

Cambridge O Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

413980569

ADDITIONAL MATHEMATICS

4037/13

Paper 1 October/November 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where *n* is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \ (r \neq 1)$$

$$S_{\infty} = \frac{a}{1-r} \ (|r| < 1)$$

2. TRIGONOMETRY

Identities

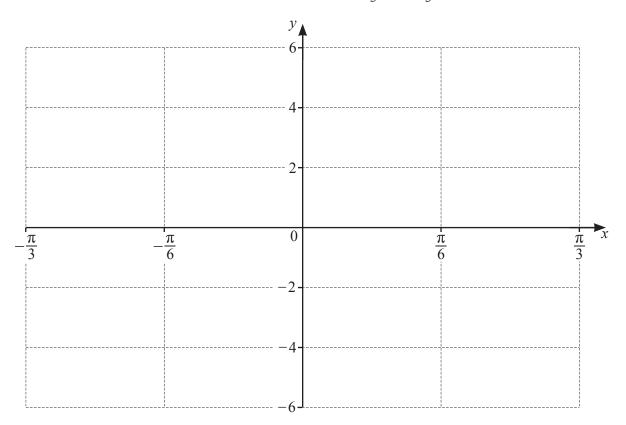
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$

1 On the axes, sketch the graph of $y = 4\sin 3x - 2$ for $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$.

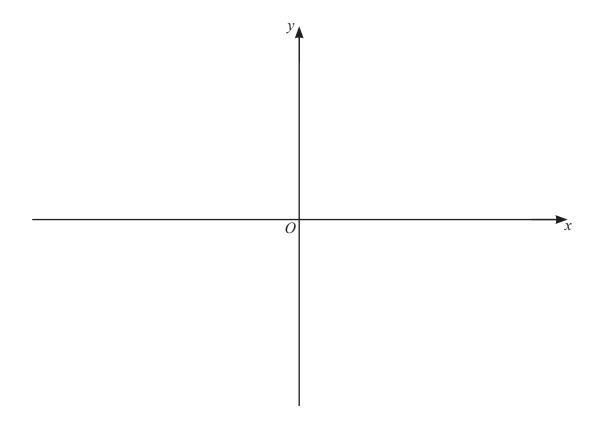




2 (a) Show that $2x^2 + x - 15$ can be written in the form $2(x+a)^2 + b$, where a and b are exact constants to be found. [2]

(b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + x - 15$. [2]

(c) On the axes, sketch the graph of $y = |2x^2 + x - 15|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]



(d) Write down the value of the constant k for which the equation $|2x^2 + x - 15| = k$ has 3 distinct solutions.

3 (a) Solve the following simultaneous equations.

$$3y - 2x + 2 = 0$$

$$xy = \frac{1}{2}$$
[3]

(b) Solve the equation $\log_3 x + 3 = 10 \log_x 3$, giving your answers as powers of 3. [4]

4	The that	polynomial $p(x)$ is such that $p(x) = ax^3 + 13x^2 + bx + c$, where a, b and c are integers. It is $p'(0) = -9$.	given
	(a)	Show that $b = -9$.	[1]
		It is also given that $3x+2$ is a factor of $p(x)$ and that when $p(x)$ is divided by $x+1$ remainder is 6.	the
	(b)	Find the values of a and c .	[4]
	(c)	Find the quadratic $q(x)$ such that $p(x) = (3x+2) \times q(x)$.	[1]
	(d)	Hence find $p(x)$ as a product of linear factors with integer coefficients.	[1]

5	A geometric principle infinity is 5.	progression	is such	that the	fifteenth	term	is equa	l to	$\frac{1}{8}$ of the	twelfth	term.	The sun	n to
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(a) Find the first term and the common ratio. [4]

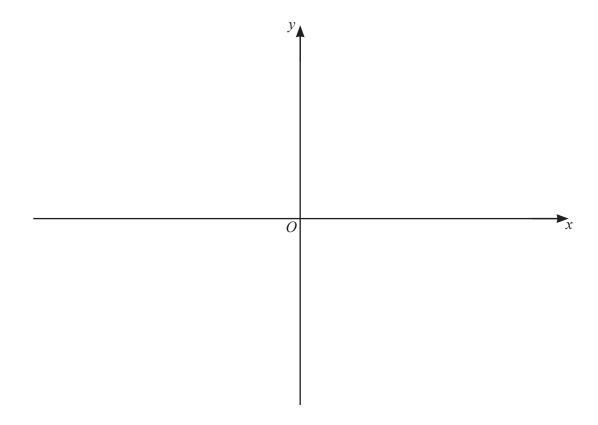
(b) Find the least number of terms needed for the sum of the geometric progression to be greater than 4.999.

6 A function f(x) is such that $f(x) = e^{3x} - 4$, for $x \in \mathbb{R}$.

(a) Find the range of f. [1]

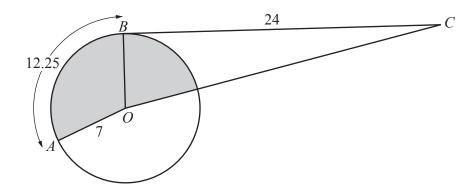
(b) Find an expression for $f^{-1}(x)$. [2]

(c) On the axes, sketch the graphs of y = f(x) and $y = f^{-1}(x)$ stating the exact values of the intercepts with the coordinate axes. [4]



7 Find the exact value of $\int_0^{\frac{\pi}{2}} (\cos 3x + 4\sin 2x + 1) dx.$ [5]

8 In this question all lengths are in metres.



The diagram shows a circle, centre O, radius 7. The points A and B lie on the circumference of the circle. The line BC is a tangent to the circle at the point B such that the length of BC is 24. The length of the minor arc AB is 12.25.

(a) Find the obtuse angle *AOB*, giving your answer in radians. [1]

(b) Find the perimeter of the shaded region. [4]

(c) Fi	ind the a	area of the shad	ded re	gion.						[2]
A 6-ch	aracter	password is to	be for	rmed	from	n the following	ng charact	ers.		
		Letters	A	В	C	D				
		Numbers	1	2	3	4				
		Symbols	*	#	\$	£				
No cha	aracter r	may be used m	ore th	an or	nce in	any passwo	ord.			
(a) (i) Find	the number of	f diffe	rent (6-cha	racter passw	ords that o	can be forme	d.	[1]
(ii) How	many of these	e 6-ch	aract	er pa	sswords end	with a syr	mbol?		[1]
			erent 6	5-cha	racte	r passwords	that includ	le all the syn	nbols, but do not sta	ert or [2]
	A 6-ch No cha (a) (ii (b) F:	A 6-character No character r (a) (i) Find (ii) How	A 6-character password is to Letters Numbers Symbols No character may be used m (a) (i) Find the number of (ii) How many of these	A 6-character password is to be for Letters A Numbers 1 Symbols * No character may be used more the (a) (i) Find the number of different 6 (b) Find the number of different 6	A 6-character password is to be formed Letters A B Numbers 1 2 Symbols * # No character may be used more than or (a) (i) Find the number of different 6 (ii) How many of these 6-character (b) Find the number of different 6-character	Letters A B C Numbers 1 2 3 Symbols * # \$ No character may be used more than once in (a) (i) Find the number of different 6-character pa (b) Find the number of different 6-character pa	A 6-character password is to be formed from the following Letters A B C D Numbers 1 2 3 4 Symbols * # \$ £ No character may be used more than once in any passwords (ii) Find the number of different 6-character passwords end (b) Find the number of different 6-character passwords	A 6-character password is to be formed from the following charact Letters A B C D Numbers 1 2 3 4 Symbols * # \$ £ No character may be used more than once in any password. (a) (i) Find the number of different 6-character passwords that of the first the first than once in the following character and the following character is a first than the fir	A 6-character password is to be formed from the following characters. Letters A B C D Numbers 1 2 3 4 Symbols * # \$ £ No character may be used more than once in any password. (a) (i) Find the number of different 6-character passwords that can be formed to the following characters. Letters A B C D Numbers 1 2 3 4 Symbols * # \$ £ No character may be used more than once in any password. (ii) How many of these 6-character passwords that can be formed to the following characters.	A 6-character password is to be formed from the following characters. Letters A B C D Numbers 1 2 3 4 Symbols * # \$ £ No character may be used more than once in any password. (a) (i) Find the number of different 6-character passwords that can be formed. (ii) How many of these 6-character passwords end with a symbol?

10 Solve the equation $\sqrt{2}\cos(3x+1.2) = 2\sin(3x+1.2)$, where x is in radians, for $-1.5 \le x \le 1.5$. [5]

11 It is given that $\int_{1}^{a} \left(\frac{3}{3x+2} - \frac{2}{2x+1} - \frac{1}{x} \right) dx = \ln \frac{1}{5}, \text{ where } a > 1. \text{ Find the exact value of } a.$ [6]

12 It is given that $y = \frac{(3x^2 - 2)^{\frac{2}{3}}}{x - 1}$, for x > 1.

(a) Write
$$\frac{dy}{dx}$$
 in the form $\frac{(3x^2-2)^{-\frac{1}{3}}}{(x-1)^2}(x^2+Ax+B)$, where A and B are integers. [5]

(b) Find the approximate increase in y as x increases from 2 to 2+p, where p is small. [2]

13	The points P and Q have coordinates $(5, -12)$ and $(15, -6)$ respectively. The point R lies on the line R the perpendicular bisector of the line R . The R -coordinate of R is R .										
	(a)	Find the <i>y</i> -coordinate of <i>R</i> .	[4]								
	(b)	The point S lies on I such that its distance from PQ is 3 times the distance of R from PQ . For coordinates of the two possible positions of S .	Find the [3]								

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