Cambridge Assessment International Education

Cambridge IGCSE™

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **October/November 2023**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	3	**B2**	**B1** for g(0) = 0 or [fg(x) =] $2\sin(e^{3x}-1)+3\cos(e^{3x}-1)$ soi
1(b)	$gg(x)=e^{3(e^{3x}-1)}-1$ oe, isw	**B1**	
1(c)	$3y=\ln(x+1)$ or $3x=\ln(y+1)$ **and** swops the variables at some point	**M1**	condone one error
	$g^{-1}(x)=\frac{1}{3}\ln(x+1)$ soi	**A1**	
	[x =] 4	**A1**	
	Alternative method		
	$x=g(\frac{1}{3}\ln 5)$ soi	**(B1)**	
	$e^{3\left(\frac{1}{3}\ln 5\right)}-1$ oe	**(M1)**	
	[x =] 4	**(A1)**	
2	Uses b^2-4ac [*0] $k^2-4(4k-15)$ [*0]	**M1**	where * is any inequality sign or =;
	$k^2-16k+60$ [*0]	**A1**	
	$(k-6)(k-10)[*0]$	**DM1**	**FT** *their* 3-term quadratic; **dep** on previous **M1**
	$6<k<10$	**A1**	Mark final answer
	Alternative method		
	$2x+k$ [= 0]	**(M1)**	
	$-\frac{k^2}{4}+4k-15[*0]$ oe or $-x^2-8x-15[*0]$ oe	**(A1)**	
	Solves or factorises $(k-6)(k-10)[*0]$ or $(x+5)(x+3)$ [*0] **and** $x=-5$, $x=-3$	**(DM1)**	**FT** *their* 3-term quadratic; **dep** on previous **M1**
	$6<k<10$	**(A1)**	Mark final answer

Question	Answer	Marks	Guidance
3(a)	Correctly eliminates $\log_2 x$ or $\log_2 y$	M1	A correct equation in $\log_2 x$ only or $\log_2 y$ only
	$x = 16$ or $y = 64$	A2	**A1** for $\log_2 x = 4$ or $\log_2 y = 6$
	$y = 64$ or $x = 16$	A2	**A1** for $\log_2 y = 6$ or $\log_2 x = 4$
	Alternative method		
	$x^3 y^2 = 2^{24}$ **and** $\frac{x^5}{y^3} = 2^2$ oe	**(M1)**	
	$y = 64$ or $x = 16$	**(A2)**	**A1** for $y^{19} = 2^{114}$ oe or $x^{19} = 2^{76}$ oe
	$x = 16$ or $y = 64$	**(A2)**	**A1** for $x^3 \times 64^2 = 2^{24}$ oe or $\frac{x^5}{64^3} = 2^2$ oe OR $16^3 \times y^2 = 2^{24}$ oe or $\frac{16^5}{y^3} = 2^2$ oe
3(b)	$2^{t+4-(1-2t)} = 2^9$ or $2^{t+4} = 2^{9+1-2t}$ oe, soi OR $t+4-(1-2t) = \log_2 512$ oe, soi or $t+4-(1-2t) = \frac{\log_a 512}{\log_a 2}$ oe	B2	**B1** for $2^{t+4-(1-2t)} = 512$ or $\frac{2^{t+4}}{2^{1-2t}} = 2^9$ soi OR $(t+4)\log_a 2 - (1-2t)\log_a 2 = \log_a 512$
	$3t + 3 = 9$ or better	M1	
	$t = 2$	A1	

Question	Answer	Marks	Guidance
4	$\frac{(x-1)^2}{x^3}=\frac{1}{x}-2x^{-2}+x^{-3}$ soi	**B2**	**B1** for $\frac{x^2-2x+1}{x^3}$ or $\frac{1}{x}-\frac{2}{x^2}+\frac{1}{x^3}$ or for any two terms correct in $\frac{1}{x}-2x^{-2}+x^{-3}$
	$\left[\ln x+\frac{2}{x}-\frac{1}{2x^2}\right]_3^5$	**B2**	**B1** for $\left[\ln x+...-\frac{1}{2x^2}\right]_3^5$ or $\left[\ln x+\frac{2}{x}+...\right]_3^5$ or $\left[k\ln x+\frac{2}{x}-\frac{1}{2x^2}\right]_3^5$ with $k \neq 0$
	$\left[\ln 5+\frac{2}{5}-\frac{1}{50}\right]-\left[\ln 3+\frac{2}{3}-\frac{1}{18}\right]$	**M1**	**dep** on at least previous **B1** for integration
	$\ln\left(\frac{5}{3}\right)-\frac{52}{225}$	**A1**	
5(a)	Correct use of $\pi r^2 h=1000$ to find an expression that can be used to eliminate h e.g. $h=\frac{1000}{\pi r^2}$ or $\pi rh=\frac{1000}{r}$	**M2**	**M1** for $\pi r^2 h=1000$ soi
	Correct substitution and completion to given answer e.g. $2\pi r^2+2\pi r\times\frac{1000}{\pi r^2}=2\pi r^2+\frac{2000}{r}$	**A1**	

Question	Answer	Marks	Guidance
5(b)	Correct derivative: $4\pi r - \dfrac{2000}{r^2}$ oe, isw	**B2**	**B1** for one correct term
	$4\pi r - \dfrac{2000}{r^2} = 0$ and solves for r	**M1**	**FT** *their* derivative providing that at least one term is correct
	$r = \sqrt[3]{\dfrac{2000}{4\pi}}$ oe, isw or 5.42 or 5.419[26…] rot to 3 or more dp	**A1**	
	Second derivative $\dfrac{d^2S}{dr^2} = 4\pi + 4000r^{-3}$ **and** When $r = 5.42$, $\dfrac{d^2S}{dr^2} > 0$ oe [hence minimum] or $4\pi + 4000(5.42)^{-3} > 0$ oe [hence minimum] or $\dfrac{d^2S}{dr^2} = 12\pi$ or 37 to 38 [hence minimum] or as $r > 0$, $\dfrac{d^2S}{dr^2} > 0$ [hence minimum] OR correctly finds the values of the first derivative at 5.42 oe $\pm h$, where h is small [hence minimum]	**A1**	**Dep** on previous mark
6(a)	Velocity: $\dfrac{8t}{4t^2-5} - 1$	**B2**	**B1** for $\dfrac{f(t)}{4t^2-5} - 1$ or for $\dfrac{8t}{4t^2-5} + g(t)$
	Correct structure of quotient rule or equivalent product rule	**M1**	**FT** *their* v if possible; must be of equivalent difficulty
	Acceleration: $\dfrac{(4t^2-5)(8)-(8t)(8t)}{(4t^2-5)^2}$ oe, isw	**A1**	**FT** $\dfrac{8t}{4t^2-5} + k$ where k is a constant
6(b)	$4t^2 - 8t - 5 = 0$ oe	**B1**	
	$(2t+1)(2t-5) = 0$	**M1**	**FT** *their* 3-term quadratic in t
	$t = 2.5$ and no other values	**A1**	**dep** on correct quadratic seen
6(c)	$\dfrac{(4\times2.5^2-5)(8)-(8\times2.5)(8\times2.5)}{(4\times2.5^2-5)^2}$ oe, soi	**M1**	Substitutes a value of $t > 2$ in an expression for a which has at least one term with a factor of $\dfrac{1}{(4t^2-5)^2}$ or $\dfrac{1}{16t^4-40t^2+25}$
	$a = -\dfrac{3}{5}$ oe only	**A1**	

Question	Answer	Marks	Guidance
7(a)	$\dfrac{BC}{\sin 60} = \dfrac{\sqrt{2}}{\sin 45}$ oe	M1	
	$BC = \sqrt{3}$ oe	A1	
	$\dfrac{1}{2} \times \sqrt{2} \times \sqrt{3} \times \sin 75 = \dfrac{3+\sqrt{3}}{4}$ OR $\text{height} = \left(\dfrac{3+\sqrt{3}}{4}\right) \times \dfrac{2}{\sqrt{3}}$ oe	M1	**FT** *their BC* providing it has not been found using the given result for sin75
	$\sin 75 = \dfrac{2(3+\sqrt{3})}{4\sqrt{6}}$ oe OR $\sin 75 = \dfrac{\left(\dfrac{3+\sqrt{3}}{4}\right) \times \dfrac{2}{\sqrt{3}}}{\sqrt{2}}$ oe $\left[= \dfrac{\dfrac{1+\sqrt{3}}{2}}{\sqrt{2}} = \dfrac{1+\sqrt{3}}{2\sqrt{2}} \right]$	A1	**dep** on all previous marks being awarded Isolates sin 75 correctly or deals with surds on LHS of correct equation e.g. $\dfrac{1}{2} \times 6 \times \sin 75 = \left(\dfrac{3+\sqrt{3}}{4}\right) \times \sqrt{6}$
	correct completion to given answer $\dfrac{\sqrt{6}+\sqrt{2}}{4}$	A1	must be convincing with an intermediate step if needed
	Alternative methods (finding *AC* first)		
	$\dfrac{1}{2} \times \sqrt{2} \times AC \times \sin 60 = \dfrac{3+\sqrt{3}}{4}$ oe	(M1)	
	$AC = \dfrac{2}{\sqrt{2}} \times \dfrac{2}{\sqrt{3}} \times \dfrac{3+\sqrt{3}}{4}$ oe	(A1)	Isolates *AC* correctly
	$AC = \dfrac{\sqrt{2}+\sqrt{6}}{2}$	(A1)	Must be convinced no calculator is being used
	$\dfrac{\sin 75}{\dfrac{\sqrt{2}+\sqrt{6}}{2}} = \dfrac{\sin 45}{\sqrt{2}}$ oe	(M1)	**FT** *their AC* May simplify to $\sin 75 = \dfrac{AC}{2}$ before inserting *their AC*
	correct completion to given answer $\dfrac{\sqrt{6}+\sqrt{2}}{4}$	(A1)	**dep** on all previous marks being awarded must be convincing with an intermediate step if needed

Question	Answer	Marks	Guidance
7(b)	$\dfrac{AC}{\frac{\sqrt{6}+\sqrt{2}}{4}}=\dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or $\dfrac{AC}{\frac{\sqrt{6}+\sqrt{2}}{4}}=\dfrac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}}$ or better	**M1**	
	$\dfrac{\sqrt{6}+\sqrt{2}}{2}$ nfww	**A1**	
8(a)	$\dfrac{\sin x}{\frac{\sin x}{\cos x}-1}-\dfrac{\cos x}{\frac{\sin x}{\cos x}+1}$	**M1**	OR $\dfrac{\sin x(\tan x+1)-\cos x(\tan x-1)}{(\tan x-1)(\tan x+1)}$
	$\dfrac{\sin x}{\frac{\sin x-\cos x}{\cos x}}-\dfrac{\cos x}{\frac{\sin x+\cos x}{\cos x}}$ OR $\dfrac{\sin x\left(\frac{\sin x}{\cos x}+1\right)-\cos x\left(\frac{\sin x}{\cos x}-1\right)}{\frac{\sin^2 x}{\cos^2 x}-1}$	**A1**	OR $\dfrac{\sin x\tan x+\sin x-\cos x\tan x+\cos x}{(\tan x-1)(\tan x+1)}$ OR $\dfrac{\sin x\left(\frac{\sin x}{\cos x}+1\right)-\cos x\left(\frac{\sin x}{\cos x}-1\right)}{\left(\frac{\sin x}{\cos x}-1\right)\left(\frac{\sin x}{\cos x}+1\right)}$
	$\dfrac{\sin x\cos x}{\sin x-\cos x}-\dfrac{\cos^2 x}{\sin x+\cos x}$ OR $\dfrac{\sin x\left(\frac{\sin x+\cos x}{\cos x}\right)-\cos x\left(\frac{\sin x-\cos x}{\cos x}\right)}{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}$	**A1**	OR $\dfrac{\frac{\sin^2 x}{\cos x}+\sin x-\sin x+\cos x}{\frac{\sin^2 x}{\cos^2 x}-1}$
	$\dfrac{\sin^2 x\cos x+\sin x\cos^2 x-\cos^2 x\sin x+\cos^3 x}{\sin^2 x+\cos x\sin x-\cos x\sin x-\cos^2 x}$ OR $\dfrac{\sin^2 x+\sin x\cos x-\cos x\sin x+\cos^2 x}{\cos x}\times\dfrac{\cos^2 x}{\sin^2 x-\cos^2 x}$	**A1**	OR $\dfrac{\frac{\sin^2 x+\cos^2 x}{\cos x}}{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}$ or $\dfrac{\frac{\sin^2 x+\cos^2 x}{\cos x}}{\frac{1-2\cos^2 x}{\cos^2 x}}$
	Fully correct justification of given answer e.g. $\dfrac{\cos x\left(\sin^2 x+\cos^2 x\right)}{\sin^2 x-\cos^2 x}=\dfrac{\cos x}{\sin^2 x-\cos^2 x}$ oe	**A1**	All steps correct and final step justified $\dfrac{1}{\cos x}\times\dfrac{\cos^2 x}{\sin^2 x-\cos^2 x}=\dfrac{\cos x}{\sin^2 x-\cos^2 x}$ oe
8(b)	$2\cos^2 x+\cos x-1[=0]$	**B2**	**B1** for $\cos x=1-\cos^2 x-\cos^2 x$ or better
	$(2\cos x-1)(\cos x+1)[=0]$	**M1**	**FT** *their* 3-term quadratic in cos$x$
	$[x=]\ 60°, 300°, 180°$ and no extras in range	**A2**	**A1** for any two correct , ignoring extras

Question	Answer	Marks	Guidance
9(a)	Derivative of e^{2x}: $2e^{2x}$ soi	**B1**	
	$x \times 2e^{2x} + e^{2x}$ isw	**B1**	**FT** *their* $2e^{2x}$
9(b)	[When $x = 1$] $y = e^2$	**B1**	
	[gradient tangent =] *their* $\left.\frac{dy}{dx}\right\vert_{x=1}$	**B1**	**FT** *their* derivative which must include at least one term in e^{2x}
	Gradient of normal = $\frac{-1}{their(3e^2)}$	**B1**	**FT** $\frac{-1}{their \left.\frac{dy}{dx}\right\vert_{x=1}}$
	$y - e^2 = \frac{-1}{3e^2}(x-1)$ oe, isw	**B1**	**dep** on 2 marks awarded in part (a) and all previous marks awarded in this part
9(c)	$\left[xe^{2x} - \frac{1}{2}e^{2x}\right]_0^2$	**M3**	**M2** for $xe^{2x} + ke^{2x}$ where $k < 0$ or $k = \frac{1}{2}$ or **M1** for $\int 2xe^{2x}dx = xe^{2x} - \int e^{2x}dx$
	$\left(2e^4 - \frac{1}{2}e^4\right) - \left(-\frac{1}{2}\right)$	**A1**	
	$1.5e^4 + 0.5$ or exact equivalent	**A1**	
10(a)	$a + 4d = 11$ oe	**B1**	
	$a + 6d = 3(a + d)$ oe	**B1**	
	Correctly eliminates one unknown and solves for a or d	**M1**	**FT** *their* linear equations in a and d providing **B1** earned.
	$d = 2,\ a = 3$	**A1**	

Question	Answer	Marks	Guidance
10(b)	$3+d=3r^2$	**B1**	
	$3+5d=3r^4$	**B1**	
	$3+5d=3\left(\frac{3+d}{3}\right)^2$ oe or $3+5\left(3r^2-3\right)=3r^4$ oe	**M1**	
	$d^2-9d[=0]$ or $3r^4-15r^2+12[=0]$	**A1**	
	d = 9 and r = 2 and no other values	**A2**	**A1** for d = 9 and no other value of d or for r = 2 and no other value of r