Cambridge Assessment International Education

# Cambridge O Level

**ADDITIONAL MATHEMATICS** **4037/23**

Paper 2 **October/November 2022**

MARK SCHEME

Maximum Mark: 80

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

| Maths-Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

## MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

### Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $2x^2 - 8x + 3x - 12 * 3x^2 - 3x + 4x - 4$ | **B1** | Correctly expands all brackets<br>* is any inequality or equals sign |
| | $[0*]\ x^2 + 6x + 8$ | **B1** | Collects terms to correct 3-term quadratic in solvable form |
| | $[0*](x+2)(x+4)$ | **M1** | Factorises or solves *their* 3-term quadratic |
| | –4 and –2 | **A1** | Correct critical values |
| | $-4 < x < -2$ mark final answer | **A1** | |
| 2 | $\frac{dy}{dx} = 2ax - 5$ | **B1** | |
| | $2a \times 2 - 5 = 7$ oe | **M1** | **FT** *their* $\left( \left.\frac{dy}{dx}\right\|_{x=2} \right) = 7$ |
| | $a = 3$ | **A1** | |
| | $7 \times 2 + b = their\ 4$<br>or $b = 2 - 4 \times their\ a$ | **M1** | **dep** on previous **M1**<br>where *their* 4 is an attempt to evaluate $y = ax^2 - 5x + 2$ using $x = 2$ and *their a* |
| | $b = -10$ | **A1** | |
| | **Alternative** | | |
| | $(-12)^2 - 4a(2-b) = 0$ oe | **(B1)** | for use of discriminant on $ax^2 - 12x + 2 - b = 0$ |
| | $144 - 8a + 4a(4a - 22) = 0$ oe<br>or $144 - (b+22)(2-b) = 0$ oe | **(M1)** | Condone one sign or arithmetic error |
| | $a^2 - 6a + 9\ [=0]$ oe<br>or $b^2 + 20b + 100\ [=0]$ oe | **(A1)** | for correct 3-term quadratic in solvable form |
| | $a = 3$<br>and $b = -10$ | **(A2)** | **A1** for $a = 3$ or $b = -10$ |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $\lg((2x-1)(x+2)) = \lg\frac{100}{4}$ oe<br>or $10^2 = 4(2x-1)(x+2)$ oe | **M2** | **M1** for one log law correctly applied within a correct equation<br>e.g. $\lg 4(2x-1)(x+2) \quad = 2$ |
| | $2x^2+3x-27[=0]$ | **A1** | Collects terms to correct 3-term quadratic in solvable form |
| | $(2x+9)(x-3)[=0]$ | **M1** | **dep** on at least **M1** previously awarded<br>Factorises *their* $2x^2+3x-27$ or solves *their* $2x^2+3x-27=0$ |
| | $x$ = 3 indicated as only valid solution | **A1** | nfww |
| 4(a) | $2k+6=8-16+6k+2$ oe | **M1** | For equating line to curve and substituting $x$ = 2, or vice versa |
| | $k$ = 3 | **A1** | |
| 4(b) | $x^3-4x^2+(2\times \mathit{their}\,k)x-4 \;\; [=0]$<br>or $x^3-4x^2+6x-4 \;\; [=0]$ | **M1** | **FT** *their k* in correct cubic |
| | $x^2-2x+2$ | **A2** | Correct quadratic factor from correct cubic<br>**A1** for a quadratic factor with two terms correct, from correct cubic |
| | $(-2)^2-4(1)(2) < 0$ oe<br>or $4 - 8 < 0$ oe<br><br>[and so $x$ = 2 is the only solution] | **A1** | Uses discriminant correctly on the correct quadratic factor |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\dfrac{\cos^2 x+(1-\sin x)^2}{(1-\sin x)\cos x}$ or $\dfrac{\cos^2 x}{(1-\sin x)\cos x}+\dfrac{(1-\sin x)^2}{(1-\sin x)\cos x}$ | M1 | Correctly takes common denominator |
| | $\dfrac{\cos^2 x+1-2\sin x+\sin^2 x}{(1-\sin x)\cos x}$ | A1 | OR $\dfrac{1-\sin^2 x+(1-\sin x)^2}{(1-\sin x)\cos x}$ |
| | $\dfrac{1+1-2\sin x}{(1-\sin x)\cos x}$ or $\dfrac{1-\sin^2 x+1-2\sin x+\sin^2 x}{(1-\sin x)\cos x}$ | A1 | OR $\dfrac{(1-\sin x)(1+\sin x)+(1-\sin x)^2}{(1-\sin x)\cos x}$ |
| | $\dfrac{2(1-\sin x)}{(1-\sin x)\cos x}=2\sec x$ or $\dfrac{2-2\sin x}{(1-\sin x)\cos x}=\dfrac{2}{\cos x}=2\sec x$ or equivalent | A1 | All steps correct and final step justified OR $\dfrac{1+\sin x+1-\sin x}{\cos x}=2\sec x$ |
| | **Alternative** ***Must work with LHS only*** | | |
| | $\dfrac{(\cos x)(1+\sin x)}{(1-\sin x)(1+\sin x)}+\dfrac{(1-\sin x)\cos x}{(\cos x)\cos x}$ | (M1) | Forms fractions with common denominator in different form |
| | $\dfrac{(\cos x)(1+\sin x)}{\cos^2 x}+\dfrac{(1-\sin x)\cos x}{\cos^2 x}$ | (A1) | Uses difference of two squares and $\sin^2 x+\cos^2 x=1$ to write fractions with a common denominator in the same form |
| | $\dfrac{2\cos x}{\cos^2 x}$ | (A1) | Combine as a single fraction and collects terms |
| | $\dfrac{2}{\cos x}=2\sec x$ | (A1) | All steps correct and final step justified |
| 5(b) | $\cos^3\dfrac{\theta}{2}=\dfrac{1}{4}$ | B1 | |
| | $\cos\dfrac{\theta}{2}=\sqrt[3]{their\ \dfrac{1}{4}}$ soi | M1 | **dep** on starting with $2\sec\dfrac{\theta}{2}=8\cos^2\dfrac{\theta}{2}$ |
| | ±101.9 awrt | A2 | and no extras in range<br>**A1** for either, ignoring extras in range<br>If **A0** then **SC1** for ±102 with no extras in range |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6 | $81+108ax+54a^2x^2+12a^3x^3$ soi<br><br>or $12a^3=\frac{3}{2}$ $b=108a$ $c=54a^2$ soi | M3 | **M2** for any 3 correct terms or 2 correct equations<br>or **M1** for any 2 correct terms, 1 correct equation or for correct but insufficiently simplified expansion e.g.<br>$3^4+4\times3^3\times ax+\frac{4\times3}{2}\times3^2\times(ax)^2$<br>$+\frac{4\times3\times2}{3\times2}\times3\times(ax)^3$ |
| | $a=\frac{1}{2}$ oe | A1 | |
| | $b$ = 54 | A1 | **FT** 108 × *their a*, providing at least **M1** awarded |
| | $c=\frac{27}{2}$ oe | A1 | **FT** 54 × (*their a*)$^2$, providing at least **M1** awarded |
| 7 | ${}^{n}\mathrm{C}_4=\frac{n!}{(n-4)!4!}$ and ${}^{n}\mathrm{C}_2=\frac{n!}{(n-2)!2!}$ soi | B1 | |
| | $\frac{n(n-1)(n-2)(n-3)}{24}=\frac{13n(n-1)}{2}$<br>or $(n-2)(n-3)=\frac{13\times24}{2}$ oe, soi | M1 | Writes in a correct form, free of factorials |
| | $n^2-5n-150$ $[=0]$ | A1 | |
| | $n$ = 15 only, nfww | A1 | **dep** on previous A1 |
| | ${}^{15}\mathrm{C}_8=6435$ only | B1 | |
| 8(a) | (Velocity vector =) $\frac{26}{\sqrt{12^2+5^2}}\begin{pmatrix}12\\5\end{pmatrix}$ oe | M2 | **M1** for $\sqrt{12^2+5^2}$ or 13 or 2 seen |
| | (Position vector =) $\begin{pmatrix}3\\-2\end{pmatrix}+t\begin{pmatrix}24\\10\end{pmatrix}$ oe | A1 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(b) | (Direction vector =) $\begin{pmatrix}4\\3\end{pmatrix}$ soi<br>or $x$ component: $\cos\alpha = \frac{4}{5}$<br>$y$ component: $\sin\alpha = \frac{3}{5}$ soi | **B1** | |
| | (Velocity vector =) $\frac{20}{\sqrt{4^2+3^2}} \times \textit{their}\begin{pmatrix}4\\3\end{pmatrix}$ oe soi<br>or $20\begin{pmatrix}\textit{their}\cos\alpha\\ \textit{their}\sin\alpha\end{pmatrix}$ soi | **M1** | |
| | (Position vector =) $\begin{pmatrix}67\\-18\end{pmatrix}+t\begin{pmatrix}16\\12\end{pmatrix}$ oe | **A2** | **A1 FT** $\begin{pmatrix}67\\-18\end{pmatrix}+t\times\textit{their}\begin{pmatrix}16\\12\end{pmatrix}$<br><br>If zero scored, **SC2** for one correct component, either $67+16t$ or $-18+12t$ |
| 8(c) | $3+24t=67+16t$ oe<br>or $-2+10t=-18+12t$ oe | **M1** | **FT** Equates *their x* components, or *their y* components from parts **(a)** and **(b)**, providing of equivalent difficulty, e.g. $a+bt=c+dt$ |
| | $t = 8$ | **A1** | **dep** on full marks in **(a)** and **(b)** |
| | (Position of meeting =) $\begin{pmatrix}195\\78\end{pmatrix}$ | **A1** | **dep** on full marks in **(a)** and **(b)** |
| 9(a) | $\frac{d}{dx}\left(e^{-2x}\right)=-2e^{-2x}$ soi | **B1** | |
| | $\frac{dy}{dx}=ke^{-2x}-2kxe^{-2x}$ oe, isw | **B1** | **FT** for use of product rule<br>$k.e^{-2x}+kx.\left(\textit{their}\frac{d}{dx}\left(e^{-2x}\right)\right)$ |
| | **Alternative** | | |
| | $\frac{d}{dx}\left(e^{2x}\right)=2e^{2x}$ soi | **(B1)** | |
| | $\frac{dy}{dx}=\frac{ke^{2x}-2kxe^{2x}}{\left(e^{2x}\right)^2}$ oe, isw | **(B1)** | **FT** for use of quotient rule<br>$\frac{k.e^{2x}-kx.\left(\textit{their}\ 2e^{2x}\right)}{\left(e^{2x}\right)^2}$ |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(b) | Equates $\frac{dy}{dx}=0$ and finds 10 – 20*x* = 0 oe | **M1** | **FT** *their* (a), provided of the form $me^{-2x}+nxe^{-2x}$ or $me^{2x}+nxe^{2x}$ |
| | $\left(\frac{1}{2}, \frac{5}{e}\right)$ oe only | **A2** | For both values:<br>$x=$ 0.5 **and** $y=5e^{-1}$<br>or 1.84 or 1.839[39...] rot to 4 or more sf<br>**A1** for $x=\frac{1}{2}$ only |
| 9(c) | $-2xe^{-2x}-e^{-2x}+c$ | **B3** | For fully correct answer<br>or **B2** for $-2xe^{-2x}-e^{-2x}$<br>or $\left[\int 4xe^{-2x}dx\right]=-2xe^{-2x}+\int 2e^{-2x}dx$<br>or **B1** for<br>$kxe^{-2x}=\int\left(ke^{-2x}-2kxe^{-2x}\right)dx$<br>or better |
| 9(d) | $-2e^{-2}-e^{-2}-\left(0-e^{0}\right)$ oe | **M1** | Correct substitution of limits into correct expression |
| | $1-\frac{3}{e^2}$ or $1-3e^{-2}$ | **A1** | |
| 10(a) | $a+(3-1)d=10$ soi | **B1** | |
| | $\frac{8}{2}\{2a+(8-1)d\}=116$ soi | **B1** | |
| | Correct method to eliminate one unknown and attempt to solve to find *a* or *d* | **M1** | **dep** on at least **B1** awarded |
| | *a* = 4 and *d* = 3 | **A2** | **A1** for either |
| 10(b) | $S_{30}=\frac{30}{2}\{2(4)+29(3)\}$<br>and $S_{11}=\frac{11}{2}\{2(4)+10(3)\}$ | **B2** | **M1 FT** *their a* and *their d* for<br>$S_{30}=\frac{30}{2}\{2(\textit{their } 4)+29(\textit{their } 3)\}$<br>or $S_{11}=\frac{11}{2}\{2(\textit{their } 4)+10(\textit{their } 3)\}$ |
| | Correct plan $S_{30}-S_{11}$ attempted | **M1** | **FT** *their a* and *their d* |
| | 1216 | **A1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | **Alternative 1** | | |
| | first term = 4 + 11×3 or 37<br>and an attempt at $S_{19}$ | **(M1)** | **FT** *their a* + 11×*their d* |
| | $\frac{19}{2}\{2(37)+(19-1)\times 3\}$ oe<br>or $\frac{19}{2}\{37+91\}$ oe | **(B2)** | **M1 FT** for *their* first term and *their d* in $\frac{19}{2}\{2(\mathit{their}\,37)+(19-1)\times \mathit{their}\,3\}$<br>or for *their* first term and *their* last term in $\frac{19}{2}\{\mathit{their}\,37+\mathit{their}\,91\}$ |
| | 1216 | **(A1)** | |
| | **Alternative 2** | | |
| | Correct sum of terms:<br>37 + 40 + 43 + 46 + 49 + 52 + 55 + 58 + 61 + 64 + 67 + 70 + 73 + 76 + 79 + 82 + 85 + 88 + 91 | **(M3)** | **M2 FT** *their a* and *their d* for sum starting with *their* 37 **and** ending with *their* 91, with at most one omission or error<br>or **M1 FT** *their a* and *their d* for sum starting with *their* 37 **or** ending with *their* 91, with at most two omissions or errors |
| | 1216 | **(A1)** | |
| 11(a) | $2\mathbf{a}+\lambda(3\mathbf{b}-2\mathbf{a})$ oe isw<br>or $3\mathbf{b}-(1-\lambda)(3\mathbf{b}-2\mathbf{a})$ oe isw | **B3** | **B1** for $\overrightarrow{PS}=3\mathbf{b}-2\mathbf{a}$ soi<br>and **B1** for correct route using $\lambda$, either $\overrightarrow{OX}=\overrightarrow{OP}+\lambda\overrightarrow{PS}$ soi<br>or $\overrightarrow{OX}=\overrightarrow{OS}-(1-\lambda)\overrightarrow{PS}$ soi |
| 11(b) | $\mu(5\mathbf{a}+2\mathbf{b})$ isw | **B2** | **B1** for $\overrightarrow{OQ}=3\mathbf{b}+5\mathbf{a}-\mathbf{b}$ oe soi |
| 11(c) | $2-2\lambda=5\mu$ and $3\lambda=2\mu$ oe | **M2** | for correctly equating scalars for both components<br>**FT** *their* **(a)** and **(b)** if possible<br>**M1 FT** for equating scalars for either component |
| | Solves to find $\lambda=\frac{4}{19}$ or $\mu=\frac{6}{19}$ | **A1** | |
| | $\lambda=\frac{4}{19}$ and $\mu=\frac{6}{19}$ | **A1** | |
| 11(d) | $\frac{6}{19}$ isw | **B1** | |
| 11(e) | $\frac{4}{15}$ isw | **B1** | |