Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

THINKING SKILLS **9694/33**

Paper 3 Problem Analysis and Solution **May/June 2018**

2 hours

Additional Materials: Electronic Calculator

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Show your working. Marks may be awarded for correct steps towards a solution, even if the final answer is not correct. Marks may be lost if working needed to support an answer is not shown.
Calculators should be used where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

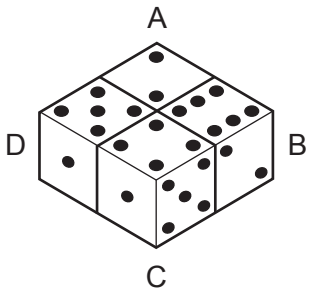
This document consists of **8** printed pages, **4** blank pages and **1** Insert.

DC (JM) 163242

CAMBRIDGE International Examinations

[Turn over

1 Archie found the drawing below on the wall of a temple ruin. He has labelled the four dice A, B, C and D.



Archie initially assumes that the four dice in the drawing are identical copies of a standard die, in which the numbers of spots on opposite faces always add up to 7.

(a) (i) How many spots are on the face of die D which is touching die C? [1]

(ii) Explain how you can deduce that there are three spots on the face of die B which is touching die C. [2]

Archie knows that the total number of spots on the four rectangular sides of an arrangement of four dice was thought to have a special significance.

(b) What are the two possible totals for the number of spots on the sides of the arrangement in the drawing that Archie has found? [3]

Archie showed the drawing to his friend George, who explained that these drawings always showed two identical standard dice and a pair of other dice, known today as Sicherman dice. In a pair of Sicherman dice, one has faces with 1, 2, 2, 3, 3 and 4 spots, while the other has faces with 1, 3, 4, 5, 6 and 8 spots. On each die the numbers of spots on opposite faces always add up to a fixed number for that die.

(c) What are the two fixed numbers for Sicherman dice? [1]

(d) Assuming that George is correct,

(i) Which two of the dice in the drawing must be the Sicherman dice? Explain your answer. [2]

(ii) What is the smallest possible total number of spots that there could be on the sides of the arrangement in the drawing? [1]

2 Trains run from Athos to Ethos, stopping at Banta, Chanta and Danter on the way. A researcher has noticed that there is a pattern in the number of people who get on and off at the different stations.

- At Banta, twice as many people get on as get off.
- At Chanta, one third of the people on the train get off, and then 30 people get on.
- At Danter, 10 more people get off than get on.

(Where the number of people getting on or off is variable, that number can be 0 and the pattern still holds.)

The researcher assumes that this pattern holds for every train that leaves Athos bound for Ethos.

At each stop, those people who are getting off the train do so before anyone gets on. All trains are empty before people get on at Athos.

On one occasion, 120 people got on the train at Athos and 24 people got on at Banta.

(a) How many people were on the train when it arrived at Ethos? [2]

(b) **(i)** What is the greatest number of people who could have made the whole journey from Athos to Ethos? [2]

(ii) What is the least number of people who could have made the whole journey from Athos to Ethos? [1]

On another occasion, 150 people got on the train at Athos and 150 people arrived at Ethos.

(c) How many people got on the train at Banta? [3]

On a third occasion, the number of people on the train between Banta and Chanta was equal to the number of people on the train between Chanta and Danter.

(d) How many people were on this train when it arrived at Ethos? [2]

3 A local store has started to make and sell cakes alongside its other products. The cakes each require 150 g flour, 200 g sugar and 2 eggs. Flour and sugar each cost 60¢ per kg and eggs cost 20¢ each. The total cost of making an individual cake is the cost of the ingredients plus $3, which covers all the other standing costs.

The store owner expected that he would be able to sell 40 cakes each week. At the end of each week, any unsold cakes are thrown away.

(a) Show that the total cost to make 40 cakes is $144.40. [2]

In the first week he made 40 cakes, which were priced at $5 each. Only 10 of the cakes were sold. When asked, the customers said that this was because the prices were too high.

In the second week 40 cakes were made, but the price was reduced to $4.50.

(b) What is the smallest number of cakes that would need to be sold so that the total money received for the cakes was higher than in the first week? [2]

In fact 20 cakes were sold in the second week. As a result, the store owner assumes that for every 5¢ by which he reduces the price of a cake, an extra customer will buy a cake. The price must always be a multiple of 5¢.

(c) If the store owner's assumption is correct, what is the maximum profit possible if he makes 30 cakes? [2]

(d) What number of cakes should the store owner make if he wants to achieve the maximum profit? [4]

In the second week (when 20 cakes were sold at $4.50 each) no customer bought more than one cake, but a number of customers commented that they would have bought a second cake if there were a discount available. Based on these comments, the store owner is considering offering a 10% discount to customers who buy two cakes. This discount would apply to **both** of the cakes. However, no customer would be allowed to buy more than 2 cakes.

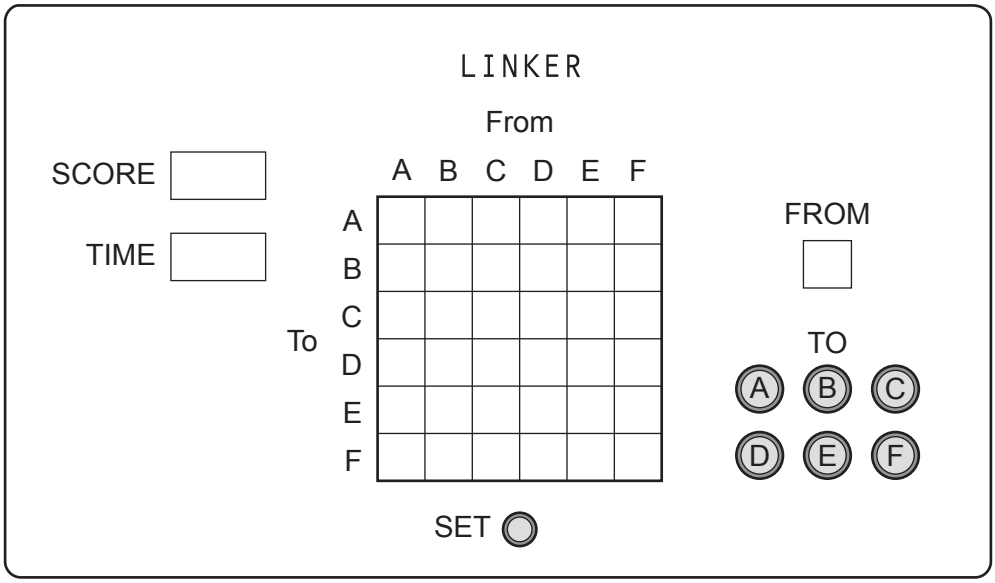
(e) (i) If all 20 customers in the second week had bought 2 cakes, what profit would the store owner have made? [2]

(ii) If this offer had been in place, how many of the customers would have had to have bought a second cake in order for the shopkeeper to make a profit? [3]

[Question 4 begins on the next page]

4 *Linker* is an electronic game in which the player tries to score as many points as possible in three minutes by linking from one letter to another successfully.

This is the device used to play *Linker*.



When the SET button is pressed, digits from 0 to 5 appear in the grid, 0 appears in the SCORE display, 3:00 appears in the TIME display and a letter (A, B, C, D, E or F) appears to the right of the grid in the FROM display.

As soon as the player presses one of the TO buttons the game begins and a link is made. The time starts to count down towards 0:00 and the relevant number of points are added to the score. The letter on the button that was pressed now appears in the FROM display ready for the next link to be made. This continues until the TIME display shows 0:00. However, if the sequence of letters entered by the player repeats the same three letters in a row, the game is over. Because of the limited time available to make as many links as possible and the threat of causing the game to end prematurely, players often do not try to score the maximum amount available from every link.

For example, suppose a player were to be given the following grid and the letter D in the FROM display:

		From					
		A	B	C	D	E	F
To	A	1	2	0	4	5	3
	B	0	5	1	3	2	4
	C	2	3	4	5	0	1
	D	4	0	2	1	3	5
	E	3	1	5	0	4	2
	F	5	4	3	2	1	0

If the player were to enter, in order, F D B B F C B B B, the links made and points scored would be:

From	*To*		
(D)	F	=	2 points
F	D	=	5 points
D	B	=	3 points
B	B	=	5 points
B	F	=	4 points
F	C	=	1 point
C	B	=	1 point
B	B	=	5 points
B	B	=	5 points

If the player's next entry were F, they would score another 4 points, but the game would be over because B B F would have occurred twice in the sequence of the player's entries. Their final score would be 35 points.

If the player's next entry were B, they would score another 5 points, but the game would be over because B B B would have occurred twice in the sequence of the player's entries. Their final score would be 36 points.

(a) Liam is playing a game of *Linker*. This is his grid.

To \ From	A	B	C	D	E	F
A	5	3	2	0	4	1
B	0	4	1	5	3	2
C	3	5	0	2	1	4
D	1	2	3	4	0	5
E	4	0	5	1	2	3
F	2	1	4	3	5	0

His starting letter in the FROM display was B.
His third entry was D, which took his score to 14.
Since then he has entered, in order, F E F C.

(i) What, in order, were his first two entries? [2]

(ii) What is his total score at present? [2]

[Question 4 continues on the next page]

(b) Inga is about to start a game of *Linker*. This is her grid.

To \ From	A	B	C	D	E	F
A	2	4	3	1	0	5
B	3	5	0	4	2	1
C	4	2	1	3	5	0
D	1	0	5	2	4	3
E	5	3	4	0	1	2
F	0	1	2	5	3	4

Her starting letter in the FROM display is A.
Her intended strategy is to score 5 points as often as possible, only scoring less when a 5-point link would end the game.

Show how Inga can score a total of 47 points for her first ten links, without causing the game to be over. [2]

For each game, the grid that appears on the screen is the following grid, or a rotation of it, with each letter from P to U replaced by a different digit from 0 to 5:

To \ From	A	B	C	D	E	F
A	P	T	R	U	S	Q
B	R	S	P	Q	T	U
C	T	Q	U	S	R	P
D	U	R	T	P	Q	S
E	Q	P	S	R	U	T
F	S	U	Q	T	P	R

Each version of the grid that appears on the screen has its own grid code, which is shown in a small display on the side of the device. This code consists of the letter P, Q, R or S, indicating the orientation of the grid, followed by the numerical values of P, Q, R, S, T and U, in order.

For example, the grid that Liam is playing with has the grid code Q251043. This means that Q is in the top-left corner and P = 2, Q = 5, R = 1, S = 0, T = 4, U = 3.

(c) (i) All the possible grid codes are programmed into the device. How many are there? [1]

(ii) Write, in order from left to right, the digits of the top row of the grid with the grid code R403215. [2]

(iii) What is the grid code for the game that Inga is about to start? [2]

(d) When Kerry plays *Linker*, he always starts by entering A A A D D D very quickly. This often gives him a reasonable score in a very short time before he settles down to give more thought to the rest of the game. On one occasion, however, he did this and scored a total of 1 point only for his first six links.

Draw a grid for which entering A A A D D D will result in a total score of 1 point only. State the code for your grid. [4]

BLANK PAGE

BLANK PAGE

BLANK PAGE

BLANK PAGE