Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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3811535200

FURTHER MATHEMATICS **9231/43**

Paper 4 Further Probability & Statistics **May/June 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{6}\left(x^{-\frac{1}{3}} - x^{-\frac{2}{3}}\right) & 1 \leqslant x \leqslant 27, \\ 0 & \text{otherwise}. \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

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The random variable Y is defined by $Y = X^{\frac{1}{3}}$.

(b) Find the probability density function of Y. [3]

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(c) Find the exact value of the median of Y. [2]

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2 Shane is studying the lengths of the tails of male red kangaroos. He takes a random sample of 14 male red kangaroos and measures the length of the tail, x m, for each kangaroo. He then calculates a 90% confidence interval for the population mean tail length, μ m, of male red kangaroos. He assumes that the tail lengths are normally distributed and finds that $1.11 \leqslant \mu \leqslant 1.14$.

Find the values of $\sum x$ and $\sum x^2$ for this sample. [6]

3 A large number of students took two test papers in mathematics. The teacher believes that the marks obtained in Paper 1 will be higher than the marks obtained in Paper 2. She chooses a random sample of 9 students and compares their marks. The marks are shown in the table.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
Paper 1	46	73	55	64	86	42	66	68	60
Paper 2	41	66	61	63	90	40	58	42	70

(a) Carry out a Wilcoxon matched-pairs signed-rank test, at the 5% significance level, to test whether the data supports the teacher's belief. [7]

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(b) State an assumption that you have made in carrying out the test in part **(a)**. [1]

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4 An inspector is checking the lengths of metal rods produced by two machines, X and Y. These rods should be of the same length, but the inspector suspects that those made by machine X are shorter, on average, than those made by machine Y. The inspector chooses a random sample of 80 rods made by machine X and a random sample of 60 rods made by machine Y. The lengths of these rods are x cm and y cm respectively. Her results are summarised as follows.

$$\Sigma x = 164.0 \quad \Sigma x^2 = 338.1 \quad \Sigma y = 124.8 \quad \Sigma y^2 = 261.1$$

(a) Test at the 10% significance level whether the data supports the inspector's suspicion. [8]

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(b) Give a reason why it is not necessary to make any assumption about the distributions of the lengths of the rods. [1]

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5 The random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = k(1 + 3t + 4t^2),$$

where k is a constant.

(a) Show that $E(X) = \frac{11}{8}$. [3]

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The random variable Y has probability generating function $G_Y(t)$ given by

$$G_Y(t) = \tfrac{1}{3}t^2(1 + 2t).$$

The random variables X and Y are independent and $Z = X + Y$.

(b) Find the probability generating function of Z, expressing your answer as a polynomial in t. [2]

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(c) Use your answer to part **(b)** to find the value of Var(Z). [3]

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(d) Write down the most probable value of Z. [1]

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6 A scientist is investigating whether the ability to remember depends on age. A random sample of 150 students in different age groups is chosen. Each student is shown a set of 20 objects for thirty seconds and then asked to list as many as they can remember. The students are graded *A* or *B* according to how many objects they remembered correctly: grade *A* for 16 or more correct and grade *B* for fewer than 16 correct. The results are shown in the table.

	Age of students		
	11−12 years	13−14 years	15−16 years
Grade *A*	25	16	19
Grade *B*	28	45	17

(a) Carry out a χ^2-test at the 2.5% significance level to test whether grade is independent of age of student. [7]

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The scientist decides instead to use three grades: grade *A* for 16 or more correct, grade *B* for 10 to 15 correct and grade *C* for fewer than 10 correct. The results are shown in the following table.

	Age of students		
	11−12 years	13−14 years	15−16 years
Grade *A*	25	16	19
Grade *B*	12	27	11
Grade *C*	16	18	6

With this second set of data, the test statistic is calculated as 10.91.

(b) Complete the χ^2-test at the 2.5% significance level for this second set of data. [2]

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(c) State, with a reason, whether you would prefer to use the result from part **(a)** or part **(b)** to investigate whether the ability to remember depends on age. [1]

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Additional page

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