final accuracy mark by giving an answer of 0.16.

(ii) Again, the allocation of one mark is indicative that very little work is needed and the inclusion of the word 'Hence' also means that the solution from part **(i)** is to be used. Most candidates did recognise that small changes were involved and were able to gain the mark available.

Answer: **(i)** 0.161 **(ii)** $0.161h$

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Question 5

Errors concerning the use of brackets were common in both parts of this question, with statements such as $(bx)^6 = bx^6$ and $(bx)^5 = bx^5$ being far too common.

(i) Most candidates were able to identify the correct term but common errors often resulted in the following statement: $924a^6bx^6 = 924x^6$. Some candidates were able to identify their error and correct it, but many were unaware that they needed to have b^6 involved.

(ii) The same problem involving powers of b also occurred in this part of the question, with many candidates realising which term was involved but omitting to take into account the correct power of b. It was intended that the result from part **(i)** be used to help with the solution of the resulting equation in part **(ii)** and many candidates were able to do this and obtain a completely correct solution.

Answer: **(ii)** $a = \frac{1}{2}$, $b = 2$

Question 6

(i) The correct use of the chain rule was made by the majority of candidates.

(ii) Unfortunately, very few candidates were able to gain any marks in this part as they incorrectly included a term in x with $\frac{3}{20x}\left(5x^2 - 125\right)^{\frac{2}{3}}$ being an all too common error. It is clear that candidates would benefit from more practice at these 'reverse differentiation' type questions.

(iii) Provided candidates had a correct form from part **(ii)**, most were able to gain a method mark for the correct substitution of the limits with some correct solutions. Again, it is important that candidates take care to use the laws of arithmetic when using their calculators to work out more complex calculations.

Answer **(i)** $\frac{20x}{3}\left(5x^2 - 125\right)^{-\frac{1}{3}}$ **(ii)** $\frac{3}{20}\left(5x^2 - 125\right)^{\frac{2}{3}} + c$ **(iii)** 5.63

Question 7

(a) Many candidates still have difficulties with questions of this type. It is necessary for the vector to be considered as the product of a magnitude and a unit direction vector. It was expected that the final answer be a vector as requested and not just the values of *a* and *b*.

(b) Many correct solutions were seen for this part with candidates correctly forming two separate equations by considering like vectors. Unfortunately, there some errors when candidates misread their own working mistaking *s* for 5. Method marks were available for a correct approach in these cases. It was also evident that some candidates were making simple arithmetic slips in the solutions of their equations. Solutions should always be checked in the original equations if possible, especially if they are not straightforward fractions or integers.

Answer: **(a)** $\begin{pmatrix} -36 \\ 15 \end{pmatrix}$ **(b)** $r = 0$, $s = -\frac{3}{2}$

Question 8

(i) Many completely correct solutions were seen, with candidates obviously being aware of the limitations that must be placed on the determinant of a matrix.

(ii) Most responses were correct, with occasional slips in either the calculation of the determinant or the signs or terms in the inverse matrix.

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(iii) It was important that candidates take notice of the word 'Hence'. It was intended that the inverse matrix found in part **(ii)** be used in the solution of the given matrix equation. Solutions which resulted from writing matrix **B** in terms of four unknowns and solving the resulting simultaneous equations were not acceptable. This question is testing syllabus objective involving the use of an inverse matrix. Provided the correct pre-multiplication of both sides of the given equation by the inverse matrix obtained in part **(ii)** was attempted, most candidates were able to gain at least two marks, with some having the occasional slip in the arithmetic calculations of the elements.

Answer: **(i)** $-6,\ 2$ **(ii)** $\frac{1}{20}\begin{pmatrix} 8 & -3 \\ -4 & 4 \end{pmatrix}$ **(iii)** $\frac{1}{20}\begin{pmatrix} 4 & 39 \\ 8 & -32 \end{pmatrix}$

Question 9

(i) It was essential that the notation $p'(0)$ be recognised as the value of $\frac{dp}{dx}$ when $x = 0$ and $p''(0)$ be recognised as the value of $\frac{d^2p}{dx^2}$ when $x = 0$. If this was not the case, then there was only one method mark available for the correct use of the factor theorem. For those candidates that did recognise the notation and make correct use of it, there were many completely correct solutions and some solutions with the occasional arithmetic slip. Unfortunately there were quite a few candidates who wrote down correct equations from differentiation but treated the substitution of $x = 0$ as $x = 1$ and subsequently were not able to find values for the constants as required.

(ii) For those candidates who had an expression for $p(x)$ with incorrect constants, a substitution of $x = \frac{1}{2}$ gave them a method mark.

Answer: **(i)** $a = 10,\ b = 43,\ c = 36$ **(ii)** 21

Question 10

This was probably the most difficult question on the paper in terms of the responses from the candidates. Too many obtained no marks at all.

(i) It was intended that the value of the constant a be found first making use of the substitution $x = 0$ and $y = 6$ into the given equation. Many candidates were able to obtain the correct value of $a = 2$. Problems arose when it came to the calculation of the constant b with many candidates being unable to deal correctly with the equation $\cos b\frac{\pi}{6} = -\frac{1}{2}$. For those candidates whose value of a was such that it did not fall into the range $0 < a \leqslant 4$, they obtained an equation which could not be solved. This should have alerted them to the fact that an error had been made and that they should go back and check their calculations. Completely correct solutions were few.

(ii) Unless the values of a and b were correct, only a method mark was available, provided $0 < a \leqslant 4$. It was intended that the equation $\cos 4x = -\frac{1}{2}$ be solved, finding the solution following $\frac{\pi}{6}$. It was essential that correct working was seen before awarding marks for this part as there were some fortuitous answers which did not gain any credit. It was also essential that the x-coordinates was given in terms of π as an exact form was required.

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(iii) It was intended that use of symmetry be made to calculate the value of x at the minimum point and hence calculate the corresponding value of y. Use of differentiation was also acceptable but more lengthy. Correct solutions were seldom seen. It was also essential that the coordinates were given in terms of π as an exact form was required.

Answer: **(i)** $a = 2,\ b = 4$ **(ii)** $\left(\frac{\pi}{3}, 0\right)$ **(iii)** $\left(\frac{\pi}{4}, -2\right)$

Question 11

(i) It was essential that the use of degrees was not made in this question. Candidates are to be discouraged from doing conversions between degrees and radians as inaccuracies and errors often occur. As the answer was given, candidates needed to show each step of their working. It was intended that an equation involving the perimeter, r and θ be formed and then after expressing r in terms of θ, a substitution into the formula for the area of a sector would yield the given result. Many candidates obtained full marks, showing each of the required steps necessary. It was also evident that there were some candidates who were unfamiliar with the part of the syllabus on circular measure.

(ii) In spite of the answer being given in part **(i)**, many candidates did not attempt this part. Of those that did, errors in the differentiation of a quotient and the subsequent equating to zero and simplification meant that few obtained full marks.

Answer: **(ii)** 6.25

Question 12

It was intended that candidates make use of their problem solving skills and formulate a plan of action to solve this problem which was given as an unstructured question. Most candidates made an attempt at it and many were able to obtain the first four marks for correctly finding the coordinates of the points *A* and *B*. Many made use of these coordinates to find the gradient of the line *AB* and hence the gradient of the line perpendicular to the line *AB*. This was unnecessary as the equation of the line joining the points *A* and *B* was given, $y = 2x + 5$, so the gradient of the perpendicular could be deduced immediately. Problems arose when it came to finding the equation of the perpendicular bisector as too many candidates made use of either the coordinates of *A* or of *B* rather than finding the coordinates of the midpoint of the line *AB*. Provided the correct equation of the perpendicular bisector had been found, most candidates went on to gain full marks.

Answer: $\left(\frac{5}{12}, \frac{5}{12}\right)$

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ADDITIONAL MATHEMATICS

Paper 4037/13
Paper 13

Key messages

In questions where a calculator is prohibited and in 'show that' questions, candidates should ensure that they show all their working in a clear and logical way. They should ensure that all necessary lines of working are included.

Candidates should be careful with their algebra in order to avoid the loss of marks through careless errors.

General comments

The paper provided a good range of responses showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues and most candidates attempted all the questions.

This paper gave candidates the opportunity to recall and use a range of mathematical techniques and to devise mathematical arguments, presenting those arguments precisely and logically. Good responses were set out clearly and demonstrated a good understanding of fundamental techniques such as the correct use of the order of operations when evaluating and manipulating mathematical expressions. Good responses also demonstrated a thorough understanding of the trigonometric, logarithmic and exponential functions, including correct manipulation, differentiation and integration of those functions.

The paper contained some questions requiring results to be shown and good responses to these demonstrated a logical progression with all steps clearly written down. In such questions candidates should be encouraged to keep their argument in a clear flow rather than having parts appearing out of order in separate places.

The paper contained questions prohibiting the use of a calculator and good responses to these questions showed full and detailed working. Candidates should be advised not to underestimate what is required to be written down in these questions.

Comments on specific questions

Question 1

(a) In the many good responses to this question the relevant term in the binomial expansion was clearly identified and equated to $-\frac{5}{8}$ and then accurately manipulated to obtain p^3 and then p. Most candidates used the correct binomial coefficient, but candidates should be aware that (px) had to be cubed to obtain p^3x^3. No credit was given for a term in a binomial expansion seen but not identified as the one required for the question.

(b) Candidates were expected to identify the term where x^8 and $\frac{1}{x^8}$, cancelled to give a numerical value. When correct this term resulted from $70 \times 2^4 \times \frac{1}{4^4}$. Many candidates successfully identified the relevant term and used a correct binomial coefficient but care was necessary with algebra and some candidates did not use powers correctly when dealing with $(2x^2)^4$ and $\left(\frac{1}{4}x^2\right)^4$. In particular,

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marks were lost if $\frac{1}{4}x^2$ was rewritten as $4x^{-2}$. Candidates should be aware that the correct term had to be identified as their final answer.

Answers :**(a)** $-\frac{1}{5}$ **(b)** $\frac{35}{8}$

Question 2

(i) This question tested candidates' knowledge of circular measure in the context of a simple proof. Candidates were expected to obtain the angle θ or the length of arc (s) in terms of r and clearly show its use in a formula for area, $\frac{1}{2}r^2\theta$ or $\frac{1}{2}rs$. Careful reading of the question was required as no credit was given for answers in terms of θ. As the answer was given, particular care had to be taken to show the steps correctly. Candidates should be reminded that the use of degree measure is not appropriate in this type of question and that appropriate formulas for arc length and area in terms of an angle in radians should be used. Candidates should be reminded of the need for clear presentation in this type of question; steps should be in sequence and brackets should be used to ensure that products are unambiguous.

(ii) Candidates were expected to differentiate the given expression for area with respect to r and many made this first step. However, to receive credit the derivative had to be equated to zero and r found. The resulting value of r had to be used to find θ. Instead, many candidates found the second derivative to show that it was in fact a maximum but this was not asked for in the question. If the question states that A has a maximum value, candidates are not expected to show that the value they find is a maximum.

Answer: **(ii)** 2

Question 3

Many candidates gave full and detailed solutions to both parts showing full working in line with the instruction not to use a calculator.

(i) Candidates were expected to use Pythagoras's Theorem with expansion of the two squared terms. There was more scope for error when using the cosine rule and candidates using that approach were less successful. Most candidates produced a full expansion that led to $\sqrt{200}$. There were some difficulties in finding the final simplest surd form with some answers left in terms of $\sqrt{8}$.

(ii) Most candidates gave a correct expression for $\tan B$ and most of them knew that the fraction had to be rationalised. There were a very few candidates who knew to rationalise but showed insufficient working to show that they had not used a calculator.

Answers: **(i)** $10\sqrt{2}$ **(ii)** $2+\sqrt{3}$

Question 4

(i) Candidates were expected to use the chain rule (function of a function); reducing the power of $(1 + \cos 3x)$ from 10 to 9 and multiplying by the derivative of $(1 + \cos 3x)$. Some good attempts using the chain rule were seen but in some of those the factor of 3 in the derivative was overlooked. Candidates would benefit from practice with this type of question as many seemed to be unfamiliar with using this differentiation technique.

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(ii) Candidates were expected to use $\frac{dy}{dx} \times \frac{dx}{dt} = \frac{dy}{dt}$ or equivalent to find $\frac{dx}{dt}$ using $\frac{dy}{dt} = 6$ and their value of $\frac{dy}{dx}$ from the previous part. There were many good solutions using values found in the first part. Some candidates confused finding $\frac{dx}{dt}$ with finding a small change in *x*.

Answers: **(i)** 30 **(ii)** $\frac{1}{5}$

Question 5

(i) There were many good responses that clearly showed the application of the change of base rule and most candidates started with $\log_9 4 = \log_3 \frac{4}{\log_3 9} = \frac{\log_3 4}{2}$. Candidates should be aware that as the result was given in the question a further step was required to show that they understood that $\log_3 2$ came from $\log_3 \sqrt{4}$. It was clear from some responses that not all candidates understood this last step as $\frac{4}{2}$ and 4 – 2 were seen. Some responses were unnecessarily complicated using other bases in intermediary steps and rarely showed enough detail to earn full marks.

(ii) Many candidates answered this question well, demonstrating that they understood the addition rule for logarithms. Candidates who obtained $\log_3 2x = 3$ usually went on to find a correct value with the occasional use of 3^2 rather than 3^3. Candidates should be aware that 'hence' in this question meant that they must use $\log_9 4 = \log_3 2$ in their solution.

Answer: **(ii)** $\frac{27}{2}$

Question 6

(i) Candidates were expected to differentiate to obtain an expression for velocity in terms of *t*, equate this to zero and solve. Some candidates did not realise that differentiation was required. Most candidates who attempted differentiation differentiated the exponential term correctly and went on to solve the equation. Most of these candidates showed a good understanding of the use of ln to solve the equation, but care was needed with the order of operations before using ln. Following the rubric on the front of the paper candidates had to give a final answer to three significant figures and a large number clearly misunderstood what had to be done in this case with premature rounding evident in many answers. Candidates should be aware which figures are significant when rounding 0.8109… .

(ii) There were many correct responses, with some candidates who did not differentiate in the first part recovering to differentiate twice in this part. Candidates generally showed a good understanding of differentiation of an exponential function.

(iii) This question was well done by those who had a suitable answer to the previous part. Most candidates had a correct order of operations and used logarithms to solve the equation $e^{-0.5t} = 0.1$. However, candidates should be aware that they should be working with more figures to have an answer to round to three significant figures. Many candidates lost the final mark through premature rounding.

(iv) There were very few good responses to this question. Good responses focused on the mathematical expression, that is, the term $e^{-0.5t}$ rather than more general considerations.

Answers: **(i)** 0.811 **(ii)** a = $3e^{-0.5t}$ **(iii)** 7.62 **(iv)** $e^{-0.5t}$ is always positive

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Question 7

Candidates showed a better understanding of vectors than in previous years but there were still candidates who did not provide a significant response to any part of this question. Candidates should pay attention to the direction of vectors and ensure they are giving an answer for $\overrightarrow{AD}$ and not $\overrightarrow{DA}$.

(i) Most candidates showed a good understanding of the question and of how to combine vectors and use the factor *m*. Candidates who tried complicated routes for $\overrightarrow{AD}$ were less successful and some candidates divided by *m*.

(ii) This part was well answered with many correct solutions. The fraction was misunderstood by some candidates and some added **a** rather than subtracting.

(iii) Many candidates made a good start by equating their two expressions for $\overrightarrow{AD}$. Few made a good attempt to combine this equation with the given relationship and few obtained a form where coefficients could be compared, or a scalar multiple found. Candidates should be aware that vectors cannot be divided and that attempts to find *m* should have been from comparison of coefficients.

Answers: **(i)** $m(\mathbf{c} - \mathbf{a})$ **(ii)** $\frac{2}{3}\mathbf{b} - \mathbf{a}$ **(iii)** $\frac{3}{8}$

Question 8

(i) About half of the responses showed a good understanding of the behaviour of the sine function and of the notation required to describe a range. Candidates should be aware that for a range the inequality should be for f(*x*) or for *y*; $5 \leqslant x \leqslant 6$ scored zero. Candidates would benefit from practice with this type of question as many candidates appeared to be guessing or just repeating the domain. Some did not take into account that the angle in the function was $\frac{x}{4}$ rather than *x*.

(ii) The majority of candidates knew what had to be done to find an inverse, but many responses showed an incomplete understanding of the order of operations for the given function and so could not reverse them effectively. Several otherwise good solutions neglected to exchange *x* and *y* as a final step. The range was often omitted. Some candidates knew that the range of the inverse function was the the same as the domain of the original function, but this had to be expressed with correct notation.

(iii) Most candidates made a good start and found the correct composite function. About half went on to provide good responses showing a good understanding of the order in which the equation had to be solved. Most responses with the order correct then went on to find a correct solution. If candidates went wrong it was usually with the first step of the solution where it was common to multiply the angle by two or to attempt to split the sine function. These and other errors in manipulation demonstrated an incomplete understanding of the order of operations. Candidates should be aware that the inverse of the sine function is $\sin^{-1}$ and this should not be confused with $\frac{1}{\sin x}$ or cosx. Candidates should be aware that as the domain of f was given in radians all angles used in this question should be in radians.

Answers: **(i)** $5 \leqslant f(x) \leqslant 6$ **(ii)** $f^{-1}(x) = 4\sin^{-1}(x-5)$, $0 \leqslant y \leqslant 2\pi$ **(iii)** π

Question 9

Candidates responded well to this question realising that a gradient had to be obtained by differentiation and nearly all candidates used *x* = 2 to find a value of *y*. A high number of candidates knew how to find the perpendicular gradient. Many candidates persevered to the end of the question and used a correct method to find the equation of a straight line given the gradient and a point on the line. Most candidates made a good

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attempt to use the quotient rule for differentiation, but marks were lost through incorrect differentiation of $\ln\left(3x^2+1\right)$ and mistakes in evaluation of the gradient function.

Most candidates gave their answer correct to two decimal places, but full marks could only be obtained by working with more figures before rounding.

Answer: $y = 2.44x - 4.23$

Question 10

(i) Many good responses were seen, and candidates clearly understood what had to be done to find the points. Most formed an equation in *x* which was more successful and easier than forming an equation in *y*. Occasional errors were to omit one of the points or to evaluate $\sqrt{4}$ as ± 4.

(ii) This part was answered well. The integration of 12 caused more problems than the other two terms. Candidates who tried to simplify the expression sometimes handicapped themselves for the next part. It was not necessary to evaluate a constant of integration and those who did so created extra work and additional chances of error.

(iii) Many good solutions to this question were seen. Two methods were used: finding the area under the curve and subtracting the area of the trapezium or subtracting the two equations then integrating and evaluating. These were equally successful, but the first method had the advantage of using the result from the previous part. Both methods required the substitution of limits which was usually well done, but care had to be taken with powers of –2. Candidates should be aware that this question required them to show all working and some solutions fell short of this or showed working inconsistent with the answer obtained. As in previous years, a significant number of candidates had difficulty identifying which areas were needed and how they should be combined.

Answers: **(i)** (2, 10) (–2, 6) **(ii)** $12x+\frac{x^2}{2}-\frac{x^3}{3}+c$ **(iii)** $\frac{32}{3}$

Question 11

(i) Nearly all candidates made a good start to this question, making correct use of the factor and remainder theorems with very few errors in substitution. Most went on to show adequate working to find one variable and then the other. Errors arose from equating the factor theorem or remainder theorem expressions to the wrong value, mistakes with signs when manipulating the equations or errors when cubing and squaring –3.

(ii) The answer had to be of the form asked for in the question. Successful candidates tended to use long division to obtain the quotient. Candidates using synthetic division using $\frac{1}{2}$ rarely succeeded in finding a correct quadratic factor as they did not compensate for having divided by $\left(x-\frac{1}{2}\right)$ rather than (2*x* – 1).

(iii) Many good solutions were seen from candidates who had the previous parts correct. A common error was to forget (2*x* – 1) in the complete factorisation. Candidates were not penalised for going on to solve the equation but it should be noted that this was not required.

(iv) There were many good solutions including some from candidates who had realised that $2\sin\theta-1=0$ could be used despite inaccuracies in previous parts.

Answers: **(i)** *a* = *b* = 6 **(ii)** $(2x-1)\left(3x^2+10x+8\right)$ **(iii)** $(2x-1)(x+2)(3x+4)$ **(iv)** $\theta = 30°,\ 150°$

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ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

Full coverage of the syllabus is required if candidates are to do well. There appeared to be candidates who had not studied all topic areas.

Candidates need to produce careful, accurate algebra and try to ensure that they do not make careless mistakes.

Candidates should remember that during their working they should be using a greater degree of accuracy than that required in their answer.

General comments

The laws of logarithms are very precise and simple when used well. Candidates often adapted them incorrectly with consequences that resulted in many marks being lost.

Manipulation of algebraic fractions was often poor due to a candidates thinking that individual terms can be cancelled.

Solution of simultaneous linear equations is often more successful by adding or subtracting multiples rather than substitution from one equation into another with terms involving fractions.

Comments on specific questions

Question 1

Most candidates were able to expand the brackets and rearrange to form a quadratic inequality and the vast majority did this accurately to obtain $x^2 > 25$. Choosing the correct regions from this proved more challenging with most candidates unable to identify these correctly. A common response was the solution that x was greater than 5 and also greater than – 5.

Answer: $x > 5$ or $x < -5$.

Question 2

This proved very straightforward for many but there were some very poor attempts from candidates who seemed not to have understood that it would be useful if the Venn diagram was filled in algebraically. A relatively common error among the least successful was to consider that there were 40 boys who played cricket only and hence 80 who played football only. Occasionally a candidate forgot to include x in the region $(F \cup C)'$, or used $x - 40$ instead of $40 - x$, or $(40 - x)^2$ instead of $2(40 - x)$.

Answer: 15

Question 3

(i) Most candidates recognised this as a quotient and used the rule correctly. The most common errors were in differentiating $\sin 2x$ usually omitting the multiple of 2 or including a negative sign. Occasionally the terms in the numerator were reversed but it was rare to see the terms added. Capable candidates often treated the expression as a product and were generally successful.

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(ii) There were some accurate answers to this part but there were many candidates who either did not realise or were not competent at dealing with expressions that required use of radian measure and as a result lost all three marks available.

Answers: **(i)** $\dfrac{3x^2\sin 2x - 2x^3\cos 2x}{(\sin 2x)^2}$ **(ii)** $y = \dfrac{3\pi^2}{16}x - \dfrac{\pi^3}{32}$ or $y = 1.85x - 0.97$

Question 4

(i) There were many successful solutions, but marks were often lost by poor algebraic manipulation and also by not working to a sufficient degree of accuracy. There were a variety of approaches when using logs with the most common being $(3x-1)\log 2 = \log 6$. Other correct expressions seen included $3x - 1 = \log_2 6$ and $3x\log 2 = \log 12$.

(ii) A number of candidates either did not notice that the logs had different bases or did not know how to deal with them. There were numerous candidates who had a limited grasp of the laws of logarithms and produced such errors as $\log(y+14) = \log y \times \log 14$ or $\log y + \log 14$ and $\dfrac{2}{\log_y 3} = 2 - \log_y 3$. It was also necessary to write 1 as $\log_3 3$ to get all terms as logs to the base 3. This fact plus the lack of knowledge that $\dfrac{1}{\log_y 3} = \log_3 y$ resulted in many candidates not obtaining the correct quadratic equation. Those who did solve the correct equation successfully invariably did not remember to discard $x = -2$.

Answers: **(i)** 1.19 **(ii)** $\dfrac{7}{3}$

Question 5

More candidates achieved successful solutions to this question than any other question on the paper. Laws of indices were generally well demonstrated and using correct powers of 2 and 3 usually resulted in two correct linear equations to solve. The most common error was to forget to multiply $p+1$ by 3, leading to the incorrect equation $3p + 1 - 2q = 11$. There were some who obtained correct equations fortuitously which could not gain credit. Another common error was to obtain $3p + 3 = 22q$.

Answer: $p = 4$, $q = 2$

Question 6

Both parts were often left out by candidates and fully correct solutions were relatively rare.

(i) There were many completely unidentifiable methods but some showed evidence of correct reasoning gone wrong. There were a number of solutions which would have been correct if they had not miscounted the letters and numbers. Other solutions wrote potentially correct products but used combinations rather than permutations.

(ii) This part was answered more successfully and many candidates were able to pick up partial marks for ${}^{10}C_7 = 120$ or ${}^{7}C_7 = 1$. There were a number who gave 84 as their answer, having overlooked the team containing none of the sisters.

Answers: **(i)** 11 340 **(ii)** 85

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Question 7

This was a challenging question with relatively few candidates scoring full marks, although many made some progress. If the candidate realised that the intention was to use the quadratic formula there was a possibility of obtaining marks particularly as the discriminant was a perfect square. Subsequent work involved rationalising the fraction to get the solutions in the required form. Some candidates having got a long way decided to reject one solution and sometimes did not write their answers in the form requested.

Answers: $x = 1 + \sqrt{3}$ or $x = -\frac{1}{2} - \frac{\sqrt{3}}{2}$

Question 8

(i) This was well done and clearly set out by many. Some candidates lost the final mark by not showing clearly how $\frac{\sin x}{\cos^2 x}$ could be simplified to $\tan x \sec x$. There were some weak attempts which showed limited knowledge of trig formulae or how to deal with fractions. Some examples include: $1 - \sin x = \cos x$ and $\frac{1}{1 - \sin x} = \frac{1}{1} - \text{cosec}\, x$.

(ii) After writing the first statement correctly many lost their way or made mistakes in finding expressions for $\tan^2 x$, $\sin^2 x$ or $\cos^2 x$. Once one of these expressions was obtained many considered only the positive root and so obtained just two solutions rather than 4.

Answers: **(ii)** 35.3°, 144.7°, 215.3°, 324.7°

Question 9

(i) Many tackled this question well realising the need to differentiate and proceeding correctly. Some found the equation of the tangent and others used the incorrect gradient, often – *m* instead of $-\frac{1}{m}$. Others did not realise that calculus was required and attempted to find the equation of the straight line from the diagram guessing the coordinates of *B*.

(ii) Most candidates gained credit here if their normal cut the *x*-axis at a value greater than 4.

(iii) The most straightforward method to answer this question was to integrate between $x = 0$ and $x = 4$ and then add on the area of the adjacent right-angled triangle. A number of candidates used limits of $x = 0$ and $x = 6$ applied to a function including the curve and line which did not result in the required area. The actual integration was usually correct but some candidates had difficulty dividing 2 by $\frac{3}{2}$.

Answers: **(i)** $y = -2x + 12$ **(ii)** $(6, 0)$ **(iii)** $14\frac{2}{3}$

Question 10

(i) The method required finding the discriminant of a quadratic obtained by eliminating *x* or *y* from the equations of a straight line and curve. Far too many sign errors were made in obtaining the quadratic in the correct form and there was often a term lost from their working. As a result the quadratic was often wrong and hence the values of *k* were also incorrect.

(ii) Credit was given for the equations of straight lines obtained with the candidate's values of *k*. However, many did not proceed as required and subsequently found where their tangents touched the curves rather than where the two straight lines intersected.

Answers: **(i)** $k = 6$ or 10 **(ii)** (2, 1)

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Question 11

(i) Most candidates were able to set up $\mathrm{gf}(x)$ correctly and subsequently expand and collect like terms. A number of candidates tried to simplify their expressions by some dubious cancelling of terms from numerator and denominator which spoilt their previously correct answers.

(ii) This was tackled confidently with a correct method in evidence and final expressions all in terms of *x*. There were the odd sign errors but the only real problem arose from those who misunderstood the notation and attempted to differentiate.

(iii) This was not as successfully attempted and the problems were entirely due to careless errors in applying the required algebra to the function given. As a result the formula often had to be applied to an incorrect quadratic. Many of those who did obtain the correct solutions did not reject $x = -1$.

Answers: **(i)** $\mathrm{gf}(x) = \frac{8x-5}{12x-10}$ **(ii)** $\mathrm{g}^{-1}(x) = \frac{x+1}{3x-2}$ **(iii)** $x = 2$

Question 12

Only a tiny minority completed this successfully and most did not even get as far as obtaining a correct triangle to work with. Very few showed the wind as from rather than to the correct direction and there seemed little awareness to show the resultant velocity as the vector sum of the original velocity plus the wind. Many triangles depicted 50° as an obtuse angle or 130° as acute. Many candidates wanted to work with a right angled triangle.

Answers: 236 km/hr, 007.5°

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ADDITIONAL MATHEMATICS

Paper 4037/23
Paper 23

Key messages

When attempting to solve a mathematical problem it is necessary, at the outset, to have a logical plan of action which is likely to lead to a successful outcome. In particular, this is essential for problems where trigonometric equations are to be solved.

The number of marks available for a question is indicative of the amount of work needed to answer it. A question carrying only two or three marks should not require a long answer with a large amount of working.

Working should always be shown so that marks for method can be awarded, even when an answer is incorrect. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator, without showing any working.

General comments

The general quality of work again varied greatly. Some candidates produced work of impressive quality, resulting in some cases in full, or almost full, marks. Others lacked the essential knowledge for mathematics of this standard.

It is necessary to point out to candidates the significance of the words 'hence' and 'state' when answering questions. A question which says "hence state…" is indicating that the answer (or answers) can and should be written down without working following the result of a previous answer. This instruction was sometimes ignored completely, and unnecessary, or even irrelevant, working was given (see **Question 3(ii)** below).

It is also necessary to state again, that in solving quadratic equations the method should always be shown, whether this is by factorisation or the use of the formula. If the equation is incorrect, Examiners can then allow a mark for method. When solutions to an incorrect equation are taken directly from a calculator method marks cannot be awarded.

It may seem obvious to remark that solving mathematical problems requires the application of a logical strategy of approach. An appropriate method has to be chosen which can be seen at the outset to lead in as efficient a way as possible to a solution. In this paper there instances of needlessly long methods being chosen (see **Question 5** below), and apparently directionless manipulations being employed, with no successful outcome (see **Question 9(c)** below).

Comments on specific questions

Question 1

Most candidates were aware that this equation has two solutions, and that to find the second solution a sign change had to be made. However this was not always done in the correct way. In an alternative approach, those who squared both sides of the equation and solved the resulting quadratic almost always obtained both solutions correctly. The most limited answers presented only the $x = 2$ solution.

Answer: $x = 2$, $x = –5$

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Question 2

Correct answers were seen most often for the first diagram, and least often for the third. A common error in the third diagram was to omit the central part of $A \cap B$. Another was to have no shading outside the sets and inside the rectangle.

Question 3

There were mixed responses to part **(i)**, caused to some extent by the negative coefficient of x^2 in the given expression. Candidates who factored this out and accounted for it later were usually successful. Much working was frequently seen in part **(ii)**, where the significance of the words “hence” and “state” had clearly not been appreciated, or perhaps the words had been ignored. The best answers used the result of part **(i)** to write down, with no working whatsoever, the two required values in part **(ii)**. Least success overall was achieved in part **(iii)**. Only the most perceptive candidates were able to make the connection between the x in part **(i)** and the z in part **(iii)**. Even those who did make this connection often omitted one of the roots in their final answer.

Answers: **(i)** $\frac{81}{4} - \left(x - \frac{7}{2}\right)^2$ **(ii)** maximum $\frac{81}{4}$ at $x = \frac{7}{2}$ **(iii)** $z = \pm\sqrt{8}$

Question 4

It was generally well understood that integration was required in both parts of this question, so almost all candidates earned some credit. However the accuracy in carrying this out was variable. A serious limitation of many answers was that the production of a constant of integration at each stage was ignored. Even when a constant was found in part **(i)**, it was sometimes ignored in part **(ii)** for the second integration.

Answers: **(i)** $\frac{dy}{dx} = x^2 - \frac{1}{(x+1)^3} + \frac{1}{8}$ **(ii)** $y = \frac{1}{3}x^3 + \frac{1}{2(x+1)^2} + \frac{1}{8}x + \frac{29}{12}$

Question 5

Almost all candidates were able to find the inverse correctly in part **(i)**. Provided the candidate was rigorous in manipulating the equations in parts **(ii)** and **(iii)** to find the unknown matrix, taking strict account of the order of operations, success usually followed. Very common errors were to use pre-multiplication in part **(ii)**, and post-multiplication in part **(iii)**. Candidates need to be aware that in such situations the use of pre- or post-multiplication, as appropriate, is a much more efficient method of solving than that of giving the unknown matrix elements of, say, *a*, *b*, *c* and *d*, and setting up four simultaneous equations. Large quantities of working using the latter method were sometimes seen, totally out of proportion to the marks available.

Answers: **(i)** $\frac{1}{5}\begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix}$ **(ii)** $\frac{1}{5}\begin{pmatrix} 0 & 5 \\ -13 & 16 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 6 & -20 \\ 8 & -20 \end{pmatrix}$

Question 6

Good concise answers to this question converted the two logarithmic equations into simple equations without logarithms, using the laws of logarithms properly, and then solved them. Others used incorrect logarithmic relationships, or treated $\log_2$ as though it was in itself an algebraic entity, and produced longer attempts with little valid mathematics.

Answer: $x = 2$, $y = 3$

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Question 7

There was very little confusion here over whether these situations involved counting permutations or combinations, and combinations formulae were most commonly used. There were many correct answers to part **(i)** and a fair number of correct answers to part **(ii)**. Candidates seem to have appreciated more readily that, to allow for the conditions relating to the twins, a multiplier of 4 was needed in part **(iv)**, than that a multiplier of 2 was needed in part **(iii)**. In both these parts, some also made the error of counting selections from 18 instead of 16 after allowance had been made for the selection from the twins. In some cases the same answer was offered, usually 11 440, for more than one of the parts **(ii)** to **(iv)**.

Answers: **(i)** 167 960 **(ii)** 11 440 **(iii)** 22 880 **(iv)** 45 760

Question 8

Most candidates found the gradient of the straight line correctly in part **(i)**. Those who recognised that the relationship between x and y had the form $y^2 = me^{2x} + c$ and that the coordinates of the given points were values of e^{2x} and y^2, and not x and y, were usually successful throughout the remainder of the question. A common error was to substitute, for example, $x = 1.5$ and $y = 5.5$. In part **(iii)** partial credit was given even if values of m and c from part **(i)** were incorrect, provided the correct form of the relationship was used.

Answers: **(i)** $y = \sqrt{3e^{2x} + 1}$ **(ii)** 34.8 **(iii)** 3.36

Question 9

Some good concise answers to part **(a)** were seen with all the working in radians, expressed as multiples of π. Occasionally when radians were expressed as decimals errors were introduced as a result of premature approximation. Another common error was to obtain the second solution by subtraction of the first from π. Working and solutions presented in degrees only were awarded no marks.

Part **(b)** was generally answered well, with the equation correctly manipulated into a correct expression for $\tan y$, and both solutions given. It was quite rare for only one solution to be given, or the second solution to be incorrect.

Answers to part **(c)** which began by rewriting the given equation correctly in terms of $\sin z$ and $\cos z$ were most likely to achieve success. Many candidates who followed this route produced clear and concise solutions. Those who opted to write $\cot z$ in terms of $\tan z$ were less likely to be successful, and some made a very serious error in their first line by replacing $\operatorname{cosec} z$ with $1 + \cot z$.

In this type of problem it is important for the candidate to have a strategy for solving: in this case to rewrite an equation containing three different trigonometric ratios as an equation containing only one. Quite often candidates were seen using their knowledge of relationships and identities to write the equation in different ways, but making little or no progress, and still ending, after filling the answer space, with an equation not reduced to a single ratio. In addition, such long undirected attempts usually contained, almost inevitably, an error in the working somewhere along the way.

Answers: **(a)** $\frac{\pi}{12}, \frac{5\pi}{12}$ **(b)** 53.1°, 233.1° **(c)** 60°, 104.5°, 255.5°, 300°

Question 10

Good knowledge of the product rule was seen in part **(i)**, with accurate differentiation of the component with the square root. Good practice was regularly seen with clear side working breaking down the right hand side of the equation into u and v components. Part **(ii)** was also answered well, with partial credit being allowed for correct working following an error in the differentiation in part **(i)**. For complete success in part **(iii)** it was necessary to have a correct result from part **(i)**, and the algebraic skills to obtain the simple quadratic equation following from setting the derivative equal to zero. Fully correct answers here were much more limited.

Answers: **(i)** $\frac{x^2}{2\sqrt{3+x}} + 2x\sqrt{3+x}$ **(ii)** $y = \frac{17}{4}x - \frac{9}{4}$ **(iii)** (0, 0), (–2.4, 4.46)

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Question 11

There was a fair number of fully correct answers to this question. In part **(i)** candidates generally knew to apply the tangency condition to the equation resulting from setting the line's equation equal to the curve's equation. Any errors which occurred were usually due to faulty algebra. The required equations were then often formed correctly in part **(ii)** and solved in pairs. Here errors sometimes occurred in finding the points of contact when, having solved one pair correctly for one coordinate, the other coordinate was found by substituting back into the wrong pair. Almost all candidates knew how to find the distance between two points in part **(iii)**. Partial credit was allowed in parts **(ii)** and **(iii)** following incorrect values of k from part **(i)**.

Answers: **(i)** 3, 11 **(ii)** tangents: $y = -5x + 8$, $y = -5x + 16$ curves: $y = 7 - 3x - x^2$, $y = 7 - 11x - x^2$
points of contact: (1, 3), (–3, 31) **(iii)** $20\sqrt{2}$

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